Chapter 9

SIMILARITY

9.1 Similar figures

Similar figures have the same shape. Similar figures are common in daily life. Building plans, surveyor's plans, road maps, town plans and photographic enlargements are all examples of similar figures. They are all cases where the original shapes have been reduced or enlarged in a given ratio. This is called the **scale** for plans and designs or the **enlargement factor** for other cases.

The scale can be calculated by finding the ratio of any length in the plan to the corresponding length in the original. For example, a block of land 120 m long is represented by a length 12 cm on the plan.

$$
\begin{aligned}
\text{Scale} &= 12 \text{ cm} : 120 \text{ m} \\
&= 120 \text{ mm} : 120\,000 \text{ mm} \\
&= 120 : 120\,000 \\
&= 1 : 1000
\end{aligned}
$$

(d) A $\frac{1}{50}$ scale model of the Boganbrey Town Hall requires 80 mL of paint to cover its outside walls. Calculate the quantity of paint needed for the outside walls of the actual Town Hall.

SOLUTION

Let the required area to be covered be A.

$$\text{Ratio of lengths} = 1:50$$
$$\text{Ratio of surface areas} = 1:50^2$$
$$= 1:2500$$

$$\boxed{\begin{array}{c} \text{Area of scale model} \\ \hline \text{Area of actual hall} \end{array}}$$

Then $\dfrac{0.08}{A} = \dfrac{1}{2500}$ $\boxed{80 \text{ mL} = 0.08 \text{ L}}$

$$A = 0.08 \times 2500$$
$$= 200$$

The Town Hall surface will require 200 L of paint.

Examples

(a) A scale model is made of a 40 m long yacht. If the scale model has a length of 20 cm, find the scale used.

SOLUTION

$$
\begin{aligned}
\text{Scale} &= 20 \text{ cm} : 40 \text{ m} \\
&= 20 \text{ cm} : 4000 \text{ cm} \\
&= 20 : 4000 \\
&= 1 : 200
\end{aligned}
$$

The scale used is 1 : 200.

(b) Beverley wishes to make a scale drawing of the Minmi Lighthouse.

She used a scale of 1 : 1000. If the actual height of the lighthouse is 58 metres, what will be the height in cm of the lighthouse in her scale drawing?

SOLUTION

Call the height of the drawing h cm.

Then

h cm : 58 m $\quad = 1 : 1000$

$\quad h : 5800 = 1 : 1000$

$$\frac{h}{5800} = \frac{1}{1000}$$

$\quad\quad 1000h = 5800$

$\quad\quad\quad h = 5.8.$

Rewrite ratios as equivalent fractions, then cross-multiply

The height on the scale drawing is 5.8 cm.

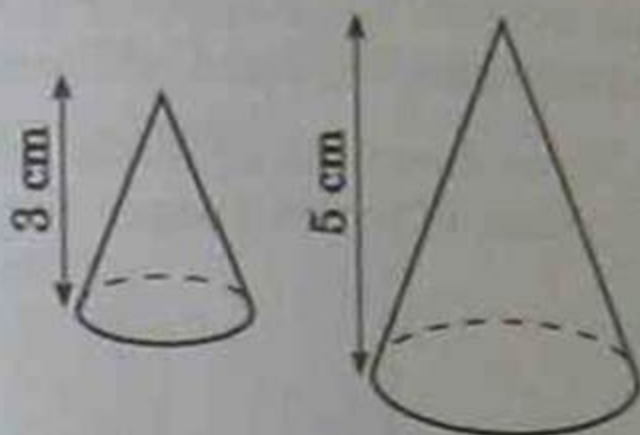## 9.5 Volumes of similar solids

Similar solids have the same shape, with the lengths of corresponding dimensions in the same ratio. Similar solids have:

- their surface areas proportional to the squares of the lengths of corresponding sides;

- their volumes proportional to the cubes of the lengths of corresponding sides.

Examples

(a)

For these similar cones, find the ratio of:

(i) their surface areas

(ii) their volumes.

SOLUTION

Ratio of heights $= \dfrac{3}{5}$

(i) Ratio of surface areas $= \dfrac{3^2}{5^2} = \dfrac{9}{25}$

(ii) Ratio of volumes $= \dfrac{3^3}{5^3} = \dfrac{27}{125}$

(b) If the volume of the larger cone in (a) is 500 cm^3, calculate the volume of the smaller cone.

SOLUTION
Let the volume be V cm^3.

$$\frac{V}{500} = \frac{27}{125}$$
$$125V = 27 \times 500$$
$$V = \frac{27 \times 500}{125}$$
$$= 108$$

The volume of the smaller cone is 108 cm^3.

(c) A scale model of a yacht displaces 800 cm^3 of water. If the scale model has a length of 10 cm, compared to the actual length of 20 m, find the volume of water displaced by the real yacht.

SOLUTION

Ratio of lengths = 10 cm : 20 m

$$= 10 : 2000$$

$$= 1 : 200$$

Ratio of volumes = $1 : (200)^3$

Let the volume of the actual yacht be V.

Then

$$\frac{800}{V} = \frac{1}{(200)^3}$$

$$\boxed{\frac{\text{plan}}{\text{actual}}}$$

$$V = 800 \times 200^3 \text{ cm}^3$$

$$= \frac{800 \times \cancel{8\,000\,000}}{\cancel{1\,000\,000}} \text{ m}^3$$

$$\boxed{\begin{array}{l} 1 \text{ m}^3 = 100 \times 100 \times 100 \text{ cm}^3 \\ \qquad = 1\,000\,000 \text{ cm}^3 \end{array}}$$

$$= 6400 \text{ m}^3$$

Actual displacement is 6400 m³.