

# Chapter 3

## REAL NUMBERS, SURDS AND INDICES

### 3.1 Real numbers

#### 3.1.1 Rational numbers

A number is rational if it can be expressed in terms of  $\frac{a}{b}$  where  $a$  and  $b$  are integers, and  $b \neq 0$ .

Examples:

Examples of rational numbers are:

$$\frac{4}{7}, -\frac{2}{3}, 1\frac{2}{3}\left(=\frac{5}{3}\right), 27\%\left(=\frac{27}{100}\right), 2\left(=\frac{2}{1}\right), \sqrt{9}\left(=3=\frac{3}{1}\right), 4:5\left(=\frac{4}{5}\right) \text{ etc.}$$



### 3.1.2 Converting fractions to decimals

Examples:

(a)  $\frac{3}{8}$    (b)  $\frac{4}{9}$    (c)  $\frac{8}{11}$

SOLUTION

(a)

$$\begin{array}{r} 0.375 \\ 8 \overline{) 3.000 } \end{array}$$

$$\therefore \frac{3}{8} = 0.375$$

(this is a terminating decimal)

(b)

$$\begin{array}{r} 0.444\dots \\ 9 \overline{) 4.000\dots } \end{array}$$

$\therefore \frac{4}{9} = 0.4$  (this is a repeating or recurring decimal)

(c)

$$\begin{array}{r} 0.7272\dots \\ 11 \overline{) 8.000\dots } \end{array}$$

$$\therefore \frac{8}{11} = 0.\dot{7}\dot{2}$$



### 3.1.3 Converting simple recurring decimals to fractions

Look at this pattern:  $\frac{1}{9} = 0.\overline{1}$ ,  $\frac{2}{9} = 0.\overline{2}$ ,  $\frac{3}{9} = 0.\overline{3}$ , and so on.

Examples: Express as rationals, that

is in form  $\frac{a}{b}$ :

- (a) 0.7 (b) 0.9 (c) 3.6

SOLUTION

(a)  $0.7 = \frac{7}{9}$



(b)  $0.\dot{9} = \frac{9}{9} = 1$  (we take  
 $0.\dot{9} = 0.9999\dots$  as equalling 1)

(c)  $3.\dot{6} = 3\frac{6}{9}$   
 $= 3\frac{2}{3}.$

### 3.1.4 Real numbers

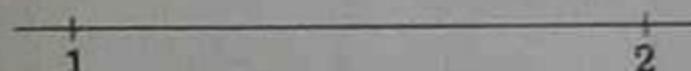
A real number can be represented on a number line and combines rational numbers with irrational numbers (such as  $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{5}$ ,  $\pi$ , etc.).

**Examples:** Using your calculator for assistance, position the following real numbers on a number line

(a)  $3, \sqrt{5}, 2.5$

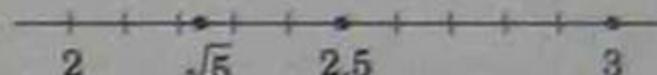


(b)  $118\%, \sqrt{2}, 7:4$



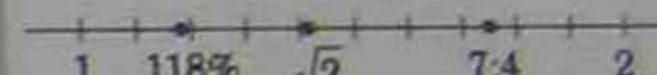
#### SOLUTION

(a) In ascending order:  $\sqrt{5} = 2.23, 2.5, 3$



(b) In ascending order:

$$118\% = 1.18, \sqrt{2} = 1.41, 7:4 = 1.75$$



## 3.2 Surds

Surds are numerical expressions which involve irrational numbers.

### 3.2.1 Approximation of surds

The calculator can be used to approximate surds.

#### Example

Arrange the following numbers in order, from smallest to largest:

3,  $\sqrt{5}$ ,  $\sqrt{7}$ , 2, 4

#### SOLUTION:

$$3 = \sqrt{9}, \sqrt{5}, \sqrt{7}, 2 = \sqrt{4}, 4 = \sqrt{16}$$

In order:  $\sqrt{4}, \sqrt{5}, \sqrt{7}, \sqrt{9}, \sqrt{16}$

That is, 2,  $\sqrt{5}$ ,  $\sqrt{7}$ , 3, 4.



### 3.2.2 Rules for surds

$$\bullet \quad \sqrt{ab} = \sqrt{a} \times \sqrt{b}$$
$$\bullet \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$
$$\bullet \quad (\sqrt{a})^2 = a$$

#### Examples

Simplify the following:

(a)  $\sqrt{48}$

(b)  $3\sqrt{12}$

(c)  $\sqrt{\frac{49}{64}}$

(d)  $(\sqrt{16})^2$

Note: We look for two factors, the first of which is a perfect square.

#### SOLUTIONS

(a) 
$$\begin{aligned}\sqrt{48} &= \sqrt{16} \times \sqrt{3} \\ &= 4 \times \sqrt{3} \\ &= 4\sqrt{3}\end{aligned}$$



(b) 
$$\begin{aligned}2\sqrt{12} &= 3 \times \sqrt{4} \times \sqrt{3} \\&= 3 \times 2 \times \sqrt{3} \\&= 6\sqrt{3}\end{aligned}$$

(c)  $\sqrt{\frac{49}{64}} = \frac{7}{8}$

(d)  $(\sqrt{16})^2 = 4^2 = 16$

### 3.2.3 Addition and subtraction of surds

We can add or subtract only like terms.

In algebra, like terms are  $3a$ ,  $7a$ ,  $2a$ , etc. — like surds are  $\sqrt{3}$ ,  $3\sqrt{3}$ ,  $-7\sqrt{3}$ , etc.

**Examples:** Simplify:

(a)  $4\sqrt{3} + 2\sqrt{3} - \sqrt{3}$

(b)  $2\sqrt{5} + \sqrt{20}$

(c)  $5\sqrt{7} - \sqrt{63} + 2\sqrt{28}$

(d)  $\sqrt{a^3} + 3a\sqrt{a}$

We may need to simplify surds before adding or subtracting.

**SOLUTIONS**

(a)  $4\sqrt{3} + 2\sqrt{3} - \sqrt{3} = 5\sqrt{3}$

(b) 
$$\begin{aligned}2\sqrt{5} + \sqrt{20} &= 2\sqrt{5} + \sqrt{4} \times \sqrt{5} \\&= 2\sqrt{5} + 2\sqrt{5} \\&= 4\sqrt{5}\end{aligned}$$

(c) 
$$\begin{aligned}5\sqrt{7} - \sqrt{63} + 2\sqrt{28} &= 5\sqrt{7} - \sqrt{9} \times \sqrt{7} + 2 \times \sqrt{4} \times \sqrt{7} \\&= 5\sqrt{7} - 3\sqrt{7} + 4\sqrt{7} \\&= 6\sqrt{7}\end{aligned}$$

(d) 
$$\begin{aligned}\sqrt{a^3} + 3a\sqrt{a} &= \sqrt{a^2} \times \sqrt{a} + 3a\sqrt{a} \\&= a\sqrt{a} + 3a\sqrt{a} \\&= 4a\sqrt{a}\end{aligned}$$

### 3.2.4 Multiplication and division of surds

We reverse our surd rules:

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

#### Examples

Simplify:

(a)  $\sqrt{5} \times \sqrt{5}$

(b)  $\sqrt{7} \times \sqrt{3}$

(c)  $2\sqrt{3} \times \sqrt{3}$

(d)  $3\sqrt{5} \times 2\sqrt{7}$

(e)  $(3\sqrt{2})^2$

(f)  $\sqrt{4a+4}$

(g)  $\sqrt{54} + \sqrt{18}$

(h)  $\frac{3\sqrt{2} \times \sqrt{6}}{\sqrt{3}}$

Expand and simplify:

(i)  $\sqrt{3}(2 - \sqrt{3})$

(j)  $4\sqrt{2}(\sqrt{2} - 1)$

#### SOLUTIONS

(a)  $\sqrt{5} \times \sqrt{5} = \sqrt{25}$   
= 5

(b)  $\sqrt{7} \times \sqrt{3} = \sqrt{21}$

(c)  $2\sqrt{3} \times \sqrt{3} = 2 \times 3 = 6$

(d)

$$3\sqrt{5} \times 2\sqrt{7} = 6\sqrt{35}$$

(e)

$$(3\sqrt{2})^2 = 3\sqrt{2} \times 3\sqrt{2}$$

$$= 9\sqrt{4}$$

$$= 9 \times 2$$

$$= 18$$

(f)

$$\begin{aligned}\sqrt{4a+4} &= \sqrt{4(a+1)} \\ &= \sqrt{4} \times \sqrt{a+1} \\ &= 2\sqrt{a+1}\end{aligned}$$

(g)

$$\begin{aligned}\sqrt{54} + \sqrt{18} &= \sqrt{\frac{54}{18}} \\ &= \sqrt{3}\end{aligned}$$

(h)

$$\begin{aligned}\frac{3\sqrt{2} \times \sqrt{6}}{\sqrt{3}} &= \frac{3\sqrt{12}}{\sqrt{3}} \\ &= \frac{3 \times \sqrt{4} \times \sqrt{3}}{\sqrt{3}} \\ &= 3 \times 2 \\ &= 6\end{aligned}$$



$$\begin{aligned}(i) \quad \sqrt{3}(2 - \sqrt{3}) &= \sqrt{3} \times 2 - \sqrt{3} \times \sqrt{3} \\&= 2\sqrt{3} - \sqrt{9} \\&= 2\sqrt{3} - 3\end{aligned}$$

$$\begin{aligned}(j) \quad 4\sqrt{2}(\sqrt{2} - 1) &= 4\sqrt{2} \times \sqrt{2} - 4\sqrt{2} \times 1 \\&= 4 \times \sqrt{4} - 4\sqrt{2} \\&= 4 \times 2 - 4\sqrt{2} \\&= 8 - 4\sqrt{2}\end{aligned}$$

### 3.2.5 Binomial products

We can use the following three rules:

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a - b)(a + b) = a^2 - b^2$



### Examples

Expand and simplify:

(a)  $(\sqrt{2} + 1)^2$

(b)  $(\sqrt{3} - \sqrt{5})^2$

(c)  $(\sqrt{2} - 1)(\sqrt{2} + 1)$

(d)  $(2\sqrt{3} - 1)(\sqrt{5} - \sqrt{2})$

### SOLUTIONS

(a) 
$$\begin{aligned}(\sqrt{2} + 1)^2 &= (\sqrt{2})^2 + 2(\sqrt{2})(1) + 1^2 \\&= 2 + 2\sqrt{2} + 1 \\&= 3 + 2\sqrt{2}\end{aligned}$$

(b) 
$$\begin{aligned}(\sqrt{3} - \sqrt{5})^2 &= (\sqrt{3})^2 - 2(\sqrt{3})(\sqrt{5}) + (\sqrt{5})^2 \\&= 3 - 2\sqrt{15} + 5 \\&= 8 - 2\sqrt{15}\end{aligned}$$

(c) 
$$\begin{aligned}(\sqrt{2} - 1)(\sqrt{2} + 1) &= (\sqrt{2})^2 - 1^2 \\&= 2 - 1 \\&= 1\end{aligned}$$

(d) 
$$\begin{aligned}(2\sqrt{3} - 1)(\sqrt{5} - \sqrt{2}) &= 2\sqrt{3}(\sqrt{5} - \sqrt{2}) - 1(\sqrt{5} - \sqrt{2}) \\&= 2\sqrt{15} - 2\sqrt{6} - \sqrt{5} + \sqrt{2}\end{aligned}$$



### 3.2.6 Rationalising the denominator

If the denominator of a fraction is irrational, we can multiply 'top and bottom' by the same surd to *rationalise* the denominator.

**Examples:** Rationalise the denominator.

(a)  $\frac{\sqrt{3} + 1}{\sqrt{2}}$

(b)  $\frac{\sqrt{2} - 1}{5\sqrt{3}}$

(c)  $\frac{2}{\sqrt{3} - 1}$

(d)  $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$

SOLUTIONS

(a) 
$$\frac{\sqrt{3} + 1}{\sqrt{2}} = \frac{\sqrt{3} + 1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{2}(\sqrt{3} + 1)}{\sqrt{4}}$$
$$= \frac{\sqrt{6} + \sqrt{2}}{2}$$

(b) 
$$\frac{\sqrt{2} - 1}{5\sqrt{3}} = \frac{\sqrt{2} - 1}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{\sqrt{3}(\sqrt{2} - 1)}{5\sqrt{9}}$$
$$= \frac{\sqrt{6} - \sqrt{3}}{15}$$

Note: We don't need to multiply by  $5\sqrt{3}$ ;  $\sqrt{3}$  will do.



$$\begin{aligned}
 (c) \quad \frac{2}{\sqrt{3}-1} &= \frac{2}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \\
 &= \frac{2\sqrt{3}+2}{\sqrt{9}-1} \\
 &= \frac{2\sqrt{3}+2}{3-1} \\
 &= \frac{2\sqrt{3}+2}{2} \\
 &= \frac{12(\sqrt{3}+1)}{2} \\
 &= \sqrt{3}+1
 \end{aligned}$$

*Note:* We multiply by the *conjugate* of the denominator — the same expression but with the opposite sign between the terms.

(d)

$$\begin{aligned}
 \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} &= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} \\
 &= \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} \\
 &= \frac{(\sqrt{3})^2 - 2(\sqrt{3})(\sqrt{2}) + (\sqrt{2})^2}{(\sqrt{3})^2 - (\sqrt{2})^2} \\
 &= \frac{3-2\sqrt{6}+2}{3-2} \\
 &= \frac{5-2\sqrt{6}}{1} \\
 &= 5-2\sqrt{6}
 \end{aligned}$$



### 3.3 Indices

For  $x^n$ ,  $x$  is the base, and  $n$  is the index.

Example:

Use your calculator to evaluate  $3^7$ .

SOLUTION: We use our  $x^y$  button.

$3^7 = 2187$ , that is,  $3 \boxed{x^y} 7 \boxed{=}$

#### 3.3.1 Factorising integers

Examples: Express as a product of their prime factors in index form:

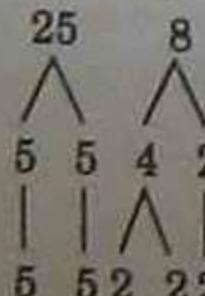
(a) 200

(b) 432



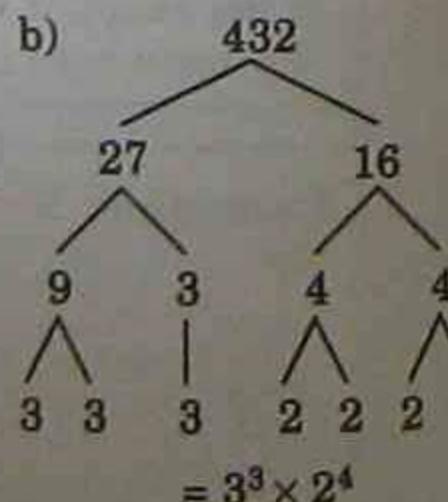
SOLUTIONS

a) 200



$$\therefore = 5^2 \times 2^3$$

b)



$$= 3^3 \times 2^4$$

### 3.3.2 Index rules

- $x^a \times x^b = x^{a+b}$
- $x^a + x^b = x^{a-b}$
- $(x^a)^b = x^{ab}$
- $x^0 = 1 \quad (\text{Note: } x \neq 0)$



**Examples:** Simplify:

(a)  $p^4 \times p^2$

(c)  $4^5 + 4^3$

(e)  $(x^2)^4$

(g)  $(4y^2)^3$

(i)  $(3a)^0 + 3a^0$

(b)  $p^6 + p$

(d)  $12a^4b + 3a^3$

(f)  $(a^{\frac{1}{4}})^3$

(h)  $4x^0$

$4^5 \div 4^3 = 4^{5-3}$

$= 4^2$

$= 16$

(d)  $12a^4b + 3a^3 = 4ab$

(e)  $(x^2)^4 = x^{2 \times 4}$

$= x^8$

(f)  $(a^{\frac{1}{4}})^3 = a^{\frac{3}{4}}$

(g)  $(4y^2)^3 = 4^3 \times (y^2)^3$

$= 64y^6$

(h)  $4x^0 = 4 \times x^0$

$= 4 \times 1$

$= 4$

(i)  $(3a)^0 + 3a^0 = 1 + 3 \times 1$

$= 1 + 3$

$= 4$

### SOLUTIONS

(a) 
$$\begin{aligned}p^4 \times p^2 &= p^{(4+2)} \\&= p^6\end{aligned}$$

(b) 
$$\begin{aligned}p^6 + p &= p^6 + p^1 \\&= p^{6-1} \\&= p^5\end{aligned}$$



### 3.3.3 Negative powers

$$\bullet \quad x^{-a} = \frac{1}{x^a}$$

Examples: Simplify:

(a)  $4^{-2}$

(b)  $\left(\frac{1}{2}\right)^{-3}$

(c)  $3x^{-2}$

(d)  $(5x^3)^{-2}$

(e)  $\frac{x^2}{x^5}$

SOLUTIONS

(a)  $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

(b)  $\left(\frac{1}{2}\right)^{-3} = \frac{1}{\left(\frac{1}{2}\right)^3} = \frac{1}{\frac{1}{8}}$

$$= 1 + \frac{1}{8}$$

$$= 1 \times \frac{8}{1}$$

$$= 8$$

Note: The negative power is the reciprocal.

$$\begin{aligned}3x^{-2} &= 3 \times x^{-2} \\&= 3 \times \frac{1}{x^2} \\&= \frac{3}{x^2}\end{aligned}$$

$$\begin{aligned}(5x^3)^{-2} &= \frac{1}{(5x^3)^2} \\&= \frac{1}{25x^6}\end{aligned}$$

$$\begin{aligned}\frac{x^2}{x^5} &= x^2 + x^5 \\&= x^{-3}\end{aligned}$$



### 3.3.4 Fractional powers

- $x^{\frac{1}{a}} = \sqrt[a]{x}$

That is,  $x^{\frac{1}{2}} = \sqrt{x}$ ,  $x^{\frac{1}{3}} = \sqrt[3]{x}$ ,

$x^{\frac{1}{n}}$  =  $n^{\text{th}}$  power of  $x$

- $x^{\frac{a}{b}} = \sqrt[b]{x^a}$



**Examples:** Simplify:

(a)  $4^{\frac{1}{2}}$

(b)  $8^{\frac{1}{3}}$

(c)  $4a^{\frac{1}{2}}$

(d)  $(16y)^{\frac{1}{2}}$

(e)  $16^{\frac{3}{2}}$

(f)  $16^{-\frac{3}{4}}$

(g)  $x\sqrt{x}$

(h)  $\frac{x}{\sqrt{x}}$

**SOLUTIONS**

(a)  $4^{\frac{1}{2}} = \sqrt{4} = 2$

(b)  $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

(c)  $4a^{\frac{1}{2}} = 4\sqrt{a}$

(d)  $(16y)^{\frac{1}{2}} = 16^{\frac{1}{2}} \times y^{\frac{1}{2}} = \sqrt{16} \times \sqrt{y} = 4\sqrt{y}$

(e)  $16^{\frac{3}{2}} = \left(16^{\frac{1}{2}}\right)^3 = 4^3 = 64$

To keep numbers small it is best to take the root first and then the power.

$$\begin{aligned}
 (f) \quad 16^{-\frac{3}{4}} &= \left( \left( 16^{-1} \right)^{\frac{1}{4}} \right)^3 \\
 &= \left( \left( \frac{1}{16} \right)^{\frac{1}{4}} \right)^3 \\
 &= \left( \frac{1}{2} \right)^3 \\
 &= \frac{1}{8}
 \end{aligned}$$

It's getting complicated — let's try the calculator!

$$\begin{aligned}
 (g) \quad x\sqrt{x} &= x \times x^{\frac{1}{2}} \\
 (h) \quad \frac{x}{\sqrt{x}} &= x + x^{\frac{1}{2}} \\
 &= x^1 \times x^{\frac{1}{2}} \\
 &= x^{1\frac{1}{2}} \\
 &= x^{\frac{3}{2}} \\
 &= x^{\frac{3}{2}} \\
 &= \sqrt{x^3}
 \end{aligned}$$



### 3.3.5 Using calculators to evaluate difficult indices

Calculators can be used to evaluate difficult indices.

**Examples:** Evaluate:

(a)  $256^{\frac{1}{8}}$

(b)  $\left(\frac{1}{16}\right)^{-\frac{3}{2}}$

(b)  $\left(\frac{1}{16}\right)^{-\frac{3}{2}} = 64$

**SOLUTIONS**

(a)  $256^{\frac{1}{8}} = 2$

Either use your  $x^{\frac{1}{n}}$  key, or use your  $x^y$  key and enter the power as a fraction.



### 3.3.6 Solving algebraic problems involving indices

**Example:** Solve for  $x$ :

(i)  $2^x = 64$

(ii)  $3^{x+2} = 81$

(iii)  $2^{2x-1} = 3125$

① Express both sides as a power of the same base number.

② Then the indices must be equal.

#### SOLUTION

(i)  $2^x = 2^6$  Both as powers of 2.  
 $\therefore x = 6$  Indices are equal

(ii)  $3^{x+2} = 3^4$  Both as powers of 3.  
 $\therefore x + 2 = 4$  Indices are equal  
 $x = 2$

(iii)  $25^{2x-1} = 3125$   
 $(5^2)^{2x-1} = 5^5$   
 $5^{4x-2} = 5^5$  Powers of 5  
 $\therefore 4x - 2 = 5$   
 $4x = 7$   
 $x = \frac{7}{4}$



## **3.4 Standard, or scientific, notation**

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### **3.4.1 Definition**

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Very large, or very small, numbers are written as a product of a number between 1 and 10, and a power of ten.



**Examples:** Express in scientific notation:

(a) 181.4

(b) 2 000 000

(c) 0.0215

(d) 0.0001

Express in ordinary, or decimal, notation:

(e)  $4.271 \times 10^2$

(f)  $3.08 \times 10^4$

(g)  $1.56 \times 10^{-2}$

(h)  $5 \times 10^{-3}$

## SOLUTIONS

(a)  $181.4 = 1\overset{1}{8}1.4,$

therefore

$= 1.814 \times 100$

2 places

$= 1.814 \times 10^2$



**Note:** The number of decimal places moved by the decimal point equals the power of ten.

(b)  $2\ 000\ 000 = 2.000\ 000$ , therefore  
 $= 2 \times 1000\ 000$  6 places

$$= 2 \times 10^6$$

(c)  $0.0215 = 0.0215$ , therefore  
 $= 2.15 \div 100$  2 places.

$$= 2.15 \times \frac{1}{100}$$

$$= 2.15 \times 10^{-2}$$



(a)

$$0.0001 = \overset{\text{****}}{0.0001}, \quad \text{therefore 4}$$

$$= 1 \div 10\ 000 \quad \text{places.}$$

$$= 1 \times \frac{1}{10\ 000}$$

$$= 1 \times 10^{-4}$$

(e)

$$4.271 \times 10^2 = 4.271 \times 100, \quad \text{therefore 2}$$

$$= 4.\overset{\text{2}}{2}7.1 \quad \text{places.}$$

$$= 427.1$$

(f)

$$3.08 \times 10^4 = 3.08 \times 10\ 000, \quad \text{therefore 4}$$

$$= 3.\overset{\text{0}}{0}800 \quad \text{places.}$$

$$= 30\ 800$$

(g)

$$\begin{aligned}
 1.56 \times 10^{-2} &= 1.56 \times \frac{1}{100} \\
 &= 1.56 \div 100, \quad \text{therefore 2} \\
 &= 0.\overline{0}1.56 \\
 &= 0.0156
 \end{aligned}$$

(h)



$$\begin{aligned}
 5 \times 10^{-3} &= 5 \times \frac{1}{1000} \\
 &= 5 \div 1000, \quad \text{therefore 3} \\
 &= 0.\overline{0}05 \\
 &= 0.005
 \end{aligned}$$


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### 3.4.2 Standard notation and the calculator

A scientific calculator can leave very large, or very small, numbers in scientific (or standard) notation.

**Examples:** Calculate, leaving your answer in scientific notation:

(a)  $13\ 364 \times 176\ 000\ 000$

(b)  $(0.2)^4$

(c)  $43 + 14\ 767$

#### SOLUTIONS

(a)  $13\ 364 \times 176\ 000\ 000$   
 $= 2.352\ 064\ 12$  (calculator display)  
 $= 2.352\ 064 \times 10^{12}$

(b)  $(0.2)^4 = 1.6 - 03$  (calculator display)  
 $= 1.6 \times 10^{-3}$

(c)  $43 + 14\ 767$   
 $= 2.911\ 898\ 151 - 03$  (calc. display)  
 $= 2.911\ 898\ 151 \times 10^{-3}$



Often we can leave the answer correct to significant figures.

**Examples:** Express in scientific notation, correct to two significant figures:

(a)

76 294 320

(b) 0.004 768 7

**SOLUTIONS**

(a)

$$7.629\ 432 \times 10^7 \approx 7.6 \times 10^7$$

(b)

$$\begin{aligned}0.004\ 768\ 7 &= 4.7687 \times 10^{-3} \\&\approx 4.8 \times 10^{-3}\end{aligned}$$

*Note:* Standard notation can be used to correct numbers to a given significance. It allows the zero problem to be avoided.



**For example:**

- (a) Write 60.432 correct to three significant figures.

$$\begin{aligned}60.432 &= \underline{6.0432} \times 10^1 \\&= 6.04 \times 10^1 \\&= 60.4 \text{ (three significant figures)}\end{aligned}$$

(b)

- Write 0.004 27 correct to two significant figures.

$$\begin{aligned}0.004\ 27 &= 4.27 \times 10^{-3} \\&= 4.3 \times 10^{-3} \\&= 0.0043 \text{ (two significant figures)}\end{aligned}$$

