

## Chapter 12

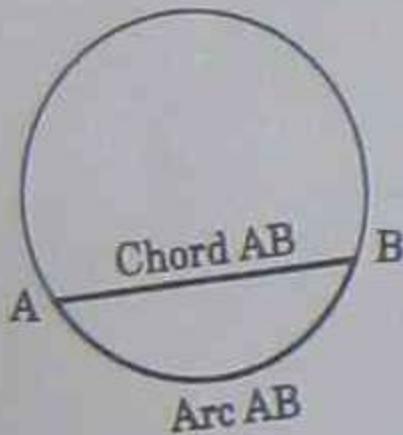
# FURTHER GEOMETRY — THE CIRCLE

### Some important points

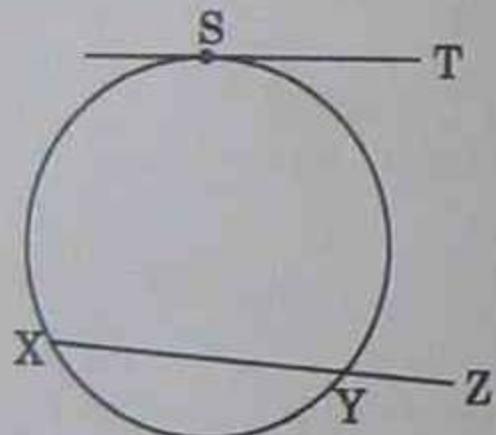
- Always draw a clear diagram and mark on the diagram all given information.
- Work from the known towards the desired result.
- If an angle is required, mark sizes of angles on the diagram as you find them. If it is a deductive question it is generally useful to label one of the angles with a pronumeral (one of the angles in the result).
- If sides are involved, consider isosceles triangles or congruent triangles.
- If you become lost, check that all the given information has been used.
- Use any *hints* given in the question.



## 12.1 Some parts of the circle



Chord AB cuts off arc AB.

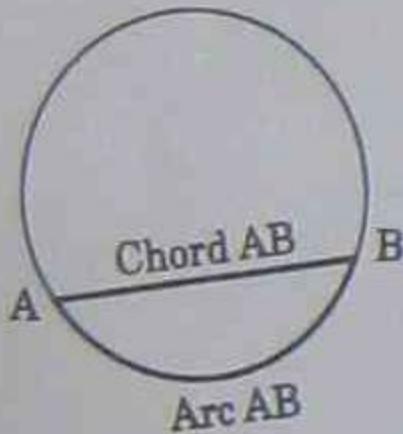


Tangent TS touches the circle at S.

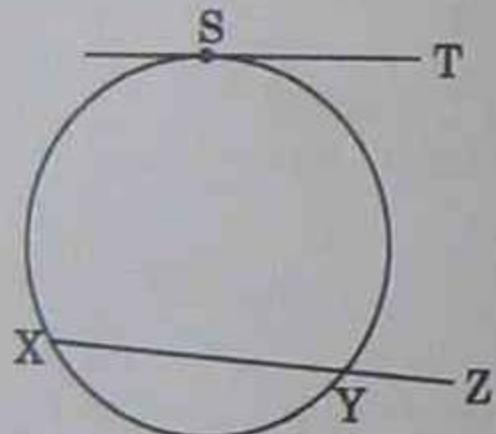
Secant XZ cuts the circle at X and Y.



## 12.1 Some parts of the circle



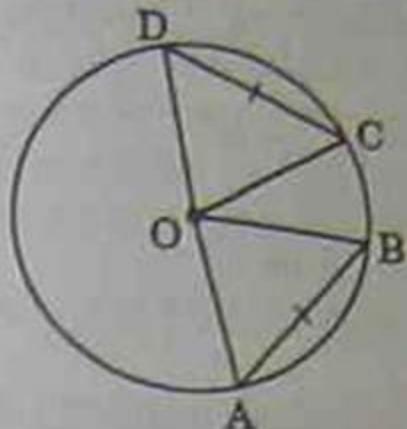
Chord AB cuts off arc AB.



Tangent TS touches the circle at S.

Secant XZ cuts the circle at X and Y.

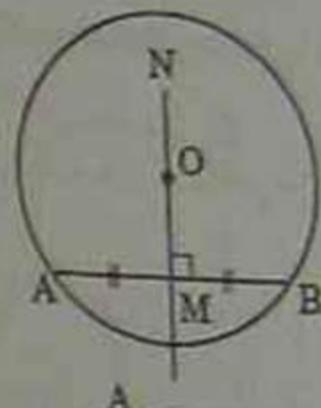




3. Equal chords of a circle subtend equal angles at the centre.  
Given  $AB = DC$ , then  $\angle AOB = \angle DOC$ .

Other interesting properties arising from these first three properties are:

- (a) The perpendicular bisector of a chord of a circle must pass through the centre of the circle. If  $AM = MB$  and  $NM \perp AB$ , then  $NB$  passes through  $O$ .

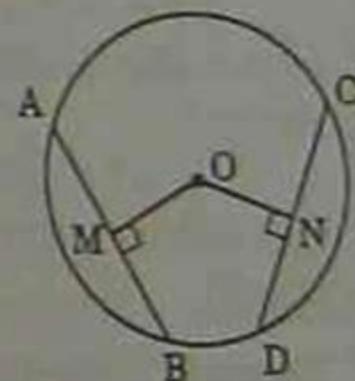


**Note:** This property is used to locate the centre of a circle passing through any three non-collinear points.

The centre is the point of intersection of the perpendicular bisectors of lines joining AB, BC and AC.



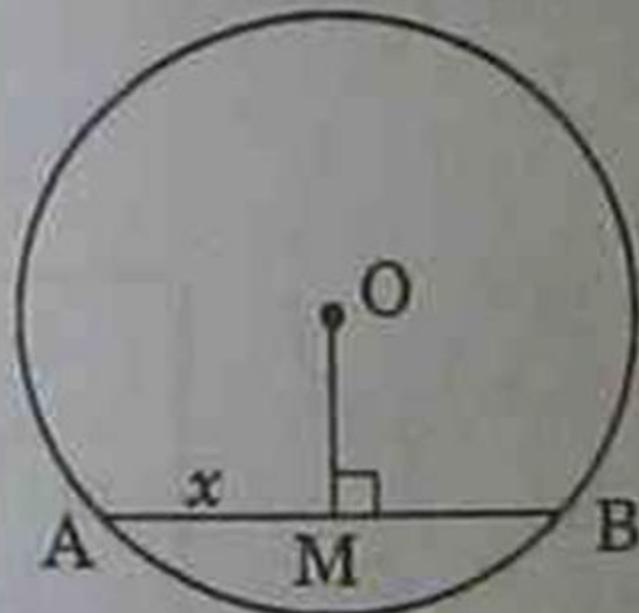
- (b) Equal chords of a circle are equidistant from the centre of the circle. Given  $AB = CD$ , then  $OM = ON$ .  
(See Exercises, Question 1)



## Simple examples

Find the value of the pronumeral in the following questions:

(a)



Given  $OM \perp AB$ ,  
O is the centre,  
 $AB = 12 \text{ cm}$ ;  
find  $x$ .

SOLUTION

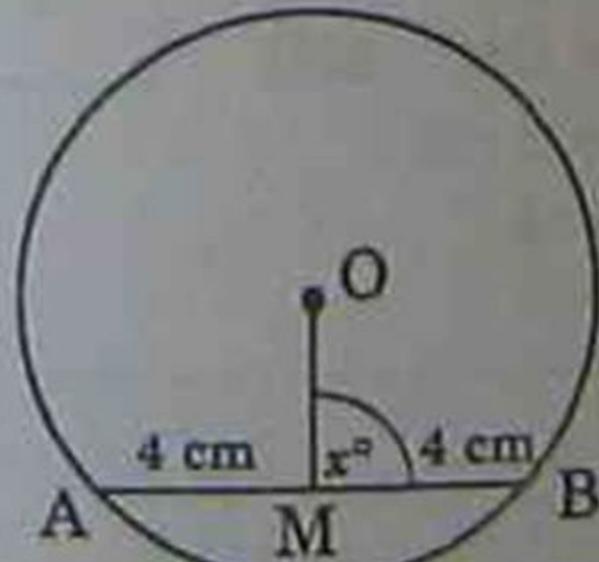
$AM = MB$  (Line from centre  $\perp$  chord)

$$\therefore AM = 6 \text{ cm}$$

$$\therefore x = 6$$



(b)

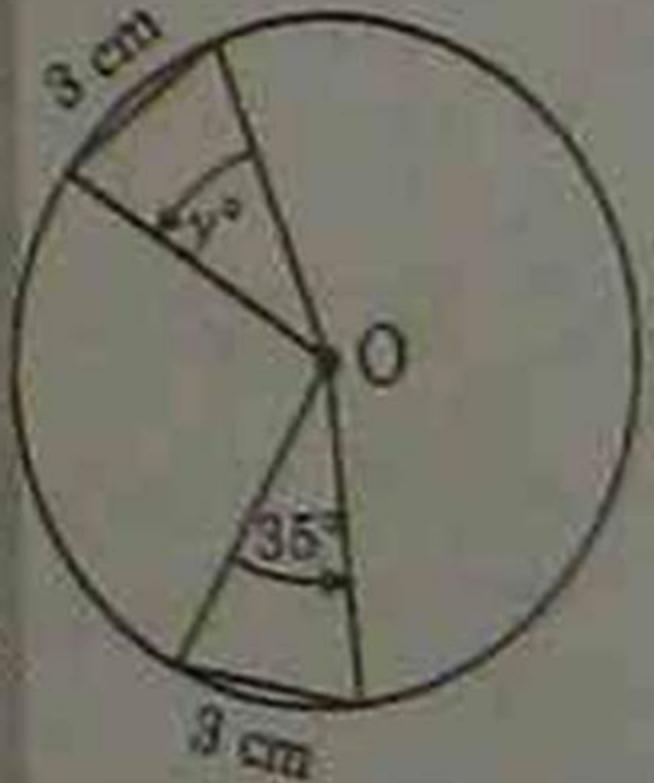


Given O is the  
centre,  
 $AM = MB = 4 \text{ cm}$ ;  
find  $x$ .

SOLUTION  
 $OM \perp AB$   
 $\therefore x = 90$



(c)

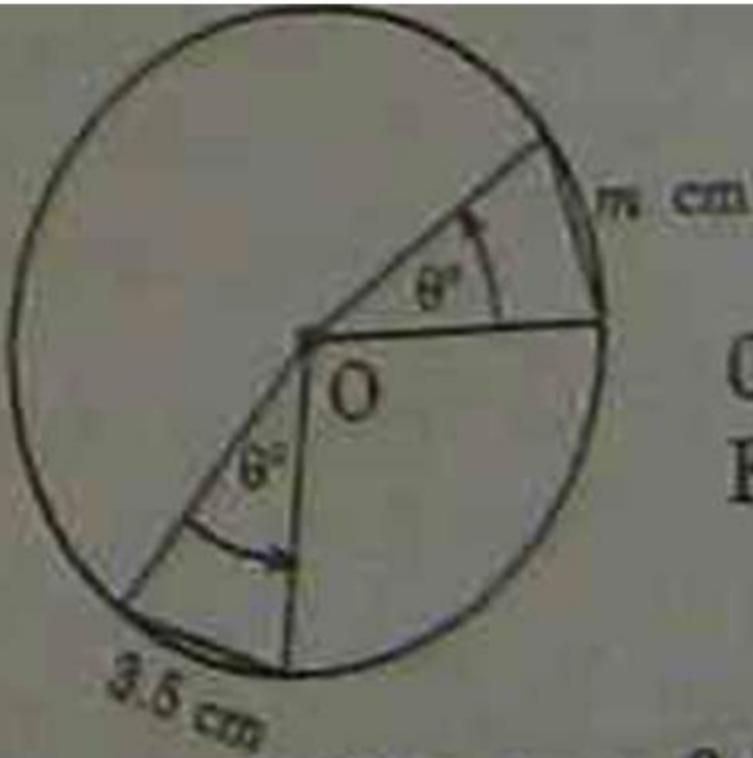


O is the centre.  
Find y.



**SOLUTION:**  $y = 35$  (Equal chords  
subtend equal angles at the centre)

(d)

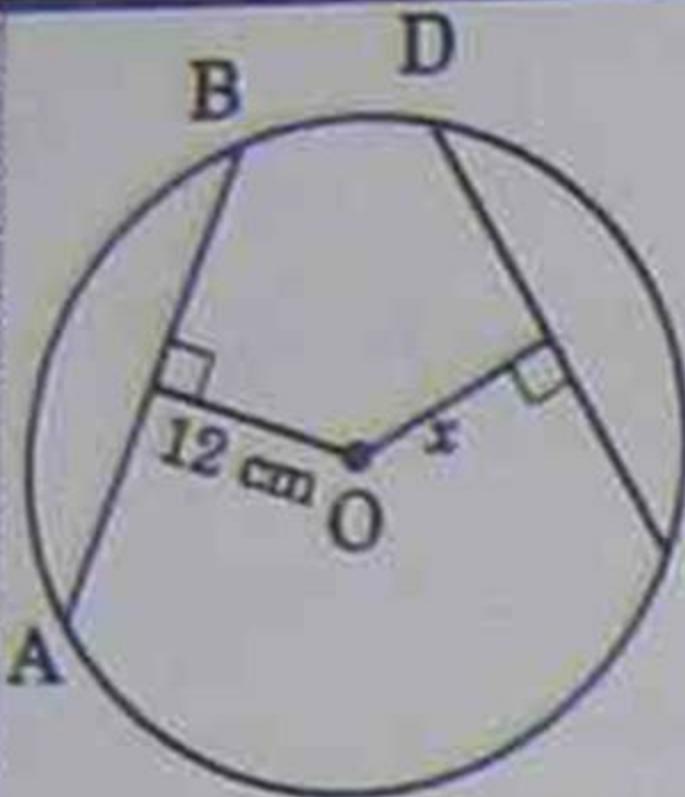


O is the centre.  
Find  $m$ .

**SOLUTION:**  $m = 3.5$  (Equal chords  
subtend equal angles at the centre)



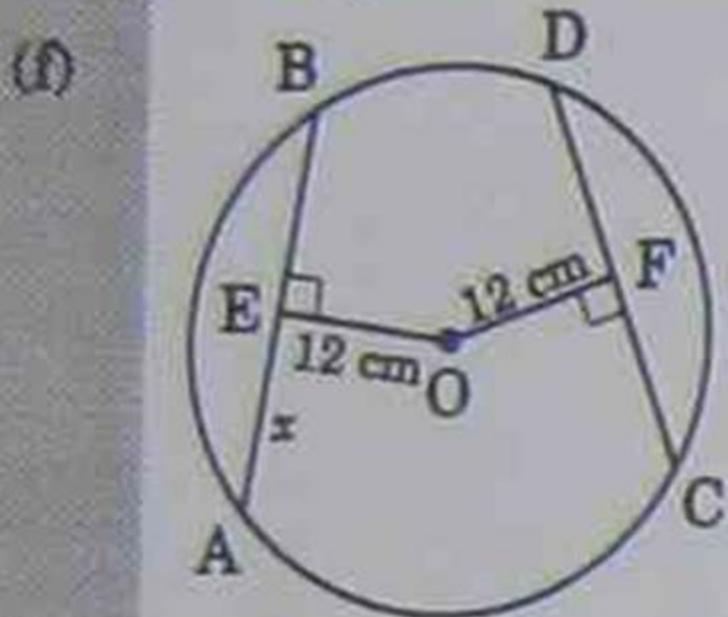
(e)



O is the centre,  
 $AB = 14 \text{ cm}$ ,  
 $CD = 14 \text{ cm}$ .  
Find  $x$ .

**SOLUTION:**  $x = 12$  (equal chords  
are equidistant from the centre)





O is the centre,  
 $EO = 12 \text{ cm}$ ,  
 $OF = 12 \text{ cm}$ ,  
 $FC = 5 \text{ cm}$ ,  
 $AE = x \text{ cm}$ .  
 Find  $x$ .

### SOLUTION

$$DC = 10 \text{ cm}$$

(perpendicular from centre to a chord  
 bisects the chord)

$$\therefore AB = 10 \text{ cm}$$

(equal chords are equidistant from the  
 centre)

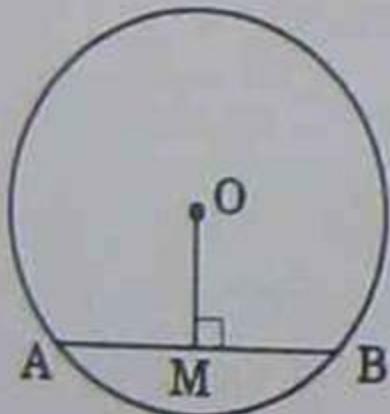
$$\therefore x = 5$$

(perp. from centre to chord bisects the  
 chord)



## Examples involving Pythagoras' Theorem

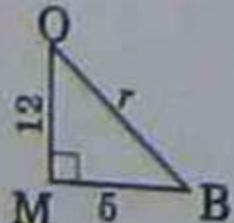
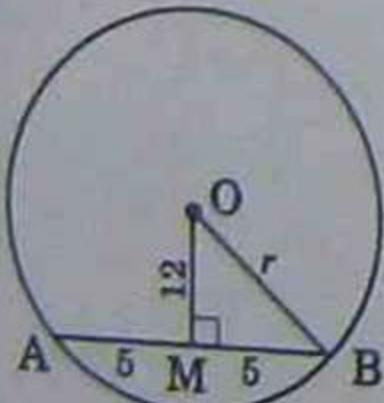
(g)



Given that a chord AB, 10 cm long, is 12 cm from the centre O, find the radius of the circle.

**SOLUTION**

Join OB and mark the information on the diagram.



Using Pythagoras' Theorem:

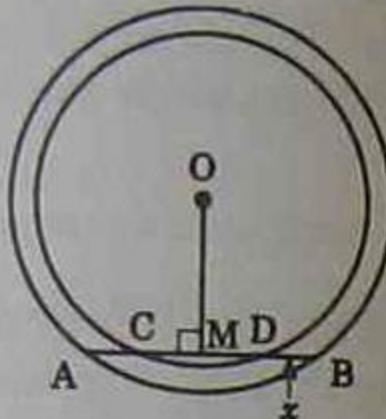
$$\begin{aligned}r^2 &= 12^2 + 5^2 \\&= 169\end{aligned}$$

$$\therefore r = \sqrt{169} \\= 13$$



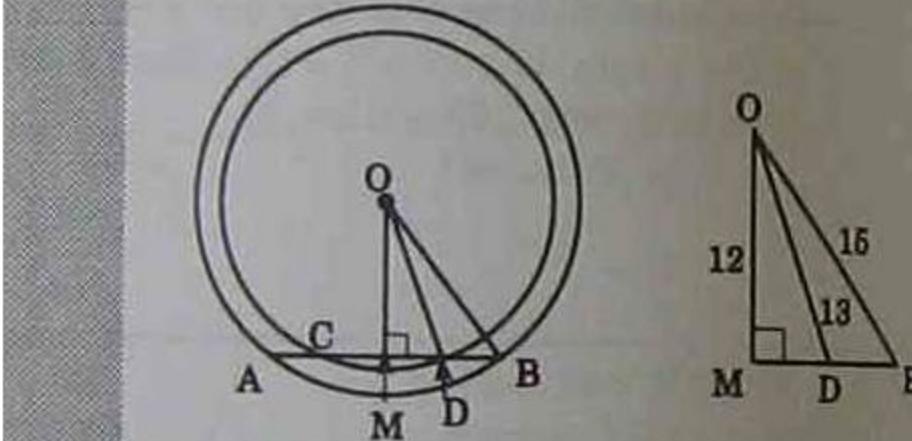
The radius of the circle is 13 cm.

- (b) A chord is drawn 12 cm from the centre of concentric circles with radii 13 cm and 15 cm. Find the length of DB in the diagram.



### SOLUTION

Join OD, OB and mark the information on the diagram.



## Using Pythagoras' Theorem:

From  $\triangle OMD$ ,

$$MD^2 + 12^2 = 13^2$$

$$\begin{aligned}\therefore MD^2 &= 13^2 - 12^2 \\ &= 25\end{aligned}$$

$$\therefore MD = 5$$

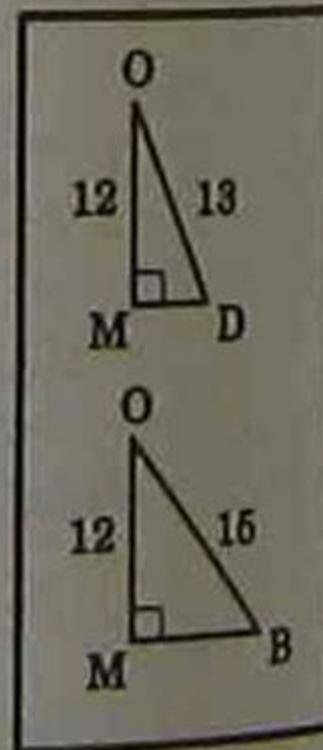
From  $\triangle OMB$ ,

$$MB^2 + 12^2 = 15^2$$

$$\begin{aligned}\therefore MB^2 &= 15^2 - 12^2 \\ &= 81\end{aligned}$$

$$\therefore MB = 9$$

$$\begin{aligned}\text{Now } DB &= BM - MD \\ &= 9 - 5 \\ &= 4\end{aligned}$$



DB is 4 cm.

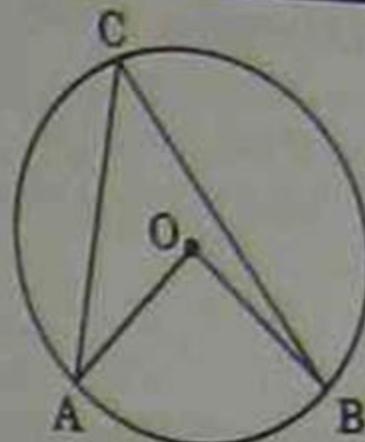
## 12.3 Properties of angles

1. The angle at the centre of a circle is double the angle at the circumference standing on the same arc.

Given O is the centre, then:

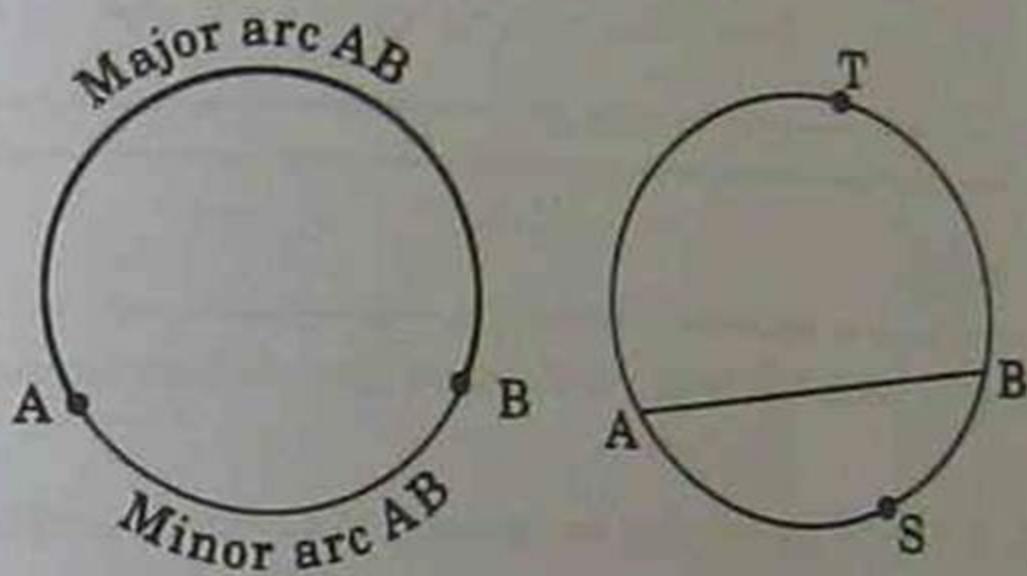
$$\angle AOB = 2 \times \angle ACB, \text{ or}$$

$$\angle ACB = \frac{1}{2} \times \angle AOB$$



## Some important notes

### (a) Arcs and chords



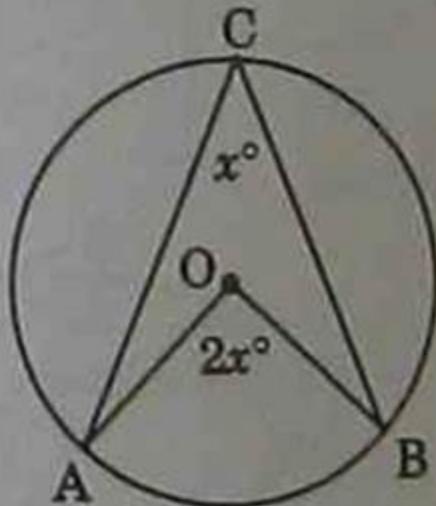
Any chord (AB in this diagram) divides the circumference into two sections. In this diagram the minor arc is ASB and the major arc is ATB.



### (b) Reflex angle at the centre

There are always two angles at the centre; one stands on the major arc — the reflex angle, while the other stands on the minor arc — the acute or obtuse angle. This second type is the most common and the easiest to see.

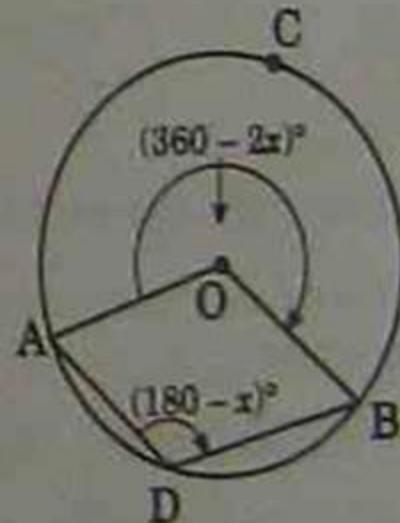
For example:



Normal case

$$\angle AOB = 2 \times \angle ACB, \text{ or}$$

$$\angle ACB = \frac{1}{2} \times \angle AOB$$



Reflex angle at the centre case

$$\text{Reflex } \angle AOB = 2 \times \angle ADB.$$

(Both stand on the major arc ACB.)

If  $\angle AOB = 2x^\circ$ , then

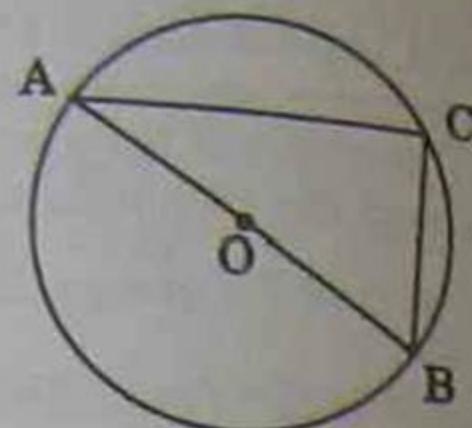
$$\text{reflex } \angle AOB = (360^\circ - 2x)^\circ$$

$$\text{and } \angle ADB = \frac{1}{2}(360^\circ - 2x)^\circ$$

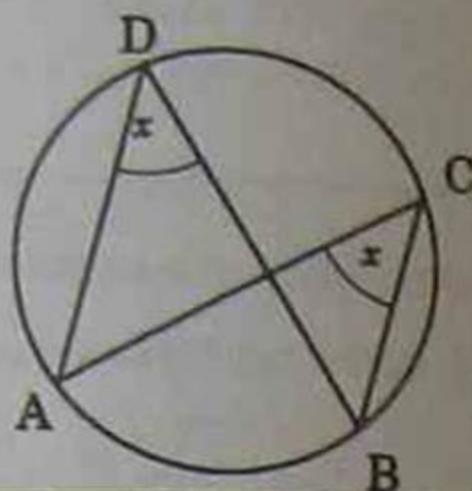
$$= (180 - x)^\circ$$

### Some important notes (continued)

2. The angle in a semicircle is a right angle. Given AB is a diameter,  $\angle ACB = 90^\circ$ .



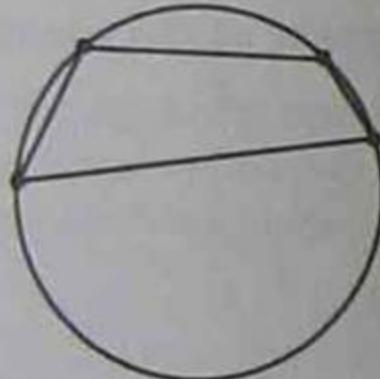
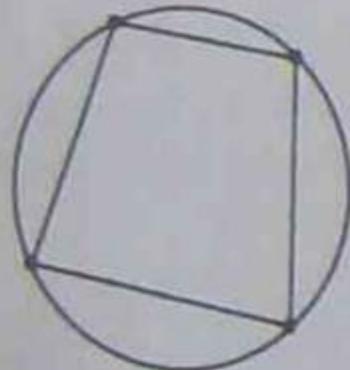
3. Angles in the same segment are equal, [that is, angles at the circumference standing on the same arc]. Given  $\angle ADB$ ,  $\angle ACB$  standing on arc AB, then  $\angle ADB = \angle ACB$ .



## 12.4 Cyclic quadrilaterals

1. A cyclic quadrilateral is a quadrilateral with all four vertices on the circumference of a circle.

Concyclic points are points through which a circle can be drawn, that is, all points lie on the circumference of a circle.



Cyclic quadrilaterals

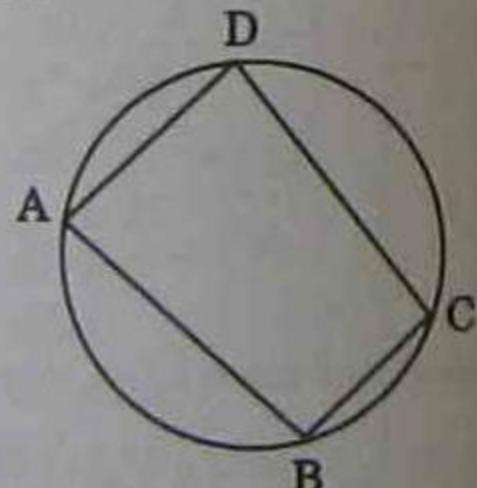
The four vertices of a cyclic quadrilateral are concyclic points.



2. Opposite angles in a cyclic quadrilateral are supplementary.

$$\angle DAB + \angle BCD = 180^\circ, \angle ABC + \angle ADC = 180^\circ.$$

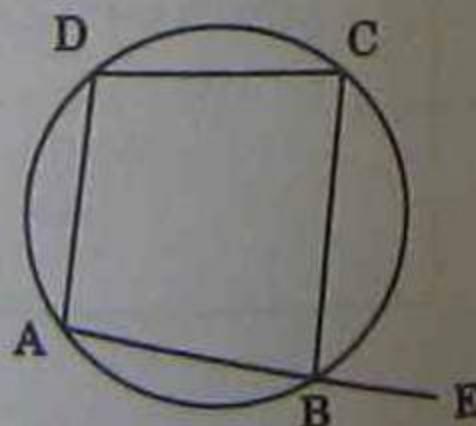
Conversely, if the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.



3. The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle. Given that ABCD is a cyclic quadrilateral:

$$\angle CBE = \angle ADC.$$

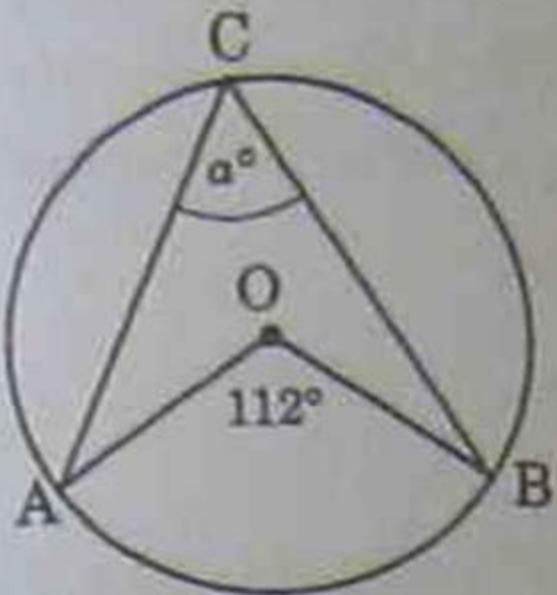
(Proof of this theorem is Question 5 in Exercises.)



## Examples

Find the value of the pronumeral in each diagram, giving adequate reasons.  
(O is the centre in each diagram.)

(a)



$$\angle AOB = 112^\circ$$

$$\angle ACB = \alpha^\circ$$

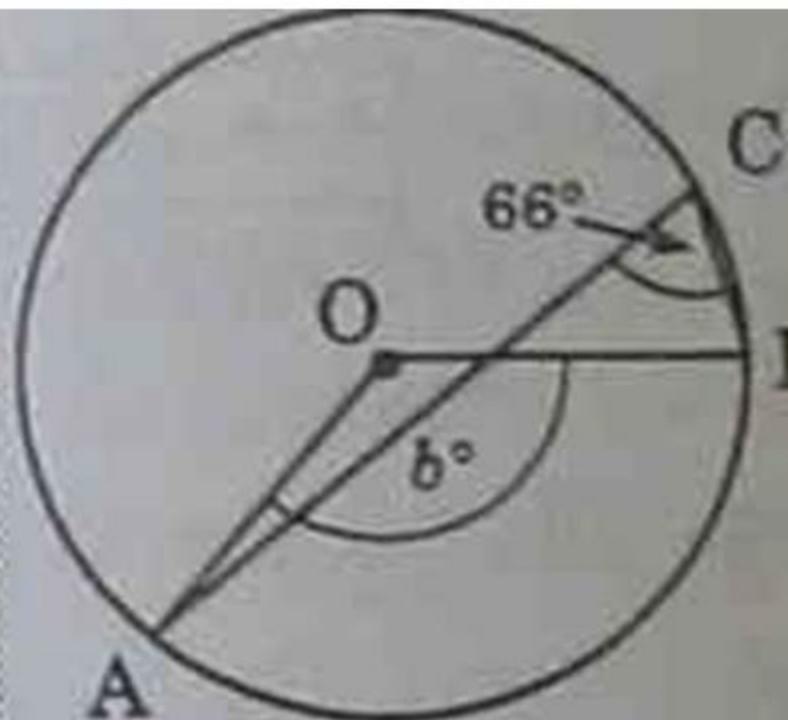


### SOLUTION

$$\alpha = \frac{1}{2} \times 112 = 56 \quad (\angle \text{ at the circum.})$$

$= \frac{1}{2} \angle \text{ at the centre on the same arc}$

(b)



$$\angle ACB = 66^\circ$$

$$\angle AOB = b^\circ$$



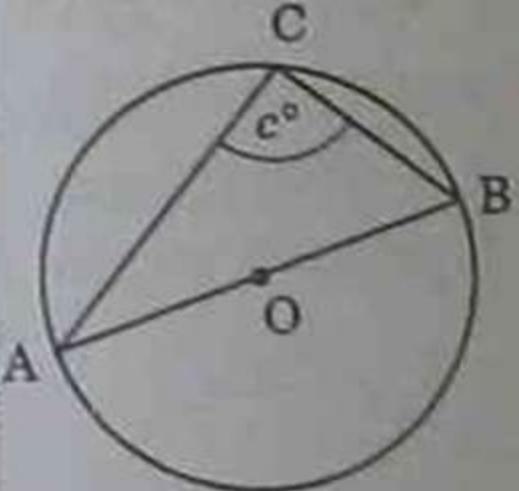
## SOLUTION

$$b = 2 \times 66 = 132$$

( $\angle$  at centre =  $2 \times \angle$  at the circumference on the same arc.)

C

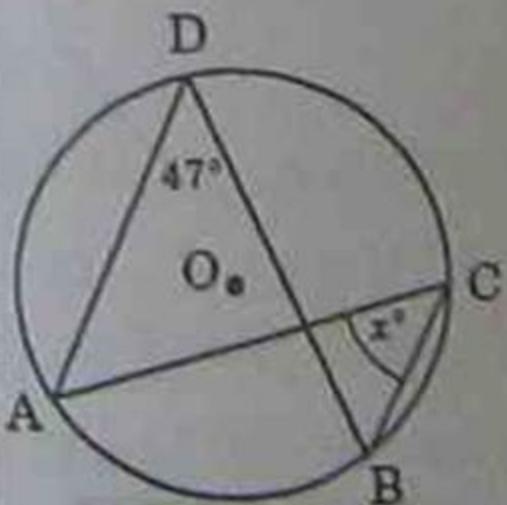
(c)



AB is diameter

SOLUTION:  $c = 90$   
(Angle in a semicircle)

(d)

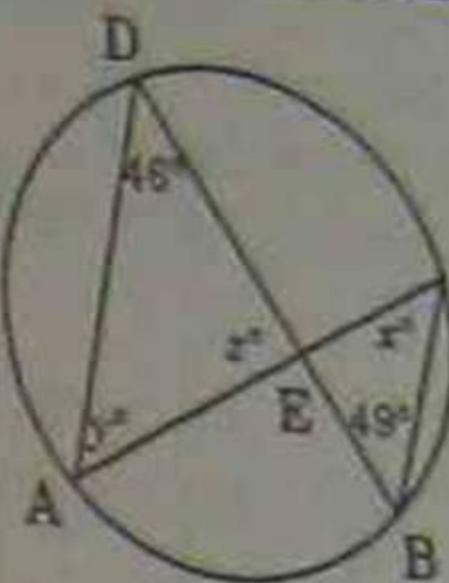


$\angle ADB = 47^\circ$   
 $\angle ACB = x^\circ$



SOLUTION:  $x = 47$   
(Angles in the same segment)

(e)



$$\begin{aligned}\angle ADB &= 46^\circ \\ \angle DBC &= 49^\circ \\ \angle DAC &= y^\circ \\ \angle ACB &= x^\circ \\ \angle AED &= z^\circ\end{aligned}$$

**SOLUTION**

$$x = 46$$

(angles in the same segment standing on arc AB)

$$y = 49$$

(angles in the same segment, on arc DC)

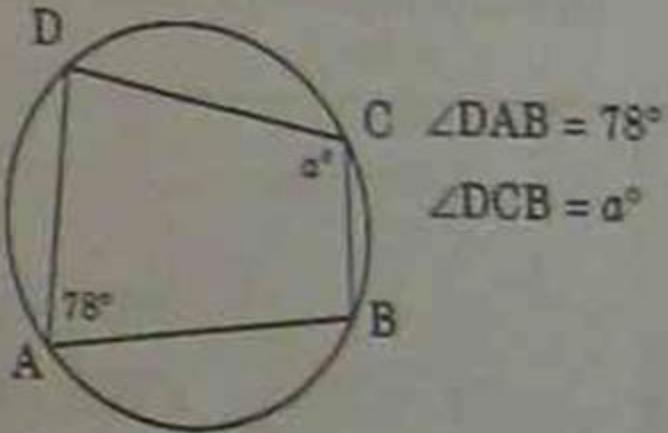
$$z = 180 - (46 + 49)$$

(angle sum of  $\triangle ADE$ )

$$= 85$$



(f)



$$\angle DAB = 78^\circ$$

$$\angle DCB = \alpha^\circ$$

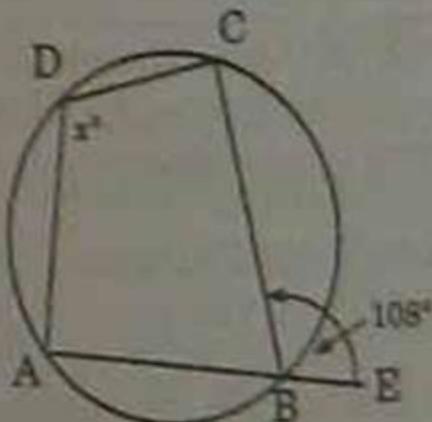
**SOLUTION**

$$\alpha = 180 - 78$$

(opposite angles of a cyclic quad'l)

$$= 102$$

(g)



AE is a straight line.

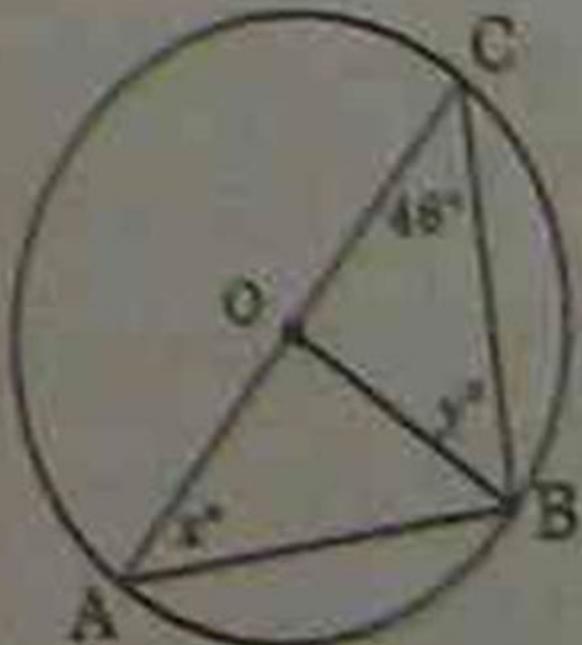
$$\angle CBE = 108^\circ$$

$$\angle ADC = x^\circ$$

**SOLUTION:**  $x = 108$ 

(exterior angle of a cyclic quad'l)

(b)



AC is a diameter.

$$\angle ACB = 46^\circ$$

$$\angle CAB = x^\circ$$

$$\angle CBO = y^\circ$$



### SOLUTION

$OC = OB$  (equal radii)

$\therefore y = 46$  (base angles, isosceles  $\triangle$ )

Continued

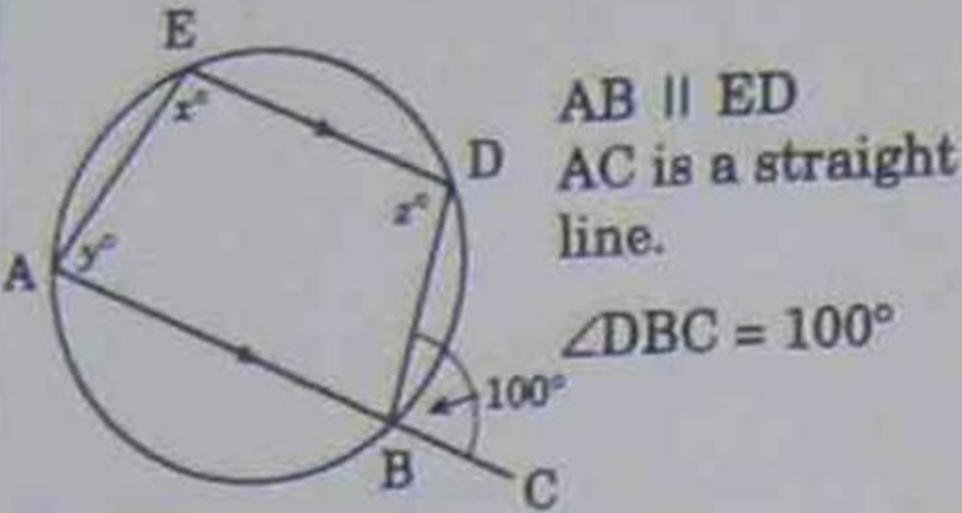
But  $\angle ABC = 90^\circ$  ( $\angle$  in a semicircle)

$$\therefore \angle ABO = 90^\circ - 46^\circ = 44^\circ$$

Also  $AO = OB$  (equal radii)

$$\therefore x = 44 \text{ (base angles, isosceles } \Delta)$$

(i)



$AB \parallel ED$   
AC is a straight  
line.

$$\angle DBC = 100^\circ$$

SOLUTION

$$x = 100$$

(exterior angle of a cyclic quadrilateral)

$z = 100$  (alternate angles,  $AB \parallel ED$ )

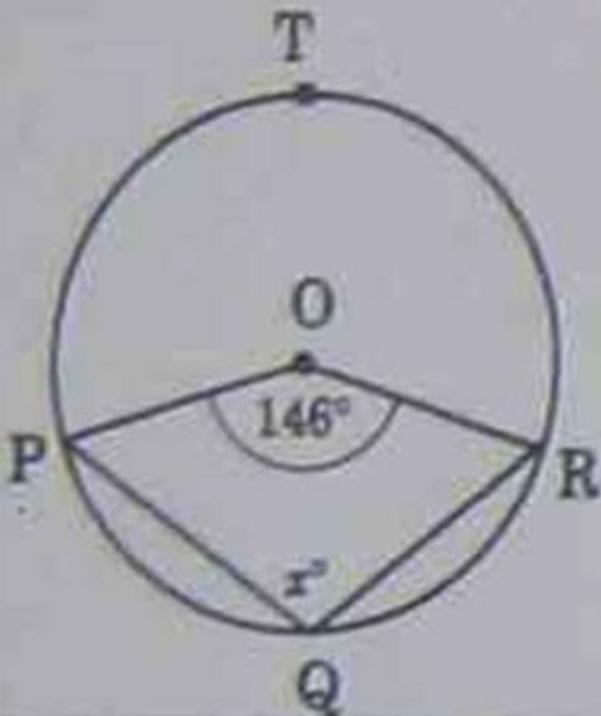
$$\therefore y = 180 - 100$$

(opposite angles of a cyclic quadrilateral)

$$= 80$$



(i)



$$\angle POR = 146^\circ$$

$$\angle PQR = x^\circ$$



*Note 1.* This is the reflex angle at the centre case.

*Note 2.* PQRO is not a cyclic quadrilateral. Only three vertices lie on the circle. The other vertex is at the centre.

SOLUTION

## SOLUTION

Reflex  $\angle POR$  stands on major arc PTR.  
 $\angle PQR$  is the angle at the circumference  
on major arc PTR.

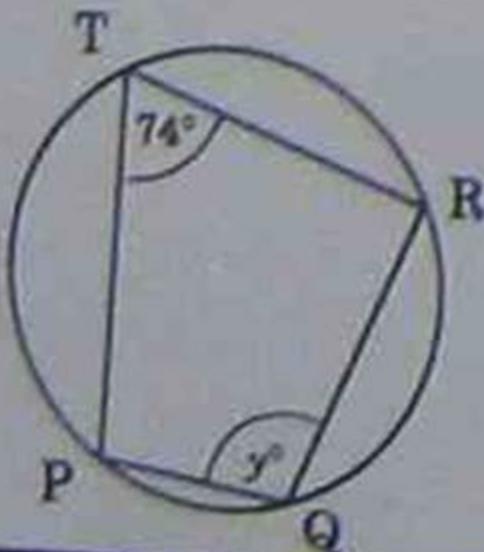
$$\angle POR = 360^\circ - 146^\circ \text{ (angles at a point)}$$

$$= 214^\circ$$

$$\therefore x = 107 \text{ (\angle at circumference)}$$

$$= \frac{1}{2} \angle \text{ at centre on major arc PTR}$$

Compare this with the following case:



This is a cyclic quadrilateral as all four vertices lie on the circle.

$$\angle PTR = 74^\circ, \angle PQR = y^\circ,$$

SOLUTION

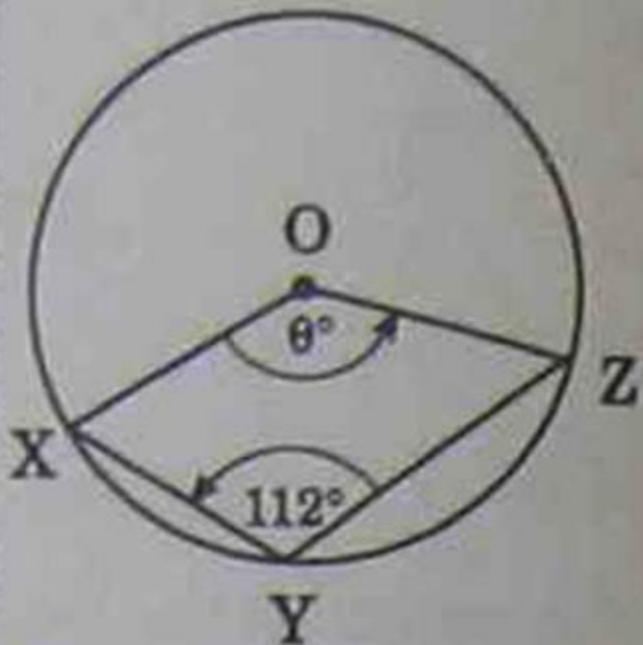
$$y = 180 - 74$$

(opposite angles of a cyclic quad)

$$\therefore y = 106$$



(k)



$$\angle XYZ = 112^\circ,$$
$$\angle XOZ = \theta^\circ$$

### SOLUTION

Reflex  $\angle XOZ = 224^\circ$

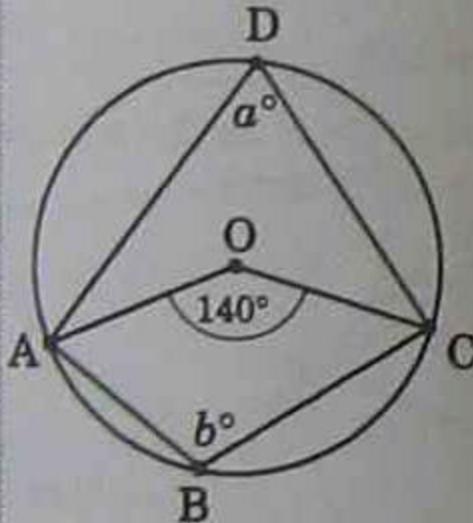
$$2 \times 112^\circ$$

( $\angle$  at centre =  $2 \times \angle$  at circumference on major arc XZ)

$$\begin{aligned}\therefore \theta &= 360 - 224 \text{ (angles at a point)} \\ &= 136\end{aligned}$$



(1)



$$\angle AOC = 140^\circ$$

$$\angle ADC = a^\circ$$

$$\angle ABC = b^\circ$$

## SOLUTION

$$a = 70$$

( $\angle$  at circumference =  $\frac{1}{2}$   $\angle$  at centre on arc AC)

$$\therefore b = 180 - 70$$

(opposite angles of cyclic quadl ABCD)

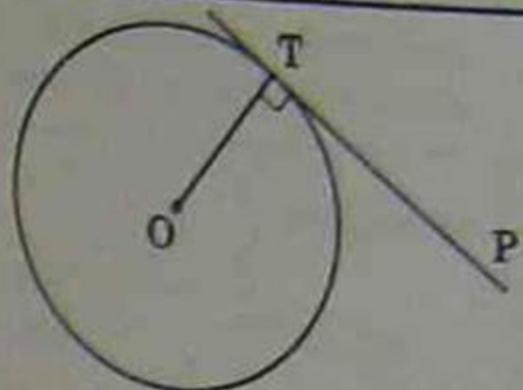
$$= 110$$



## 12.5 Properties of tangents

1. The angle between a tangent and a radius drawn to the point of contact is a right angle.

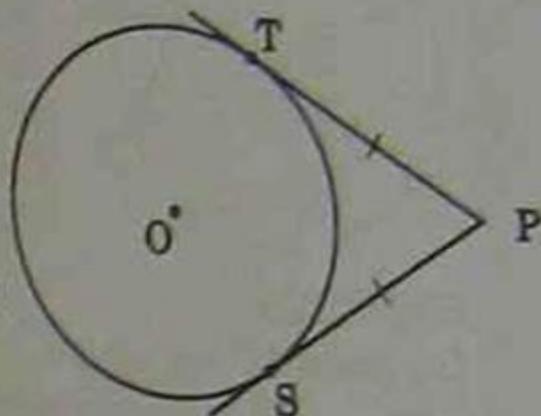
$PT$  is a tangent at  $T$ ,  $O$  is the centre; then  
 $\angle OTP = 90^\circ$ .



2. The lengths of tangents drawn from an external point are equal.

$PT$  and  $PS$  are tangents; then  $PT = PS$ .

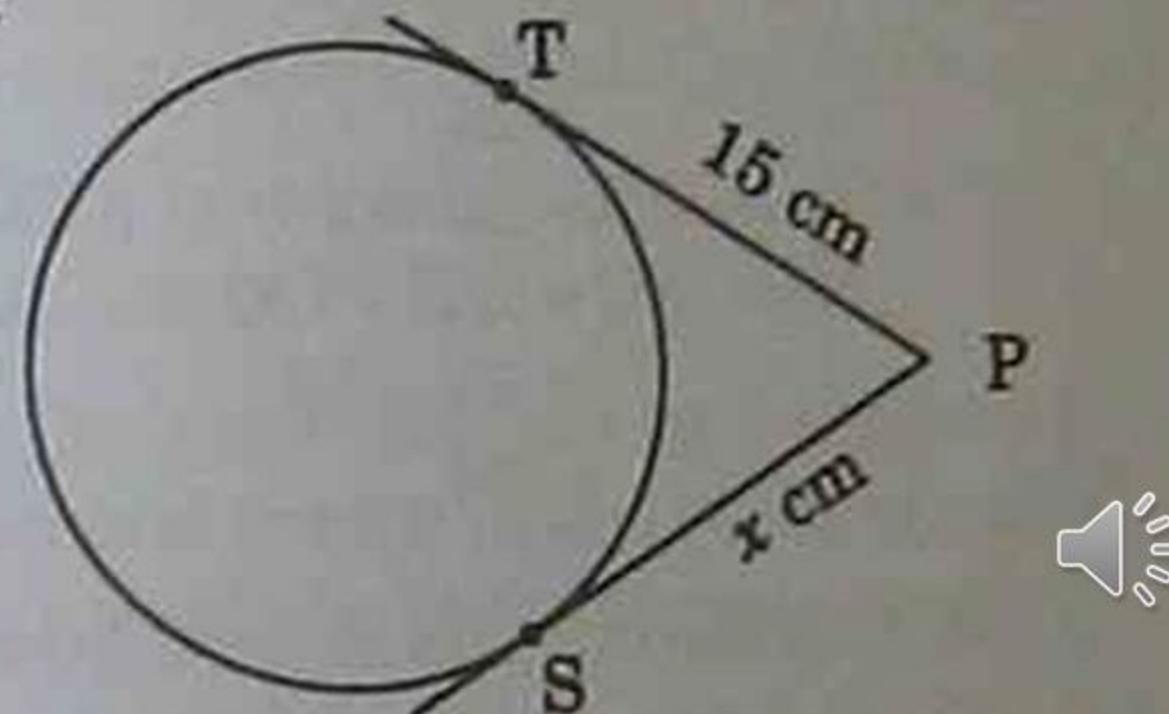
[Congruent triangles are used after joining  $OT$  and  $OS$ .]



## Examples

In Examples (a) – (c), find the value of  $x$ .

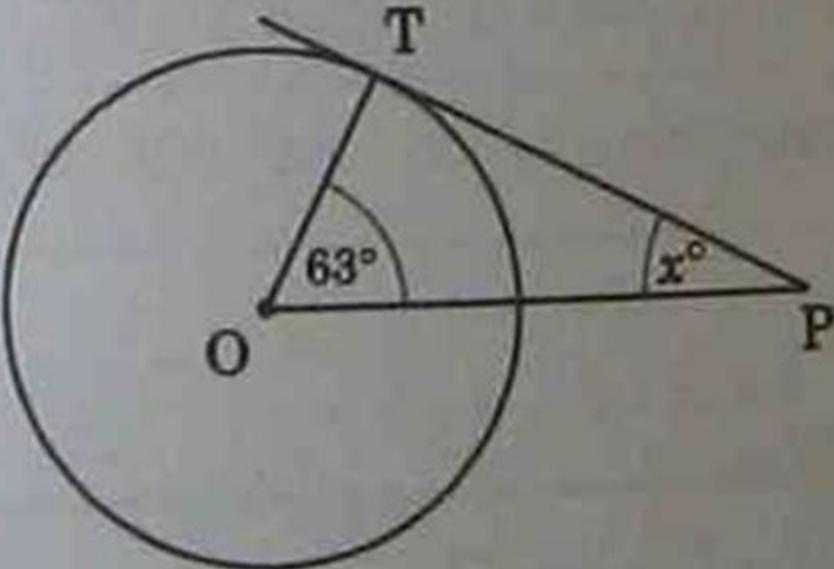
(a)



PT, PS are tangents,  $PT = 15 \text{ cm}$ .

SOLUTION:  $x = 15 \text{ cm}$  (tangents drawn from external point)

(b)



O is centre. PT is a tangent touching  
at T.  $\angle TOP = 63^\circ$ ,  $\angle OPT = x^\circ$

### SOLUTION

$$\angle OTP = 90^\circ$$

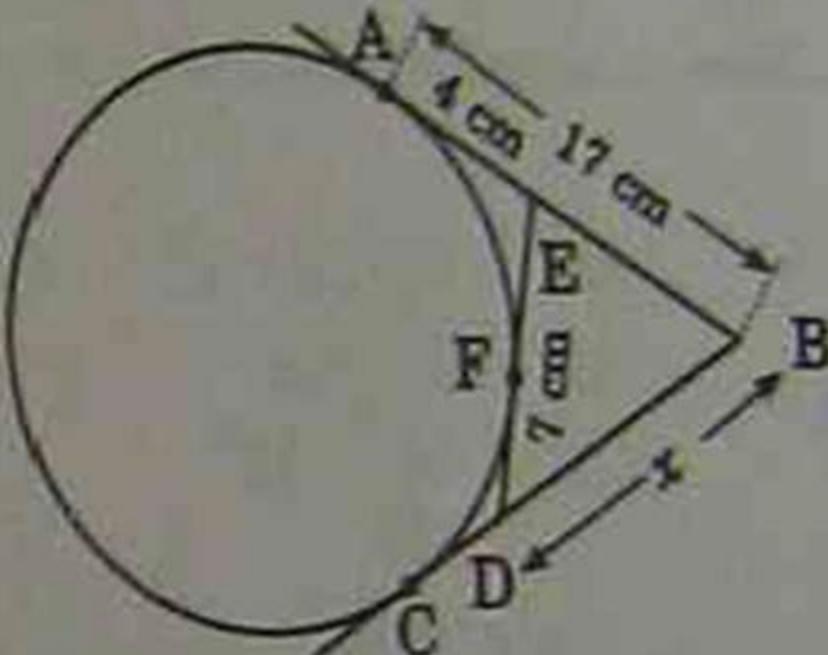
(radius drawn to a point of contact)

$$\therefore x = 90 - 63 \text{ } (\angle \text{ sum of } \Delta)$$

$$x = 27$$



(c)



AB, BC and ED are tangents touching  
at A, C and F respectively.

AB = 17 cm, AE = 4 cm, ED = 7 cm,  
BD = x cm. Find x.

Draw clear diagrams. Mark all  
known information on the diagram.  
Use all the given data.

## SOLUTION

Look for:



$AB = BC$  (tangents from point B)

$$\therefore BC = 17 \text{ cm}$$

Also  $AE = EF$  (tangents from point E)

$$\therefore EF = 4 \text{ cm}$$

Then  $FD = 3 \text{ cm}$  ( $ED = 7 \text{ cm}$ )

But  $FD = DC$  (tangents from point D)

$$\therefore DC = 3 \text{ cm}$$

Then  $BD = 14 \text{ cm}$  ( $BC = 17 \text{ cm}$ )

$$\therefore x = 14$$

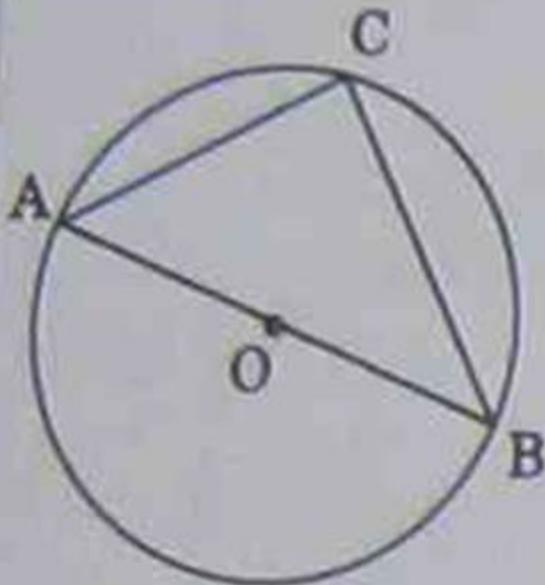


## 12.6 More applications of Pythagoras' Theorem

*Remember:* Whenever a right angle occurs in a triangle, there are opportunities to use either Pythagoras' Theorem or trigonometry. Here Pythagoras' Theorem is considered.

### Examples

(a)



AB is a diameter of a circle, centre O, and radius 20 cm.

Given

$BC = 32 \text{ cm}$ ,  
find the length  
of chord AC.

### SOLUTION

Let chord  $AC = x \text{ cm}$ .

Now  $AB = 40 \text{ cm}$  (radius = 20 cm)

Also  $\angle ACB = 90^\circ$  (angle in a semicircle)



Now, using Pythagoras' Theorem:

$$AB^2 = BC^2 + AC^2$$

$$40^2 = 32^2 + x^2$$

$$\therefore x^2 = 40^2 - 32^2$$

$$= 576$$

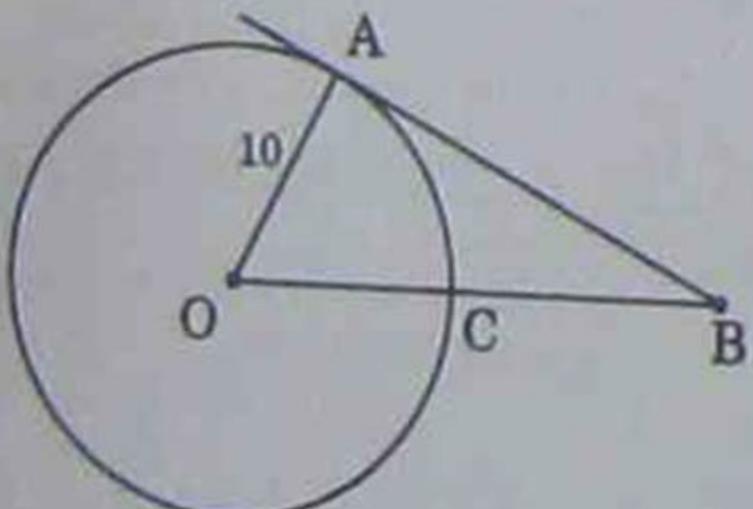
$$\therefore x = \sqrt{576}$$

$$= 24$$

$\therefore$  AC is 24 cm long.

$\therefore$  AC is 24 cm long.

(b)



AB is a tangent to a circle, centre O, radius 10 cm. If AB is 24 cm, find the length of BC.

## SOLUTION



To find BC, we must first find BO.

$$\angle OAB = 90^\circ$$

(angle between the radius and the tangent to the point of contact)

Let OB =  $y$  cm.

Using Pythagoras' Theorem in  $\triangle OAB$ :

$$OB^2 = OA^2 + AB^2$$

$$\begin{aligned}y^2 &= 10^2 + 24^2 \\&= 676\end{aligned}$$

$$\therefore \begin{aligned}y &= \sqrt{676} \\&= 26\end{aligned}$$

$$OB = 26 \text{ cm}$$

$$\begin{aligned}\text{Now } BC &= OB - OC \\&= 26 - 10 \\&= 16\end{aligned}$$



(OC is a radius and is thus 10 cm)

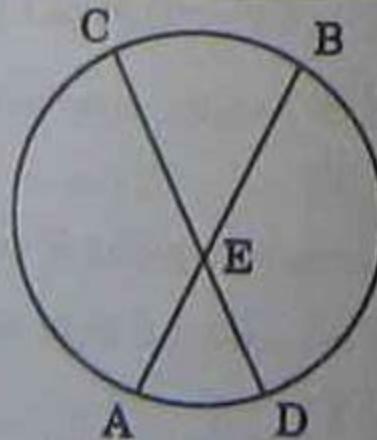
$\therefore$  BC is 16 cm long.

## 12.7 Ratio theorems (Proved using similar triangles)

1. The products of intercepts of intersecting chords are equal, that is:

$$AE \cdot EB = DE \cdot EC$$

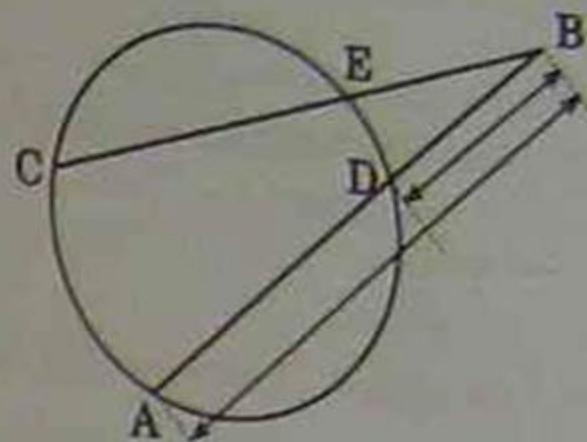
$$\boxed{AE \times EB = DE \times EC}$$



2. An extension of (1) concerning secants from an external point B is illustrated in the diagram.

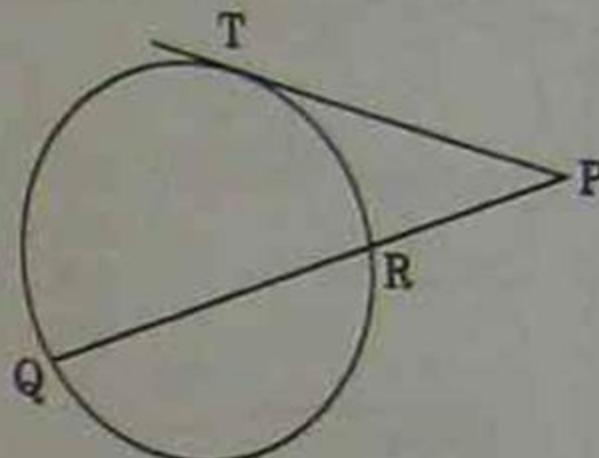
This is a case of chords produced intersecting at an external point. Chords AD and CE are extended to meet at B. Then  $AB \times BD = CB \times BE$

(End-point A to intersection B  $\times$  intersection B to other end point D)



3. The square of the length of the tangent is equal to the product of the intercepts of a secant drawn from an external point.

$$PT^2 = QP \times PR$$



## 12.8 Angle in the alternate segment

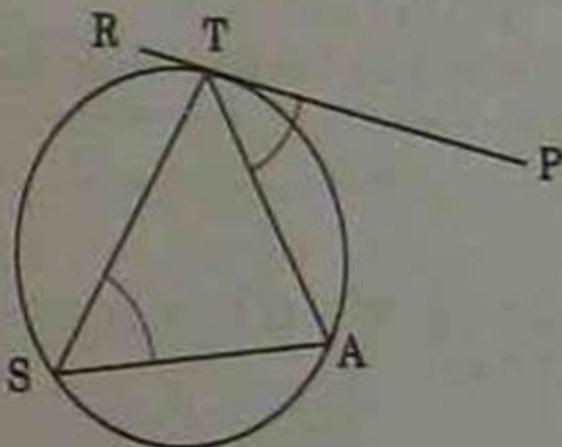
- PT is a tangent at T. AT is a chord drawn to the point of contact.  $\angle TSA$  is the angle at the circumference subtended by chord AT.

$\angle TSA$  is the angle in the alternate segment relative to  $\angle PTA$ .

- An angle formed by a tangent to a circle with a chord drawn to the point of contact is equal to any angle in the alternate segment.

From the diagram,  $\angle PTA = \angle TSA$ .

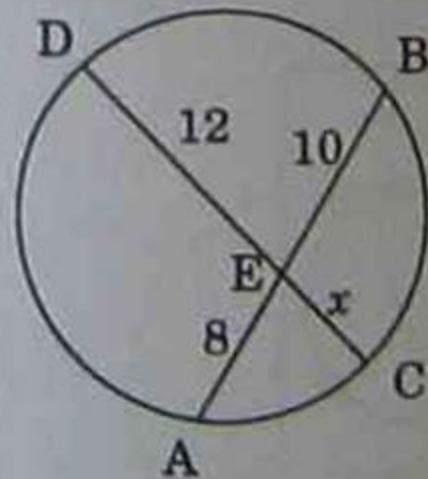
Also note that  $\angle RTS = \angle TAS$  for the same reason (tangent RT, chord TS).



## 12.9 Worked examples

Find the value of the pronumeral in each diagram. All lengths are in cm.

(a)



$$\begin{aligned}AE &= 8 \\EB &= 10 \\DE &= 12 \\EC &= x\end{aligned}$$

**SOLUTION**

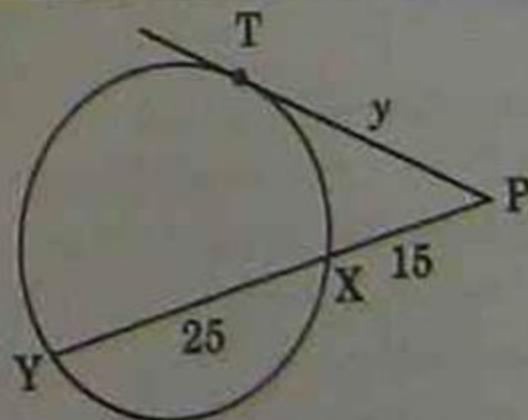
$$12 \times x = 8 \times 10$$

(intercepts of intersecting chords)

$$12x = 80$$

$$x = \frac{80}{12} = 6\frac{2}{3}$$

(b)



$$\begin{aligned}XY &= 25 \\XP &= 15 \\PT &= y\end{aligned}$$

Find the exact value.

Note:  $YP = 40$



## SOLUTION

$$y^2 = 40 \times 15$$

(product of intercepts of secant = square of tangent.)

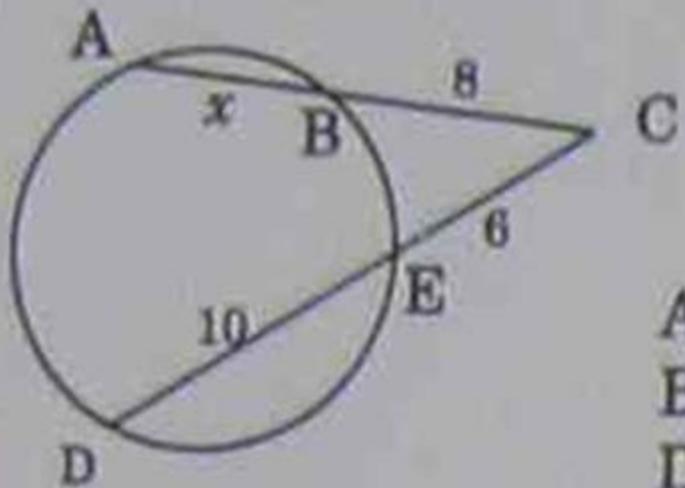
$$= 600$$

$$\therefore y = \sqrt{600}$$

$$= 10\sqrt{6}$$



(c)



$$\begin{aligned}AB &= x \\BC &= 8 \\DE &= 10 \\EC &= 6\end{aligned}$$

*Note:*

$$AC = x + 8$$

$$DC = 16$$



## SOLUTION

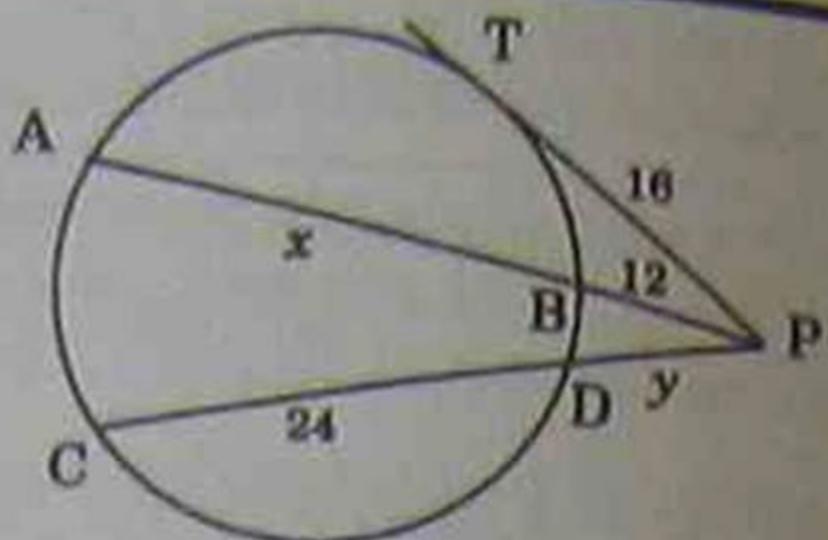
$$(x+8) \times 8 = 16 \times 6$$

(intercepts of intersecting chords)

$$\therefore x + 8 = 12$$

$$x = 4$$

- (d)  $AB = x$ ,  $BP = 12$ ,  $CD = 24$ ,  $DP = y$ , and  $TP = 16$ .



SOLUTION



$$16^2 = (x + 12)12$$

(product of intercepts of secant = square of the tangent)

## SOLUTION

$$16^2 = (x + 12)12$$

(product of intercepts of secant = square of the tangent)

$$\frac{256}{12} = x + 12$$

$$21\frac{1}{3} = x + 12$$

$$\therefore x = 9\frac{1}{3}$$

$$16^2 = (y + 24)y$$

(product of intercepts of secant = square of the tangent)

$$\therefore 256 = y^2 + 24y,$$

$$\text{that is, } y^2 + 24y - 256 = 0$$

$$(y + 32)(y - 8) = 0$$

$$\therefore y = -32 \text{ or } 8,$$

$$\text{that is, } y = 8$$

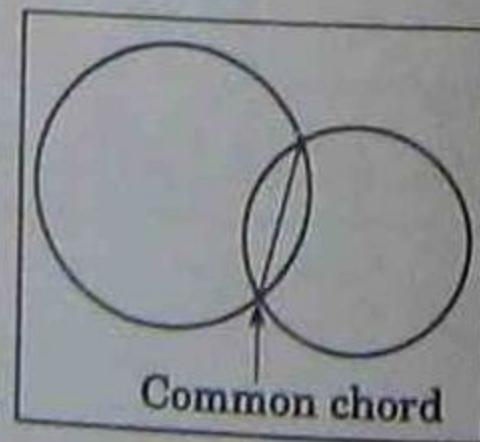
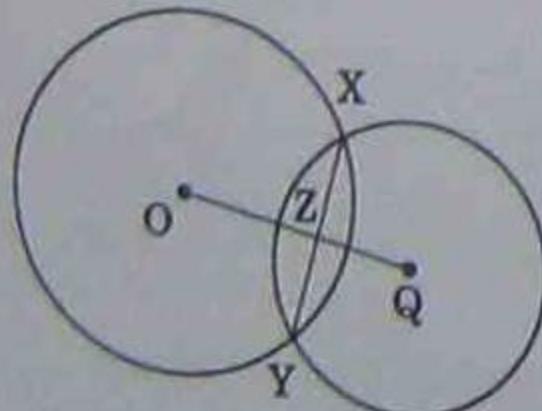
[Length must be positive.]



## 12.10 Circles that touch

### 12.10.1 Intersection of two circles

When two circles intersect, the line joining their centres bisects their common chord at right angles.



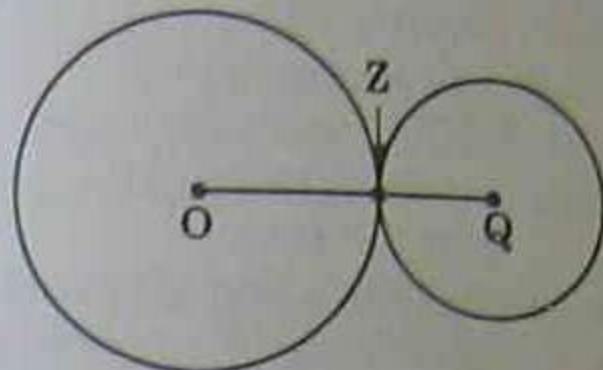
Two circles, centres O and Q intersect at X and Y. The line joining the centres OQ intersect the common chord XY at Z.

Then  $XZ = ZY$  and  $XY \perp OQ$ . (This is proven using congruent triangles.)

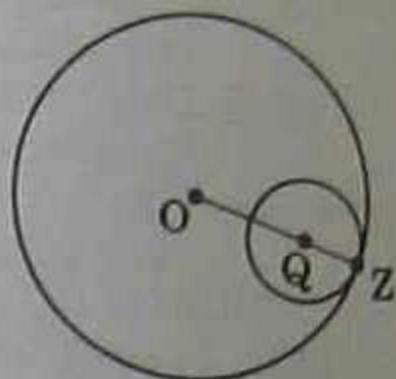


### 12.10.2 Point of contact of two circles

When two circles touch, their centres and the point of contact are collinear.



Two circles with centres O and Q touch at Z. OQ is a straight line and Z lies on OQ (or OQ produced), that is, O, Q and Z are collinear.



(This is proved by drawing the common tangent at Z.)



## 12.11 Worked examples using deductive reasoning

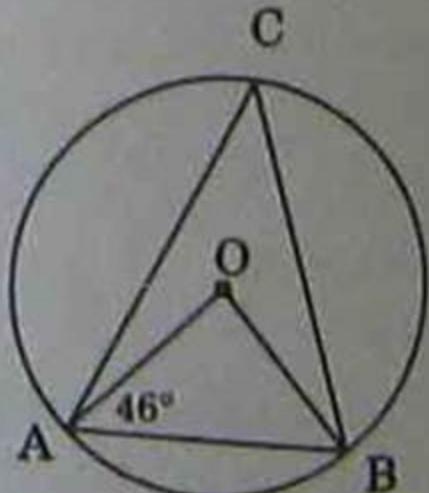
- Always draw a clear diagram.
- Mark on the diagram all the given information.
- Work from the known towards the unknown.
- If you become lost, check that you have used all the data.
- Any known facts can be used — you cannot use the fact that you are asked to prove.

In all diagrams, O is the centre of the circle.



## Examples

- (a)  $\angle OAB = 46^\circ$ . Find  $\angle ACB$ .



## SOLUTION

$AO = OB$  (equal radii)

$\therefore \angle ABO = 46^\circ$

(base angles of isosceles  $\Delta$ )

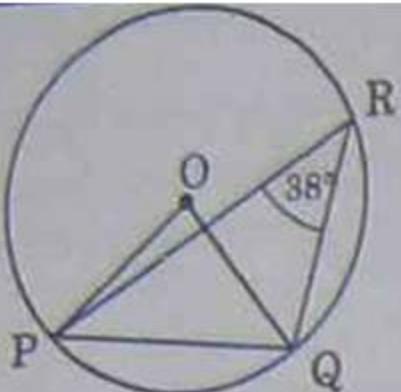
$$\begin{aligned}\therefore \angle AOB &= 180^\circ - (46^\circ + 46^\circ) \\ &= 88^\circ \text{ (angle sum of } \Delta)\end{aligned}$$

$\therefore \angle ACB = 44^\circ$

(angle at circumference  $= \frac{1}{2} \angle$  at  
centre on arc AB)



(b)



$\angle PRQ = 38^\circ$ ,  
 $\angle OQR = 47^\circ$ .  
 Find  $\angle OPR$ .

### SOLUTION

$$\angle POQ = 76^\circ$$

( $\angle$  at centre =  $2 \times \angle$  at circumference  
 on arc PQ)

But  $OP = OQ$  (equal radii)

$\therefore \angle OPQ = \angle OQP$

(base angles, isosc.  $\Delta$ )

$$\angle OQP = \angle OPQ$$

$$= \frac{1}{2}(180^\circ - 76^\circ)$$

$$= 52^\circ$$

Then  $\angle PQR = \angle OQP + \angle OQR$

$$= 52^\circ + 47^\circ$$

$$= 99^\circ$$

Then  $\angle RPQ = 43^\circ$  ( $\angle$  sum of  $\Delta PRQ$ )

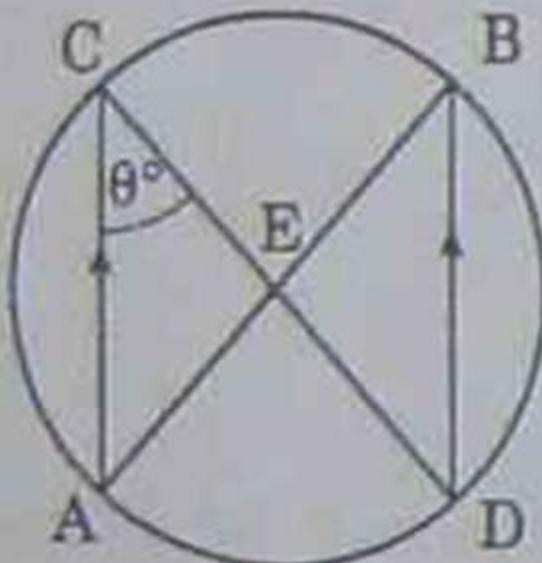
But  $\angle OPR = \angle OPQ - \angle RPQ$

$$= 52^\circ - 43^\circ$$

$$= 9^\circ$$



(c)



$AC \parallel BD$ . Prove that  $\angle ACE = \angle CAE$ .

SOLUTION

Let  $\angle ACE = \theta^\circ$

$\therefore \angle EDB = \theta^\circ$

(alternate angles,  $AC \parallel BD$ ),

and  $\angle CAB = \theta^\circ$

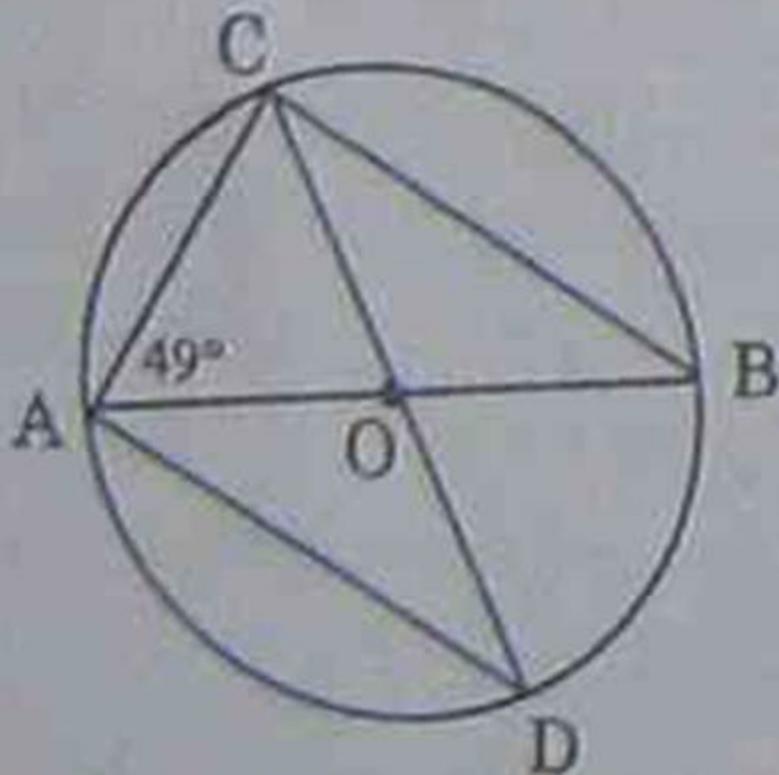
(angles in the same segment on arc CB)

Then  $\angle ACE = \angle CAE$  (both  $\theta^\circ$ )



(f)

(d)



Given that AB, CD are diameters,  
 $\angle CAB = 49^\circ$ . Find  $\angle ADC$ .



## SOLUTION

$AO = OC$  (equal radii)

$$\angle ACO = 49^\circ$$

(base angles of isosceles  $\Delta$ )

But  $\angle ACB = 90^\circ$

(angle in a semicircle)

$$\therefore \angle OCB = 41^\circ (90^\circ - 49^\circ)$$

Also,  $OC = OB$  (equal radii)

$$\therefore \angle OBC = 41^\circ$$

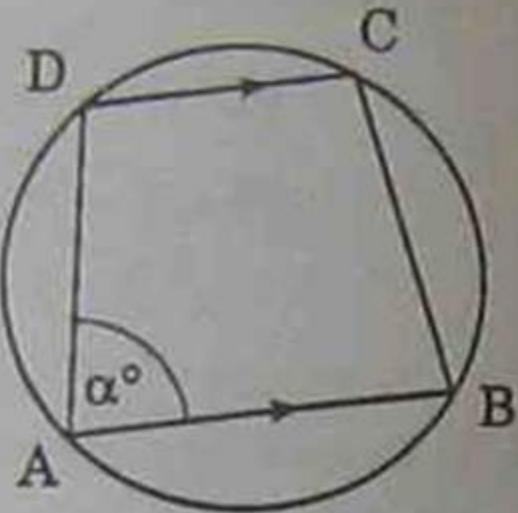
(base angles of isosceles  $\Delta$ ),

and  $\angle ADC = 41^\circ$

(angles in the same segment)



(e)



Given that  $AB \parallel DC$  and  $ABCD$  is a cyclic quadrilateral, prove that  $\angle ADC = \angle BCD$ .

### SOLUTION

$$\text{Let } \angle DAB = \alpha^\circ$$

$$\therefore \angle ADC = (180 - \alpha)^\circ$$

(co-interior angles,  $AB \parallel CD$ ),

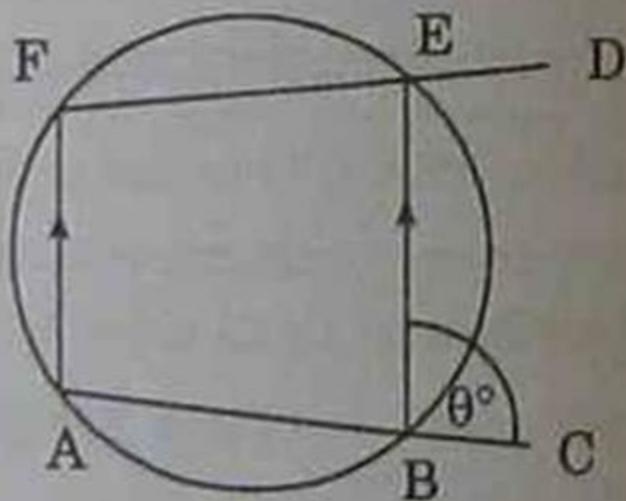
$$\text{and } \angle DCB = (180 - \alpha)^\circ$$

(opposite angles of a cyclic quad'l)

$$\therefore \angle ADC = \angle BCD \text{ [both } (180 - \alpha)^\circ]$$



(f)



Given that  $AF \parallel BE$ , and  $ABEF$  is a cyclic quad'l, prove that  $\angle CBE = \angle DEB$

### SOLUTION

Let  $\angle EBC = \theta^\circ$

$\therefore \angle FAB = \theta^\circ$

(corresponding angles,  $AF \parallel BE$ ),

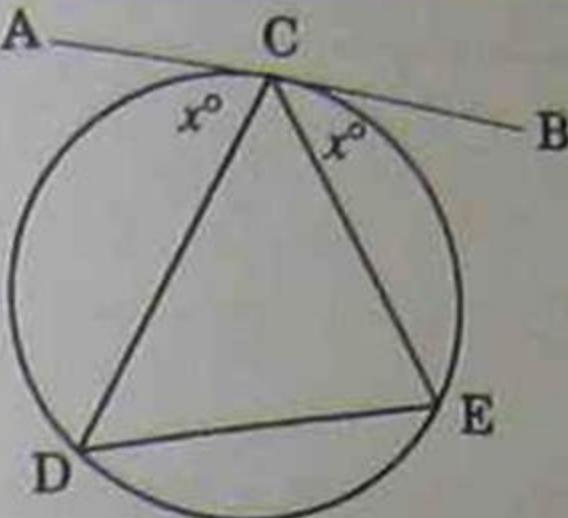
and  $\angle DEB = \theta^\circ$

(External  $\angle$  of a cyclic quad'l)

$\therefore \angle CBE = \angle DEB$  (both  $\theta^\circ$ )



(g)



Given that AB is tangent at C and  
 $\angle ACD = \angle BCE$ , prove that  $DE \parallel AB$ .

### SOLUTION

Let  $\angle ACD = x^\circ$

Then  $\angle BCE = x^\circ$  (data)

But  $\angle BCE = \angle CDE$

(angle in the alternate segment)

$\therefore \angle CDE = x^\circ$

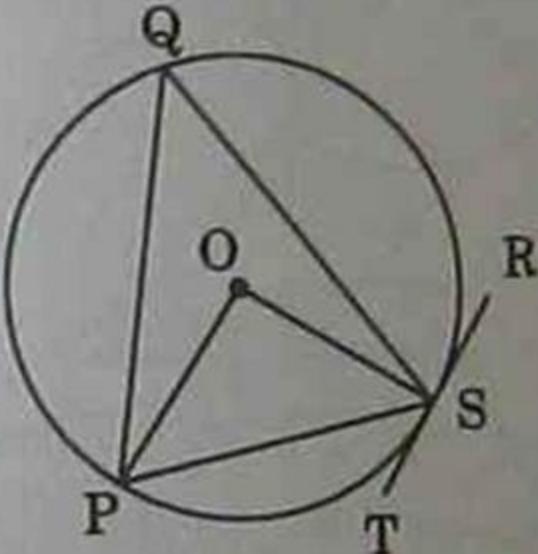
Then  $\angle ACD = \angle CDE$  (both  $x^\circ$ )

$\therefore AB \parallel DE$

(a pair of alternate angles are equal).



(h)



Given that TR is a tangent at S, prove that  $\angle POS = 2 \times \angle PST$ .

### SOLUTION

Let  $\angle PST = \theta^\circ$

Now  $\angle PQS = \angle PST$

(angle in alternate segment)

$\therefore \angle PQS = \theta^\circ$

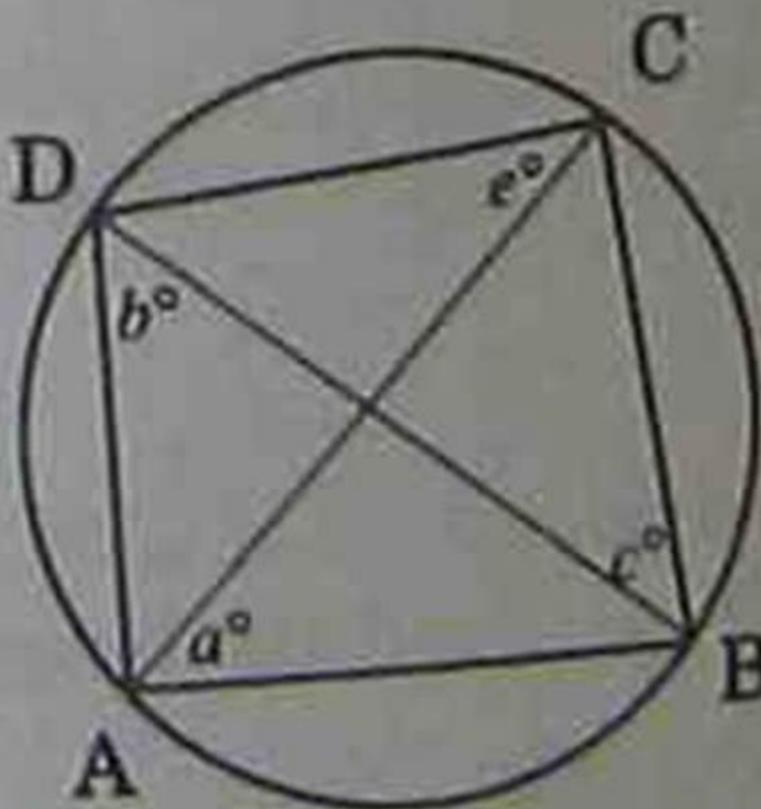
But  $\angle POS = 2 \times \angle PQS = 2\theta^\circ$

(angle at centre =  $2 \times$  angle at circum. on arc PS)

$\therefore \angle POS = 2 \times \angle PST$



(i)



Given that ABCD is a cyclic quadrilateral and  $\angle CAB = a^\circ$ ,  $\angle ADB = b^\circ$ ,  $\angle DBC = c^\circ$ ,  $\angle ACD = e^\circ$ , prove that  $a + b + c + e = 180^\circ$ .

## SOLUTION

$$\angle BDC = a^\circ$$

(angles in same segment)

and

$$\angle ABD = e^\circ$$

(angles in same segment).

$$\text{But } \angle ADC + \angle ABC = 180^\circ$$

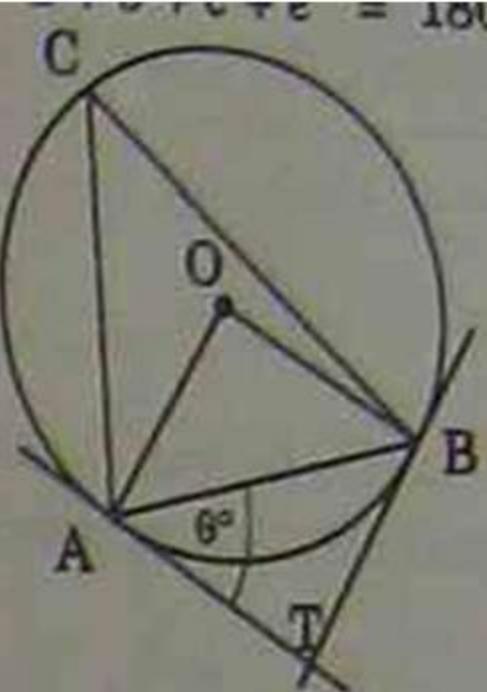
(opposite angles of a cyclic quadrilateral)

$\therefore$

$$a + b + c + e = 180$$



(j)



Given that TA and TB are tangents to a circle centre O and  $\angle TAB = 6^\circ$ , prove that ATBO is a cyclic quadrilateral.

### SOLUTION

$$TA = TB$$

(tangents from an external point)

$$\therefore \angle TBA = 6^\circ$$

(base angles of isosceles  $\Delta$ )



Then  $\angle ATB = 180^\circ - 2\theta$

(angle sum of  $\Delta$ )

But  $\angle TAB = \angle ACB$

(angle in alternate segment)

$\therefore \angle ACB = \theta^\circ$

and  $\angle AOB = 2\theta^\circ$  (angle at centre =  
2  $\times$  angle at circum. on arc AB)

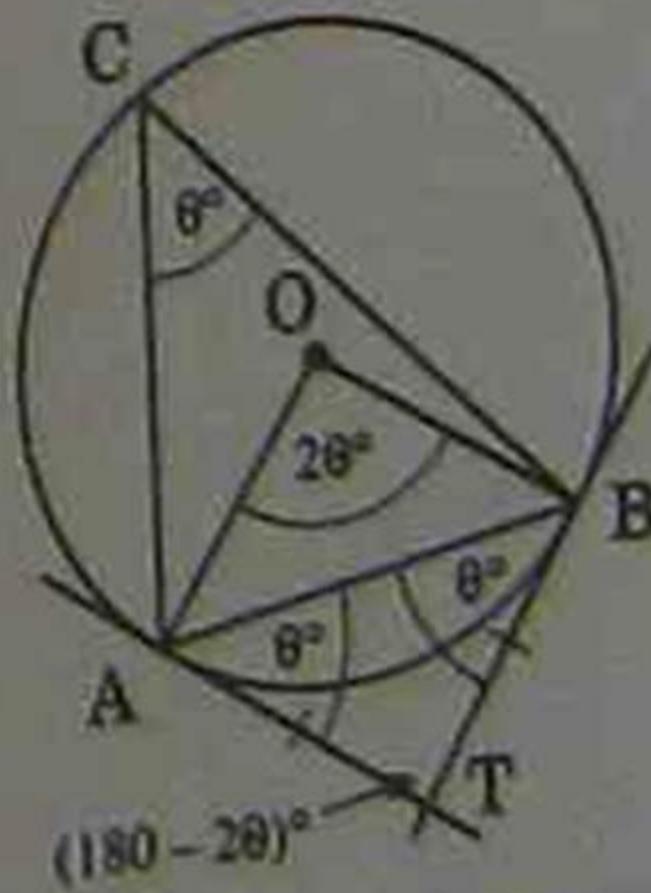
Now

$$\angle AOB + \angle ATB = 2\theta^\circ + 180^\circ - 2\theta^\circ = 180^\circ$$

$\therefore$  ATBO is cyclic quadrilateral  
(opp. angles supplementary).



This is what your diagram should look like before you attempt to write down any steps of your working:



Remember:

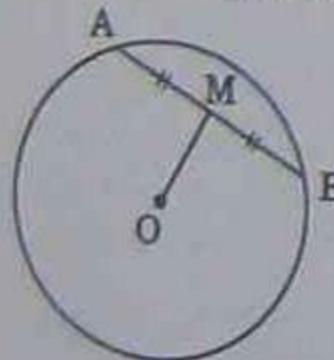
- Make sure all data are used.
- Mark information on the diagram.
- Mark angles on the diagram as you find them.
- Work from the known towards the unknown.

To do all this you must have a clearly drawn diagram.



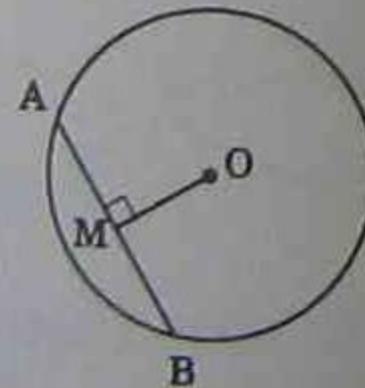
## 12.2 Properties of chords

All properties in this section rely on the use of congruent triangles.



1. A line from the centre of a circle to the midpoint of a chord meets the chord at right angles.

Given  $AM = MB$ , then  $OM \perp AB$ .



2. A perpendicular drawn to a chord from the centre of a circle bisects the chord.

Given  $OM \perp AB$ , then  $AM = MB$ .

