$$= \$134.64$$

$$\text{Interest rate} = \frac{\text{interest}}{\text{amount borrowed}} \times 100\%$$

$$\text{Interest rate} = \frac{\$134.64}{\$612} \times 100$$
$$= 22\%.$$

Interest rate is 22% per annum.

1.4 Saving and borrowing

1.4.1 Simple interest

$SI = PRt$, where SI = simple interest, P = principal, R = annual interest rate expressed as a decimal, t = number of years.

Note: $R = \frac{r}{100}$ when r is expressed as a percentage.

Examples

Find the simple interest gained on the following:

(a) $1600 at $12\frac{1}{2}$% p.a. for 3 months.

SOLUTION

3 months is $\frac{3}{12}$ of a year

$$SI = PRt$$
$$= \$1600 \times 0.125 \times \tfrac{3}{12}$$
$$= \$50$$

Interest gained is $50.

(b) $500 at 4.75% p.a. for 18 months.

SOLUTION

$\frac{18}{12} = 1\frac{1}{2}$
$= 1.5$

$$SI = PRt$$
$$= \$500 \times 0.0475 \times 1.5$$
$$= \$35.625$$
$$= \$35.63 \text{ [to the nearest cent]}$$

Interest gained is $35.63.

(c) $760 at 9% p.a. for 1 day.

SOLUTION

$365\frac{1}{4}$ days in each year,

$\therefore \dfrac{1}{365\frac{1}{4}}$ of a year

$$SI = PRt$$
$$= \$760 \times 0.09 \times \frac{1}{365\frac{1}{4}}$$
$$= \$0.187\,268\,993$$
$$= \$0.19 \text{ [to the nearest cent]}$$

Interest gained is $0.19.

1.4.2 Compound interest

This is more realistic than simple interest mentioned above and assumes depositors will gain interest on their interest as their money lies untouched in their account, rather than withdrawing interest on the day it is earned.

For example, \$1000 gaining compound interest of 12% per annum over three years can be calculated the following way:

$\$1000 \times 1.12 \times 1.12 \times 1.12$

deposit + interest for first year

Year 1's total + interest for second year

Year 2's total + interest for third year,

that is, total after 3 years $= \$1000 \times 1.12 \times 1.12 \times 1.12$

$= \$1000 \times 1.12^3$

$= \$1404.93$ (to the nearest cent)

Compound interest $= \$1404.93 - \$1000 \leftarrow$ $\boxed{\text{\$1000 is the original deposit}}$

$= \$404.93.$

> *Remember:* To increase \$1000 by 12% we could find 12% of \$1000 and add to \$1000 — it is easier to simply multiply: \$1000 × 1.12

We can use the formula:

$$A = P(1+R)^n$$

where A = accumulated Amount

P = principal

R = annual interest rate as a decimal.

n = number of years (or time periods).

Note: $R = \frac{r}{100}$ when r is expressed as a percentage.

Examples

expressed as a percentage.

Examples

Terry invested $42 000 in a savings account which attracted an 8.5% interest rate compounded annually.

Find out how much money Terry has in the account after 4 years.

Continued

SOLUTION

$$A = P(1 + R)^n$$

$$= \$42\,000(1 + 0.085)^4$$

$$= \$42\,000 \times 1.085^4$$

$$= \$58\,206.07 \quad \text{(to the nearest cent)}$$

Terry has $58 206.07 in his savings account.

(b) Mercia works as a manager of a clothing factory. Her pay conditions involve an increase of 6% every year. If this year she is paid a salary of \$48 320, what will her salary be in seven years time?

SOLUTION

$$A = P(1+R)^n$$

$$= \$48\,320(1+0.06)^7$$

$$= \$48\,320 \times 1.06^7$$

$$= \$72\,655.41 \text{ (to the nearest cent)}$$

Mercia will be paid a salary of \$72 655.41

(c) Susan can choose between two accounts: the first offering simple interest at a rate of 12% and the second compound interest at a rate of 8%. If she had \$400 to invest for three years, which account should she use to gain the greater amount of interest?

SOLUTION

$$\text{Simple interest} = \$400 \times 0.12 \times 3 = \$144$$

For compound interest,

$$A = 400(1.08)^3 = \$503.88$$

(to the nearest cent)

$$\text{Compound interest} = \$503.88 - \$400 = \$103.88.$$

Susan would gain more interest by using the simple interest account.

(d) Noel deposits \$1000 in an account which offers interest at 6% per annum *compounded monthly*. He invests the money for two years. Find his balance after two years.

SOLUTION: As the interest is determined monthly, there will be 24 payments over the two years. Also, the rate is 6% per annum, that is $\frac{6}{12} = 0.5\%$ per month.

$$\boxed{\text{Note: } 0.5\% = 0.005}$$

$$\begin{aligned} A &= P(1+R)^n \\ &= \$1000(1+0.005)^{24} \\ &= \$1000(1.005)^{24} \\ &= \$1127.16. \end{aligned}$$

Noel will have \$1127.16 in his account after two years.

1.5 Depreciation

Whereas compound interest is calculating values that are increasing, depreciation is the opposite — that is, calculating values that are decreasing.

We can use the formula: $A = P(1 - R)^n$

where A = final value
P = initial value
R = annual depreciation rate expressed as a decimal.
n = number of years (or time periods).

Note: $R = \dfrac{r}{100}$ when r is expressed as a percentage.

Examples

(a) Lloyd buys a second-hand car for \$14 400. If it depreciates in value at 13% per annum, find its value after four years.

SOLUTION

$$\begin{aligned} A &= P(1-R)^n \\ &= \$14\,400(1-0.13)^4 \\ &= \$14\,400(0.87)^4 \\ &= \$8249.73 \text{ (to the nearest cent)} \end{aligned}$$

Lloyd's car is valued at \$8249.73.

(b) Christine purchases a computer valued at \$2100. If it depreciates at 20% per annum, how much will it depreciate in its third year?

SOLUTION: $A = P(1-R)^n$

Value after two years:

$$\$2100(1-0.2)^2 = \$2100(0.8)^2 = \$1344$$

Value after three years:

$$\$2100(1-0.2)^3 = \$2100(0.8)^3 = \$1075.20.$$

Amount of depreciation

in third year: $= \$1344 - \$1075.20 = \$268.80$

The computer depreciates \$268.80 in its third year.

(c) Helen purchases a television set for \$1000. If it depreciates at 20% per annum, how many years will elapse until it is valued at \$512?

SOLUTION:

$$A = P(1-R)^n$$

$$512 = 1000(1-0.2)^n$$

$$512 = 1000(0.8)^n$$

$$0.8^n = \frac{512}{1000}$$

$$0.8^n = 0.512$$

By trial and error, using the $\boxed{x^y}$ key:

$n = 3.$

> You could also take logs of both sides (See Chapter 13.)

After three years the television set is valued at \$512.

1.6 Loans

1.6.1 Loans involving flat-rate interest

Loans involving flat-rate interest involve simple interest which is calculated on the orig-
nal amount borrowed, regardless of the amount of money owing.

Examples

(a) Peter borrowed \$4000 from the bank and was charged flat-rate interest of 11% per annum. He was allowed to repay the loan in 36 monthly instalments. Find the amount of each instalment.

SOLUTION

Note: This is similar to Example (c) in Section 1.3.2.

Interest $= \$4000 \times 0.11 \times 3$
$= \$1320$

Monthly instalment
$= (\$4000 + \$1320) \div 36$
$= \$147.78$ (to the nearest cent)

Peter repays \$147.78 in each instal

ment was \$300. If she was charged fla
rate interest of 10% per annum, find he
original loan.

SOLUTION

Total repayments $= \$300 \times 24$
$= \$7200$

Let original loan $= L$

Interest
$= L \times 10\% \times$

$$L + L \times 10\% \times 2 = 7200$$
$$L + L \times 0.1 \times 2 = 7200$$
$$L + 0.2L = 7200$$
$$L(1 + 0.2) = 7200$$
$$L(1.2) = 7200$$
$$1.2L = 7200$$

1.6.2 Loans involving reducible interest

The vast majority of loans involve reducible interest, that is, interest is calculated on the amount owing at specified times throughout the loan period.

Examples

(a) Brad borrows \$9000 from a bank and agrees to meet the annual repayments of \$2100. He is charged interest at 14% per annum reducible. How much will he owe the bank after two repayments?

SOLUTION: Each year Brad is charged interest on the balance owing and then makes his annual repayment.

Amount owing after one year
$= \$9000 \times 1.14 - \$2100$
$= \$8160.$
Amount owing after two years
$= \$8160 \times 1.14 - \$2100$
$= \$7202.40.$

Brad still owes \$7202.40 at the end of two years.

(b) Carolyn decides to borrow \$72 000 for a home loan. At the end of each month, interest is calculated before the monthly repayment is made. The interest rate is 15% per annum, monthly reducible. She decides to repay the loan at \$950 per month. How much will she owe after her first monthly repayment?

SOLUTION: 15% per annum means $\frac{15}{12}$% per month, that is, 1.25% per month.

$\boxed{\textit{Note: } 1.25\% = 0.0125}$

Amount owing after one month
$= \$72\,000 \times 1.0125 - \$950$
$= \$71\,950.$

[Note that after repaying \$950, only \$50 has come off the borrowed amount.]

1.7 Sales

1.7.1 Sales discount

Examples

(a) At a sale offering discounts of 15% for cash, Ross purchased a CD player selling for $220. Find the cash price paid by Ross.

SOLUTION

100% – 15%
= 85%

Cash price = 85% of $220
= 0.85 × 220
= $187

Ross will pay $187.

Note: We could have found 15% of $220, and subtracted this from $220.

(b) Paul, a plumber, purchases supplies from a hardware store and gains a trade discount of 12%. His purchases total $145 and he is allowed a further discount of 8% for paying cash. Find his final bill.

SOLUTION

Cost after trade discount
= 88% of $145
= $127.60

Cost after further discount
= 92% of $127.60
= $117.39 (to the nearest cent)

Paul has to pay $117.39.

[*Note:* This is different to simply adding the two discounts.]

(c)

20% off

STOREWIDE SALE

Elene paid $38.40 for a cutlery set at a sale which offered 20% discount on all goods. How much did Elene save on the original price?

SOLUTION: Elene's $38.40 represents 80% of the original price.

80% of original price = $38.40
20% of original price = $38.40 ÷ 4
= $9.60
100% of original price = $9.60 × 5
= $48

The original price was $48.

> *Note:* We could have brought it down to 10%, or 1%, etc., rather than 20%.

1.7.2 Sales profit and loss

1.7.2 Sales profit and loss

Examples

(a) A dealer buys a car for \$7200 and resells it for \$9600. Find the profit expressed as a percentage of the cost price.

SOLUTION:

$$\text{Profit} = \$9600 - \$7200$$
$$= \$2400$$
$$\text{Profit as \% (cost)} = \frac{2400}{7200} \times 100$$
$$= 33.\dot{3}\%$$

Profit is $33\frac{1}{3}\%$.

Note:

- Profit as % (cost)

$$= \frac{\text{profit}}{\text{cost}} \times 100\%$$

- Profit as % (selling)

$$= \frac{\text{profit}}{\text{selling}} \times 100\%$$

(b) Jordie purchased a case of tomatoes for \$4.60 and sold them, making a profit of 24% on the cost price. Find the price at which Jordie sold the case.

SOLUTION: Price $= 1.24 \times \$4.60$

$= \$5.70$ (to the nearest cent)

Jordie sold the case for \$5.70.

(c) A shoe shop marks up the price of shoes by 50%. If Lyndy buys a pair of shoes from the shop for \$60, what was the shop's profit?

SOLUTION

As the shop's cost price = 100%
shop's selling price = 150%.
150% of cost price = $60
1% of cost price = $60 + 150
= $0.40 (to the nearest cent)
100% of cost price = $0.40 × 100
= $40 (to the nearest cent)
Store profit = $60 – $40
= $20

Store profit is $20.