

Chapter 6

COORDINATE GEOMETRY

6.1 Important formulae

Three very important formulae used in coordinate geometry are:

1. The distance formula: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

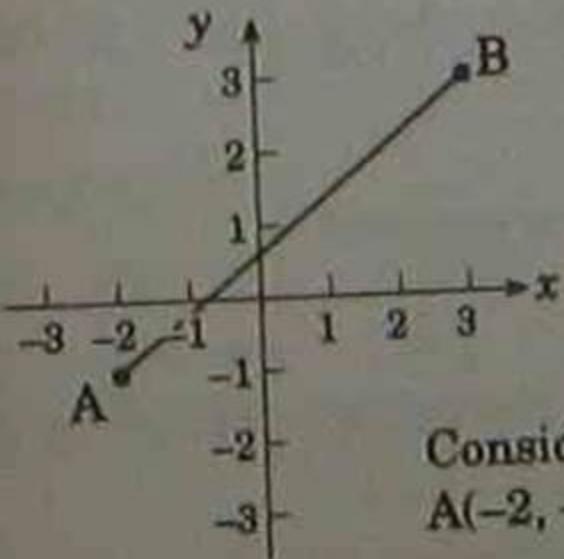
(Based on Pythagoras' Theorem)

2. The midpoint formula: $MP = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

3. The gradient formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

The distance formula is used to calculate the straight-line distance between points (x_1, y_1) and (x_2, y_2) . The midpoint formula is used to find the coordinates of the midpoint of the line segment joining two points. While the gradient formula is used to find the gradient

Examples



Consider the points
A(-2, -1) and B(3, 3).

Calculate:

- (a) Distance AB
- (b) Midpoint of AB
- (c) Gradient of AB.

SOLUTION

Put $(x_1, y_1) = (-2, -1)$; $(x_2, y_2) = (3, 3)$.

$$\boxed{\begin{array}{ll} x_1 = -2, & x_2 = 3 \\ y_1 = -1, & y_2 = 3 \end{array}}$$



Note: This is simply an application of Pythagoras' Theorem.

$$\begin{aligned}(a) \quad d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\&= \sqrt{(-2 - 3)^2 + (-1 - 3)^2} \\&= \sqrt{(-5)^2 + (-4)^2} \\&= \sqrt{25 + 16} \\&= \sqrt{41}\end{aligned}$$

AB is $\sqrt{41}$ units long.

Note: This is the exact distance AB.

$$\begin{aligned}(b) \quad MP &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\&= \left(\frac{-2 + 3}{2}, \frac{-1 + 3}{2} \right) \\&= \left(\frac{1}{2}, 1 \right)\end{aligned}$$

Average of
x-coordinates,
average of
y-coordinates.

Midpoint is $\left(\frac{1}{2}, 1 \right)$.

Continued



(c)

$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1} \\&= \frac{3 - (-1)}{3 - (-2)} \\&= \frac{4}{5}\end{aligned}$$

Gradient is $\frac{4}{5}$.

Difference in y
Difference in x

Note: The gradient is always left as a simple fraction. Do not convert either to a decimal or to a mixed number.

Remember when putting negative numbers on the calculator that it is the $+/_-$ button you use. For example, $3 - (-1)$ would involve

3 \square 1 $+/_-$ $=$

$(-3 - 2)^2$ is calculated:

3 $+/_-$ \square 2 $=$ INV $\sqrt{x^2}$



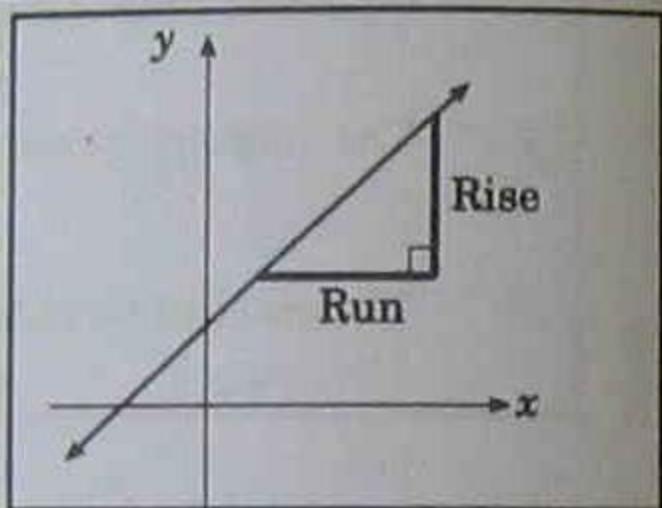
6.1.1 A note on gradient

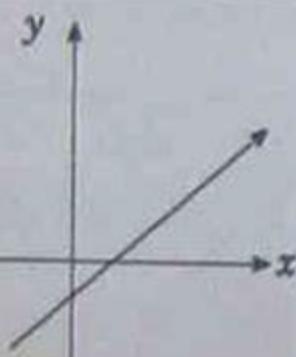
The gradient or slope of a line can be thought of

as $\frac{\text{vertical rise}}{\text{horizontal run}}$ or $\frac{\text{rise}}{\text{run}}$.

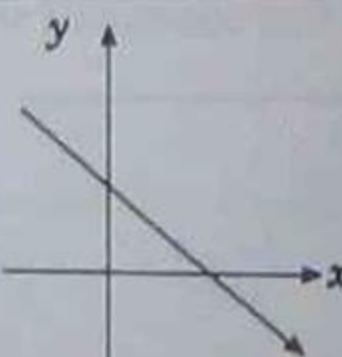
This is the reason why the y -values are on top in the formula.

The gradient of a line can be either positive or negative. A line sloping upward to the right has a positive gradient; one sloping downward to the right has a negative gradient.





Positive gradient



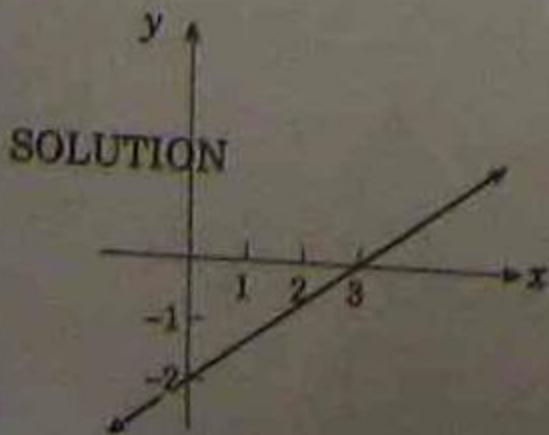
Negative gradient

The gradient can be found from the diagram by looking at the triangle formed by the line and the coordinate axes.



Examples

(a)

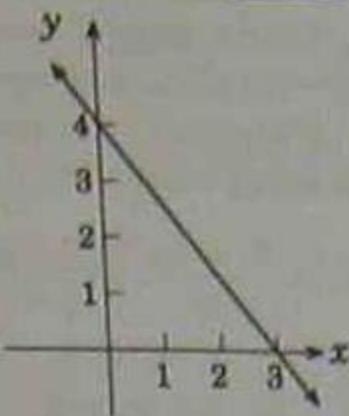


Rise = 2 (\uparrow)
Run = 3 (\rightarrow)
Direction is positive (\nearrow)

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{2}{3}.$$



(b)

**SOLUTION**

$$\text{Rise} = 4 \quad (\uparrow)$$

$$\text{Run} = 3 \quad (\rightarrow)$$

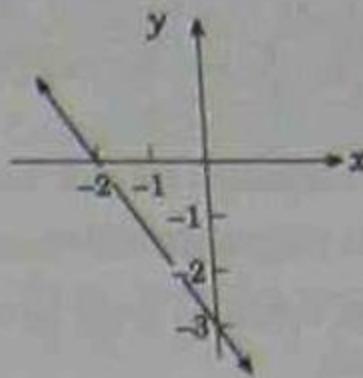
Direction is negative (\searrow)

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

$$= -\frac{4}{3}.$$



(c)

**SOLUTION**

$$\text{Rise} = 3$$

$$\text{Run} = 2$$

Direction is negative (\searrow)

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

$$= -\frac{3}{2}.$$

Remember: Take rise and run as positive lengths and then consider the sign of the gradient by observing the direction of the line.

6.1.2 Use of the formulae

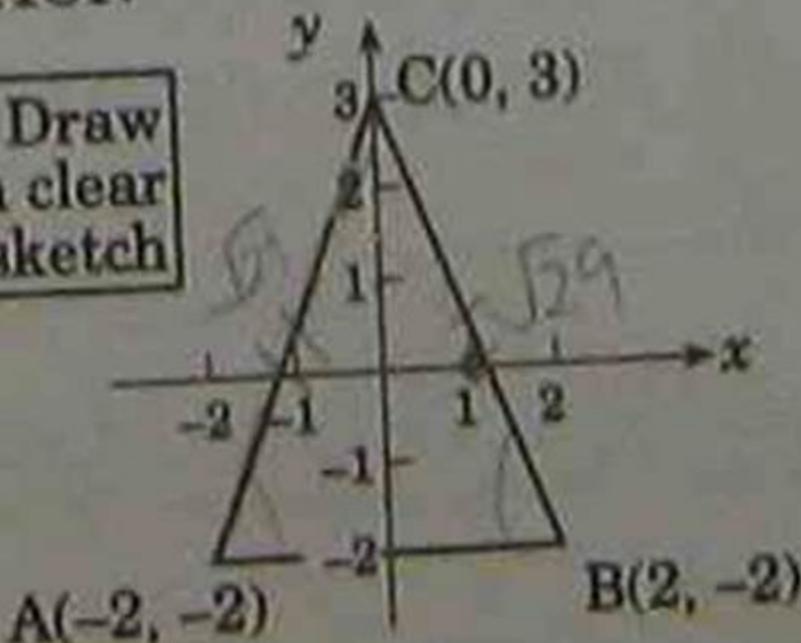
Examples

- (a) Plot the points A(-2, -2), B(2, -2) and C(0, 3) on a number plane. Prove that:
- Length AC is $\sqrt{29}$ units.
 - $\triangle ABC$ is isosceles.
 - The midpoint of AB lies on the y-axis.



SOLUTION

Draw
a clear
sketch



(i)
$$\begin{aligned} d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(-2 - 0)^2 + (-2 - 3)^2} \\ &= \sqrt{4 + 25} \\ &= \sqrt{29} \text{ units} \end{aligned}$$

AC is $\sqrt{29}$ units.



(ii) For $\triangle ABC$ to be isosceles, two sides must have the same length.
Check the length of BC.

$$\begin{aligned}BC &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\&= \sqrt{(2 - 0)^2 + (-2 - 3)^2} \\&= \sqrt{4 + 25} \\&= \sqrt{29} \text{ units,} \\&\therefore AC = BC \text{ (both } \sqrt{29}).\end{aligned}$$

that is, $\triangle ABC$ is isosceles.

(iii)

$$\begin{aligned}\text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\&= \left(\frac{-2 + 2}{2}, \frac{-2 + -2}{2} \right) \\&= (0, -2).\end{aligned}$$

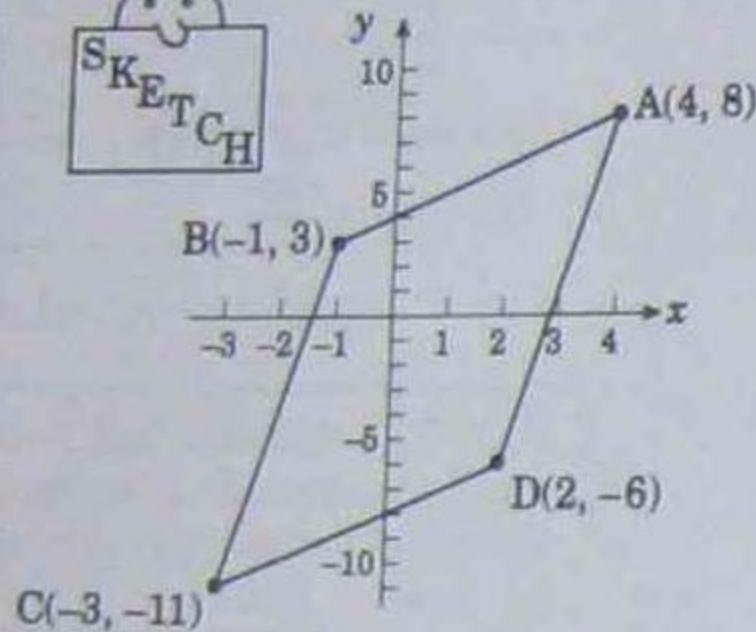
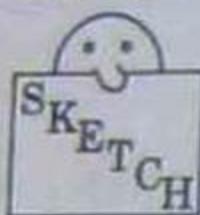
The point $(0, -2)$ lies on the y-axis as the x-coordinate is 0.



(b) The points $A(4, 8)$, $B(-1, 3)$, $C(-3, -11)$ and $D(2, -6)$ are the vertices of a parallelogram. Show that:

- The opposite sides are equal.
- The diagonals bisect each other.
- The opposite sides have equal gradients.

SOLUTION



(i) Using the distance formula:

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ in each case.

$$AB = \sqrt{(-1 - 4)^2 + (3 - 8)^2} = \sqrt{50}$$

$$CD = \sqrt{(-3 - 2)^2 + (-11 + 6)^2} = \sqrt{50}$$

$$\therefore AB = CD$$

$$AD = \sqrt{(4 - 2)^2 + (8 + 6)^2} = \sqrt{200}$$

$$BC = \sqrt{(-1 + 3)^2 + (3 + 11)^2} = \sqrt{200}$$

$$\therefore AD = BC.$$



The opposite sides are equal.

(ii) This example requires the midpoint of each diagonal, AC and BD. If they are the same, the diagonals must bisect each other. Using

$$MP = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \text{ in each case}$$

$$\begin{aligned} MP_{AC} &= \left(\frac{-3 + 4}{2}, \frac{-11 + 8}{2} \right) \\ &= \left(\frac{1}{2}, -\frac{3}{2} \right) \end{aligned}$$

$$\begin{aligned} MP_{BD} &= \left(\frac{-1 + 2}{2}, \frac{3 - 6}{2} \right) \\ &= \left(\frac{1}{2}, -\frac{3}{2} \right) \end{aligned}$$



therefore the diagonals intersect at $\left(\frac{1}{2}, -\frac{3}{2} \right)$, the midpoint of each diagonal.

The diagonals bisect each other.

(iii) Using in each case $m = \frac{y_2 - y_1}{x_2 - x_1}$,

$$m_{AB} = \frac{8 - 3}{4 - (-1)} = \frac{5}{5} = 1$$

$$m_{CD} = \frac{-11 - (-6)}{-3 - 2} = \frac{-5}{-5} = 1$$

$$m_{AD} = \frac{8 - (-6)}{4 - 2} = \frac{14}{2} = 7$$

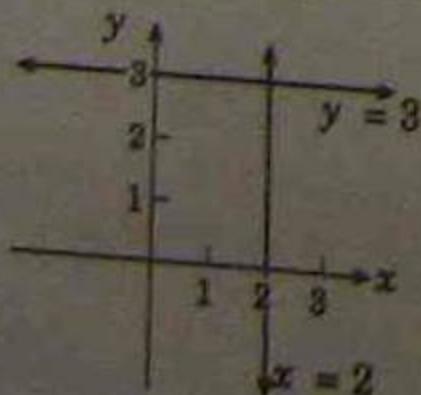
$$m_{BC} = \frac{3 - (-11)}{-1 - (-3)} = \frac{14}{2} = 7.$$



The opposite sides have equal gradients.

6.2 Forms of the straight-line equation

6.2.1 Lines parallel to the coordinate axes



Line parallel to x -axis: $y = b$

Line parallel to y -axis: $x = a$

The diagram shows the lines $y = 3$, parallel to the x -axis and $x = 2$, parallel to the y -axis.

