

## Processes of science: Data analysis

### Background

The scientific method of solving the problems of the world involves the processes of describing, explaining and predicting scientific events. Each of these processes is likely to require the analysis of data. Data analysis involves gathering and interpreting information, usually by graphical means.

## Performing a data analysis by drawing a graph

### Equipment

HB pencil; rubber; protractor; ruler of 30.0 cm calibrated in mm; drawing compass; 1 mm graph paper 25 cm  $\times$  18 cm; scientific calculator.

## Skills required to draw graphs

Check that you can:

- (a) draw up a table;
- (b) describe the quantities in each column and/or row of your table by name, symbol and unit;
- (c) record information in your table;
- (d) draw a high quality graph to relate columns and or rows of data in the table. A high quality graph should:
  - (i) have a heading (or caption) e.g. 'Displacement of car versus time';
  - (ii) be constructed using a suitable scale, so that the graph covers most of the graph paper;
  - (iii) describe the quantity plotted on each axis by:
    - name, e.g. 'displacement of car along a straight road'
    - symbol, e.g. 's'
    - unit, e.g. 'm'
    - contain an accurate plot of each set of points as  $\odot$  or  $\ominus$  or  $\square$ ;

(iv) be drawn as a line of best fit, i.e. a smooth line through the points, all of which are on the line or equally spread either side of the line. Plotted points well off the line need to be checked and if necessary disregarded;

(v) not be drawn as a full line outside the data range;

(vi) be extrapolated by drawing a dotted line;

(c) identify:

(i) the slope,  $m = \text{rise} / \text{run}$ ;

(ii) intercepts on  $y$  and  $x$  axes;

(iii) the area under the graph.

## Skills required to interpret graphs

1. CHECK THAT YOU CAN INTERPRET a straight-line graph which shows a linear relationship between data:

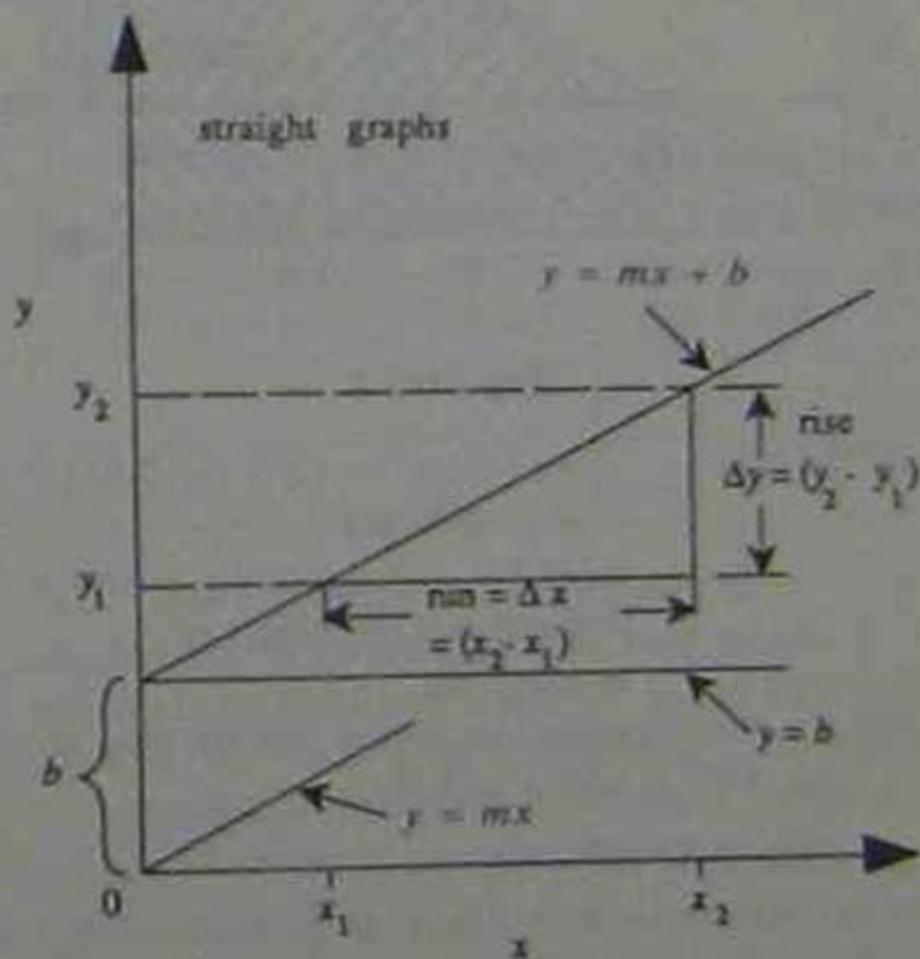


Fig. POS 1

xii STUDYMATE 2 UNIT PHYSICS

Determine from a straight-line graph:

(a) the slope or gradient,  $m = \Delta y / \Delta x$ ;

(b) the intercept on the  $y$  axis when  $x = 0$ ,  $y = b$ ;

(c) the area between the graph and the horizontal axis.

### EXAMPLE

$v \text{ (ms}^{-1}\text{)}$	7.0	12	19	25
$t \text{ (s)}$	1.0	2.0	3.0	4.0

Use the table of data giving the velocity  $v$  of an object at various times  $t$ , and

- plot a graph of  $v$  (vertical axis) against  $t$ ;
- calculate the slope (gradient) of the graph in (a).

Show your working and state the units of the slope. What is the physical significance of the slope?

- Extrapolate your graph and determine the intercept on the  $v$  axis. What is the physical significance of this intercept?
- Calculate the area under the graph from  $t = 1.0 \text{ s}$  to  $t = 4.0 \text{ s}$ . State the physical significance of the area.
- Write an equation for the line you have drawn.
- What is the change in  $s$  from  $t = 1.0 \text{ s}$  to  $t = 4.0 \text{ s}$ ?

*Answer*

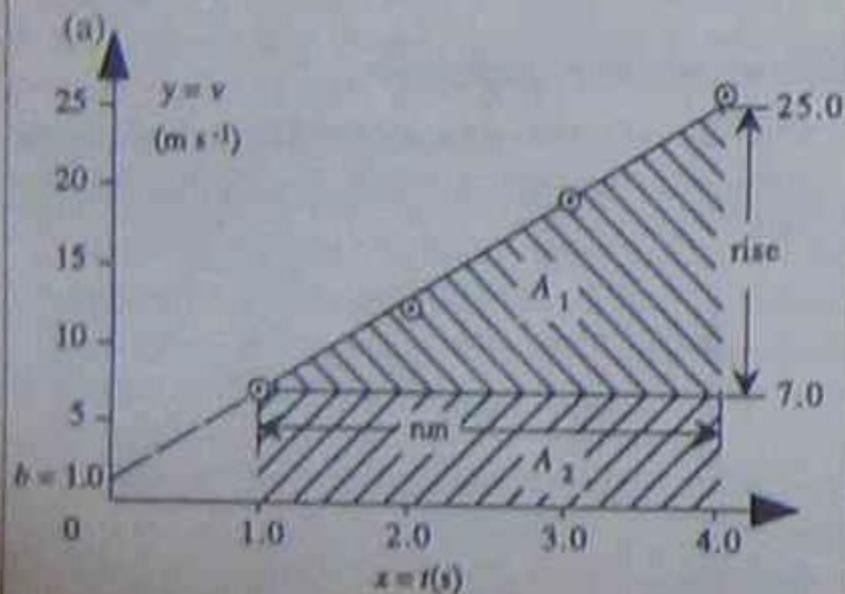


Fig. POS 2

(b) Slope = rise/run =  $(25 - 7.0) / (4.0 - 1.0) \text{ m s}^{-2}$   
 $= +18/3.0 = +6.0 \text{ m s}^{-2}$ ; a +ve slope means the object is accelerating.

(c)  $1.0 \text{ m s}^{-1}$ ; means the object was moving at  $1.0 \text{ m s}^{-1}$  at time  $t = 0.0 \text{ s}$ .

(d) Area  $A = A_1 + A_2$   
 $(A_1 = 1/2 \times 3.0 \times 18 \frac{\text{m}}{\text{s}} \times \text{s}$   
 $= 27 \text{ m}$   
 $A_2 = 3.0 \times 7 \text{ m} = 21 \text{ m})$

$$= (27 + 21) \text{ m}$$

$= 48 \text{ m}$ . This is numerically equal to the displacement of the object from  $t = 1.0 \text{ s}$  to  $t = 4.0 \text{ s}$ .

(e) The graph is a straight line of the form  $y = mx + b$  with  $y = v$ ;  $m = 6.0$ ;  $x = t$  and  $b = 1.0$ . The equation of the line is  $v = 6.0t + 1.0$ .

(f)  $48 \text{ m}$ —see answer to part (d).

## EXAMPLE

A system experiences a constant accelerating force. The acceleration of the system changes with its mass as shown in the following table:

mass $m$ (kg)	10	20	30	40
acceleration $a$ ( $\text{m s}^{-2}$ )	24	12	8	6
$1/a$ acceleration $1/a$ ( $\text{s}^2 \text{m}^{-1}$ )	0.042	0.083	0.125	0.166

- (a) Plot a fully labelled graph of  $m$  (vertical axis) against  $1/a$ .
- (b) Calculate the slope  $k$  of the graph in (a). What are the units of  $k$ ?
- (c) Write an equation between  $k$ ,  $m$  and  $a$ .
- (d) Compare your equation from (c) with the mathematical expression of Newton's Second Law and state the physical significance of  $k$ .

*Answer*

- (a) See Figure POS 5.

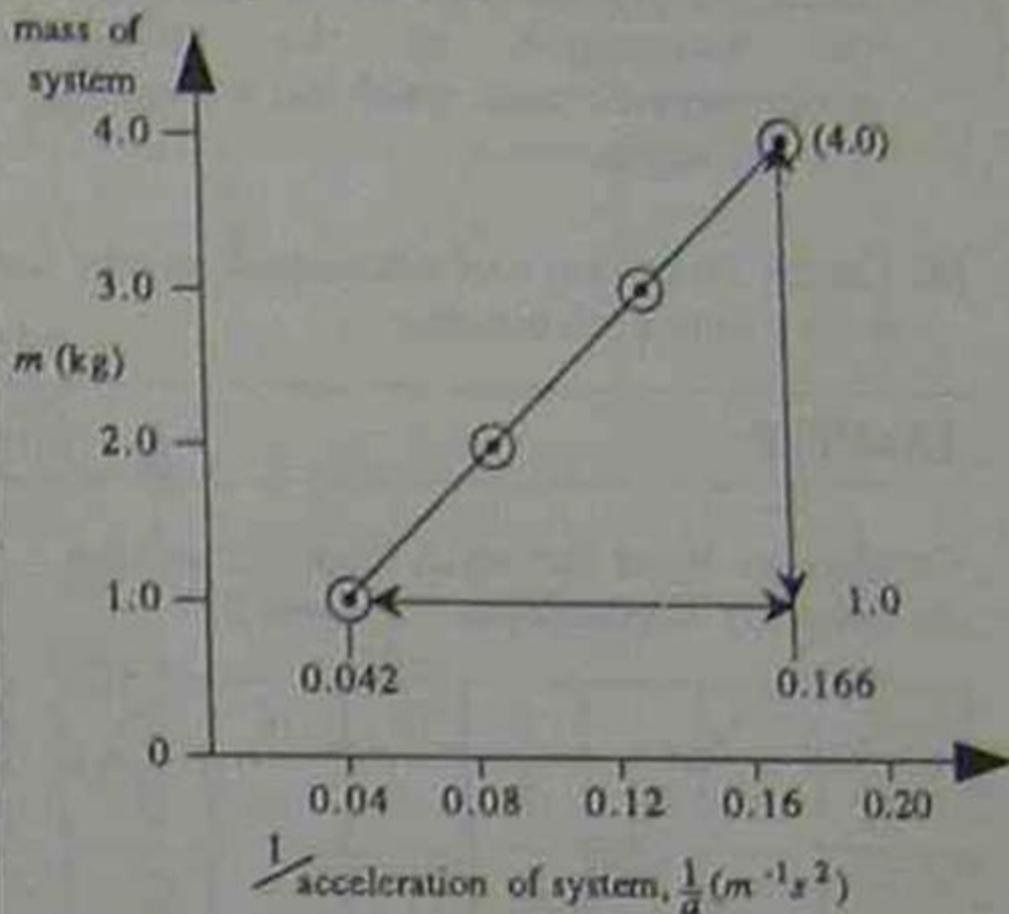


Fig. POS 5

$$\begin{aligned} \text{(b) } k &= (4.0 - 1.0)/(0.166 - 0.042) \\ &= 3.0/0.124 = 24. \end{aligned}$$

Units of  $k$  are  $m \text{ kg s}^{-2}$  or newton.

$$\text{(c) } k = ma \text{ or } 24 = ma.$$

(d) From Newton's Second Law,  $k$  is the net acceleration force.

## EXAMPLE

Consider an object that starts from rest and has the following displacements  $s$  at times  $t$ :

$s$ (m)	0	1	4	9	16	25
$t$ (s)	0	1	2	3	4	5
$t^2$ (s <sup>2</sup> )	0	1	4	9	16	25

- Plot  $s$  (vertical axis) against  $t$ .
- Plot  $s$  (vertical axis) against  $t^2$ .
- What is the slope of the graph in (b)?
- Determine the acceleration of the object.
- Is the acceleration in (d) constant? Explain.

*Answer*

(a)

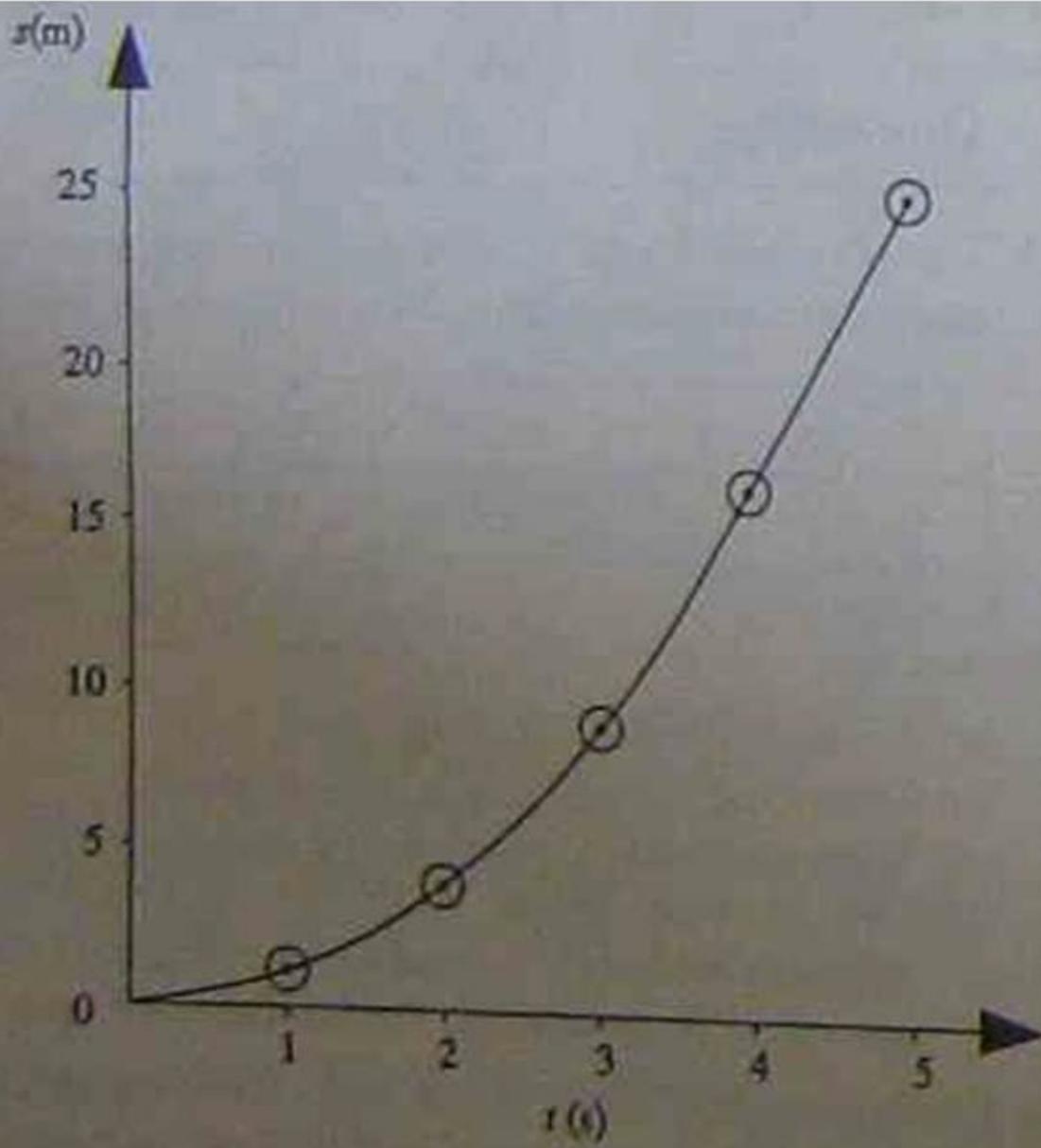


Fig. POS 6

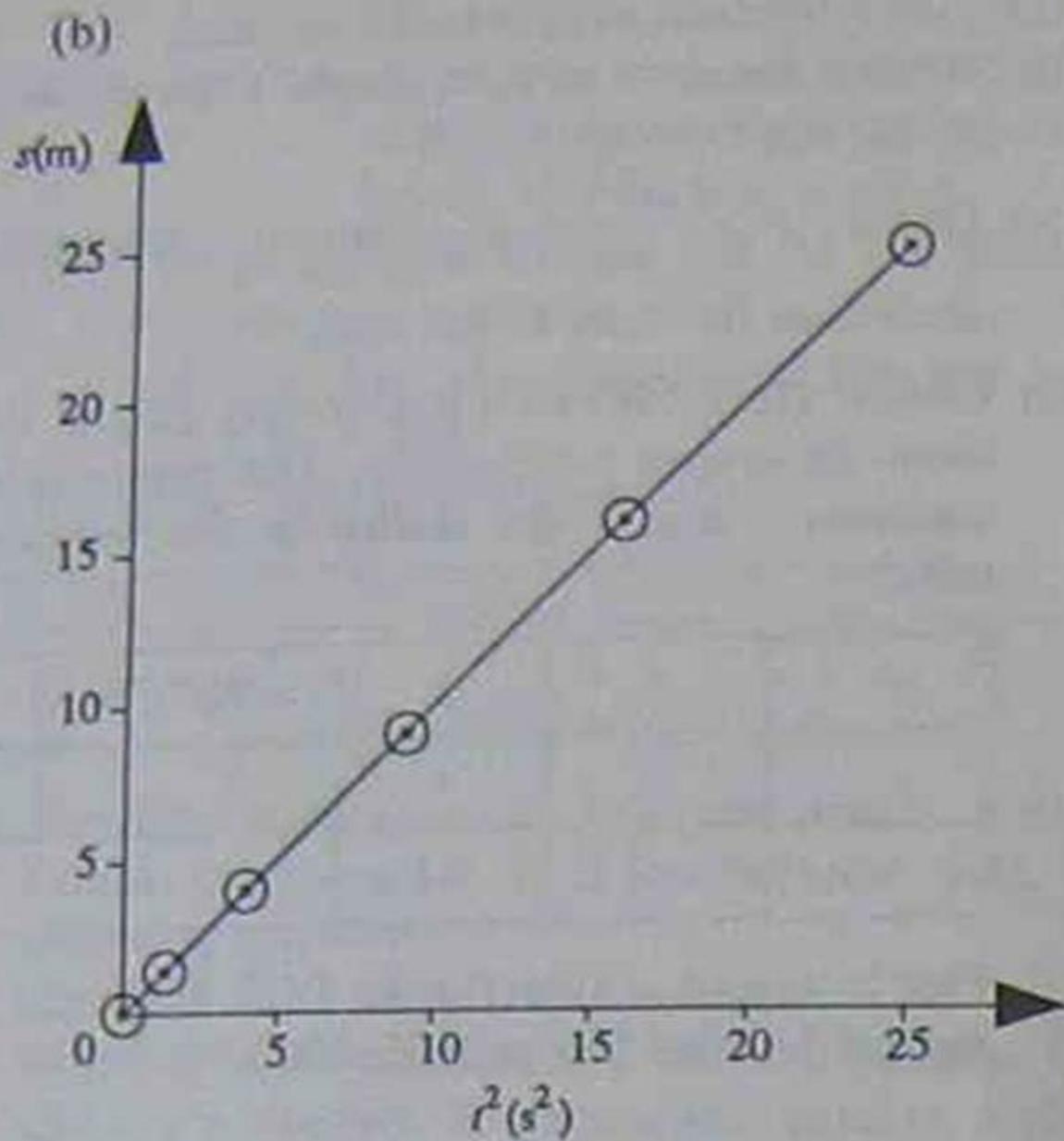


Fig. POS 7

Fig. POS 7

(c) Slope  $m = \text{rise/run} = (25 - 0)/(25 - 0)$   
 $= 1.0 \text{ m s}^{-2}$ .

(d) Equation of the straight line through the origin in Figure POS 7 is the equation of motion  $s = 1/2 at^2$ . Compare this with the equation of a straight line,  $y = mx$ . It can be seen that the slope of the graph:

$$m = a/2, \text{ therefore}$$

$$a = 2 \times m = 2 \times 1.0 = 2.0 \text{ m s}^{-2}.$$

(e) Yes; constant slope in  $s$  versus  $t^2$  graph means constant acceleration.

## EXAMPLE

The electrostatic force  $F$  between two charges  $q_1$  and  $q_2$  a distance  $d$  apart in air is given as shown in the following table:

$F$ (N)	1.0	2.0	3.0	4.0
$d$ (m)	1.0	0.71	0.58	0.50
$1/d^2$ ( $m^{-2}$ )	1.0	2.0	3.0	4.0

- (a) Plot a fully-labelled graph of  
(i)  $F$  against  $d$ ; (ii)  $F$  against  $1/d^2$ .
- (b) Calculate the slope  $k$  of the graph in (a)(ii).
- (c) Given:  
 $F = \kappa q_1 q_2 / d^2$ , where  $\kappa = 9 \times 10^9$   
SI units and  $q_1 = q_2 = q$ , graphically  
determine  $q$ .

Answer

(a) (i)

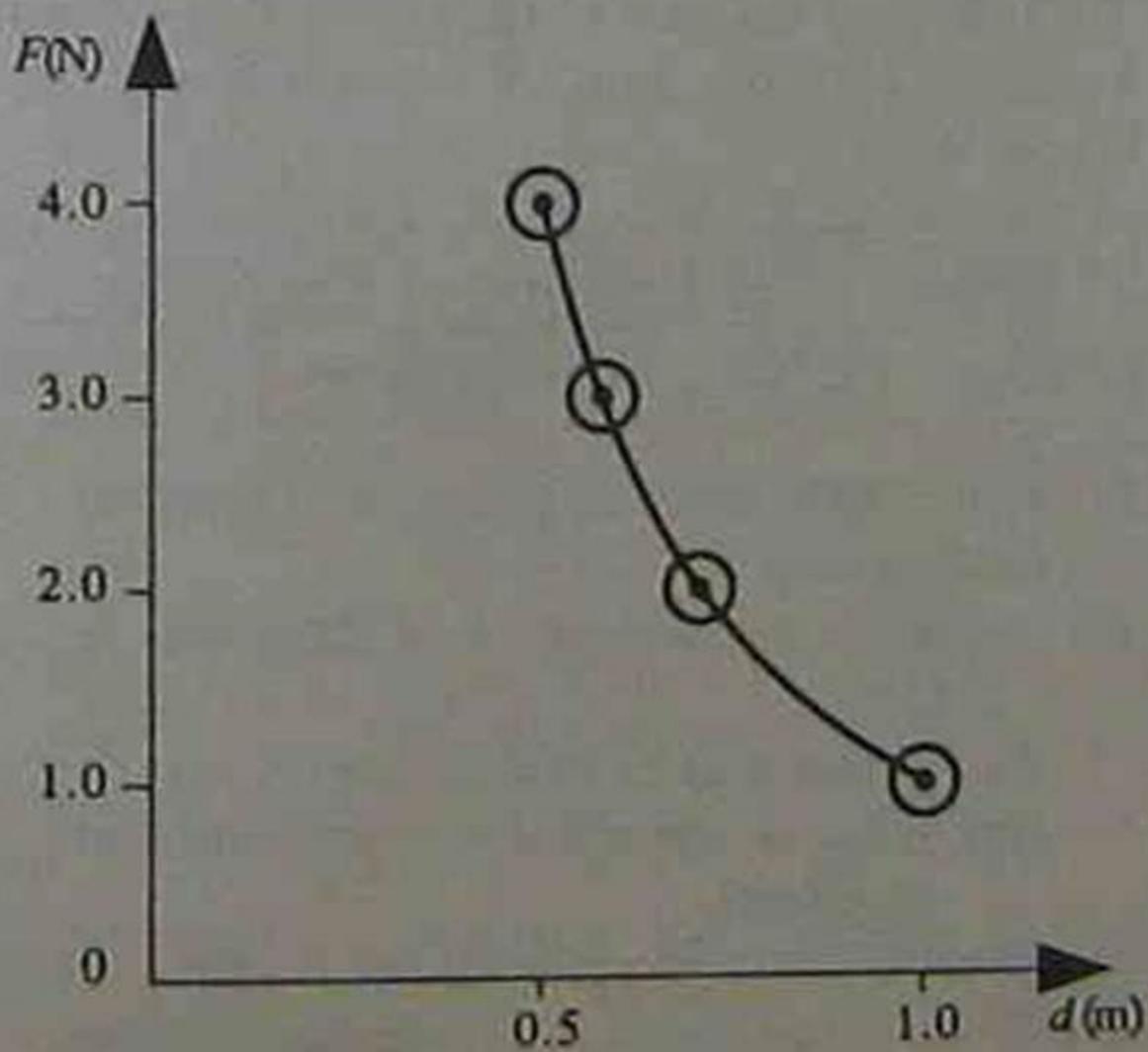


Fig. POS 9

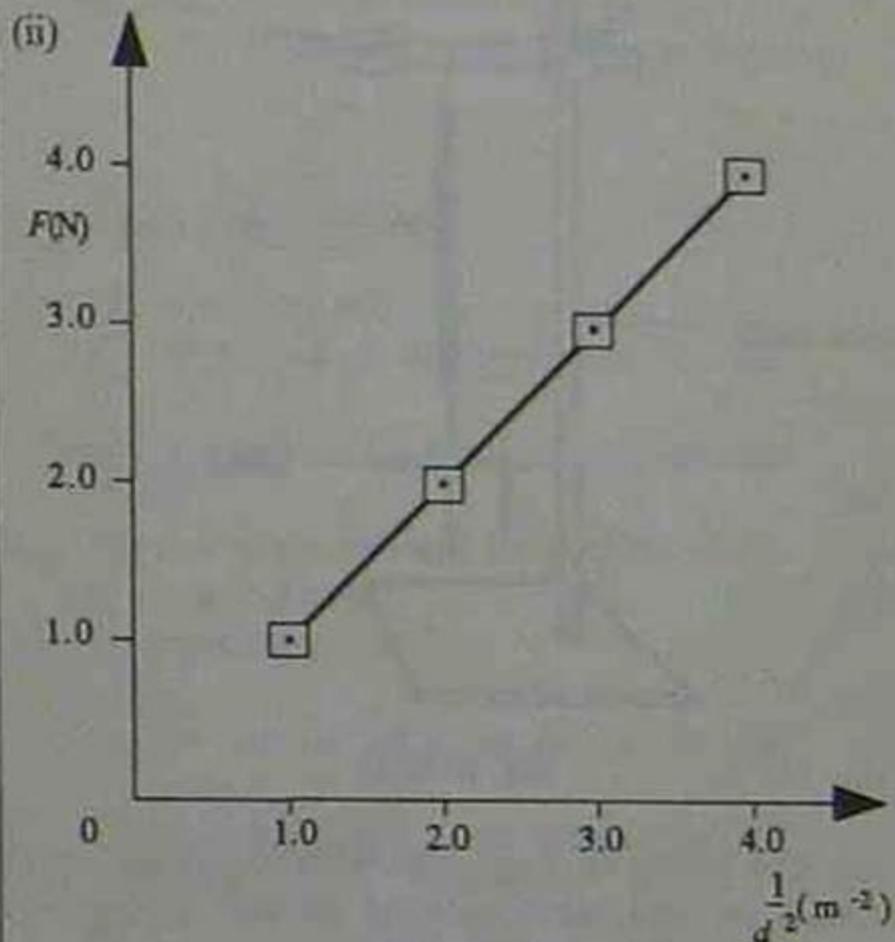


Fig. POS 10

(b) Slope  $k = \text{rise/run}$   
 $= (4.0 - 1.0) / (4.0 - 1.0) \text{ N m}^2$   
 $= 1.0 \text{ N m}^2$

(c) Since slope of graph  $k = \kappa q_1 q_2 = \kappa q^2$   
 $\therefore 1.0 = 9 \times 10^9 q^2$ , thus  $q = 1 / \sqrt{9 \times 10^9}$   
 $= 10 \mu\text{C}$

## CHAPTER 1

Describing motion I: Motion  
in one dimension*SYMBOL AND UNIT SUMMARY*

<i>Symbol</i>	<i>Quantity</i>	<i>Unit</i>
$s$	displacement	m
$v$	final velocity	$\text{m s}^{-1}$
$u$	initial velocity	$\text{m s}^{-1}$
$a$	acceleration	$\text{m s}^{-2}$
$t$	time	s

# Displacement

In one-dimensional motion there are only two possible directions, backward and forward. Usually we make one of these directions positive and the other negative.

Displacement is a quantity that gives the position of an object by means of a direction and a distance from a reference point. Displacement depends on the relative positions of the reference point and the object, and does not depend on the path of the object in reaching its final position.

Displacement is usually represented by the symbol  $s$ .

## Average velocity

Average velocity,  $v_{ave}$ , is found by dividing the total displacement,  $\Delta s$ , by the total time,  $\Delta t$ , taken, and includes a direction.

$$\therefore v_{ave} = \frac{\Delta s}{\Delta t}$$

When a body changes velocity uniformly from  $u$  to  $v$ :

$$v_{ave} = \frac{(u + v)}{2}$$

$$\text{then } v_{ave} = \frac{(u + v)}{2} = \frac{\Delta s}{\Delta t} \text{ or } \frac{s}{t}$$

Average speed is the total distance travelled divided by the total time taken.

The unit of average speed and average velocity is the metre per second ( $\text{m s}^{-1}$ ), but other convenient units such as kilometres per hour ( $\text{km h}^{-1}$ ) may be used.

## EXAMPLE

A runner takes 4 minutes to jog 300 m east and then 400 m west. Calculate:

- (a) average speed;
- (b) average velocity.

*Answer*

(a) Distance covered = 700 m

$$\begin{aligned}\text{average speed} &= \frac{700}{240} \\ &= 2.9 \text{ m s}^{-1}\end{aligned}$$

- (b) Let east be +ve, west be -ve.

$$\begin{aligned}\text{Displacement} &= 300 \text{ m east} + 400 \text{ m west} \\ &= +300 \text{ m} - 400 \text{ m} \\ &= -100 \text{ m}\end{aligned}$$

i.e. 100 m west.

$$\begin{aligned}\text{Average velocity} &= \frac{100}{240} \\ &= 0.42 \text{ m s}^{-1} \text{ west}\end{aligned}$$

To convert kilometres per hour to metres per second, first multiply by 1000 because there are 1000 m in one kilometre, then divide by 3600 because there are 3600 s in an hour.

To convert metres per second to kilometres per hour, first multiply by 3600 and then divide by 1000.

# Graphing displacement

The motion of an object can be displayed as a graph of displacement against time. The vertical axis is used for displacement and the horizontal axis is used for time. A graph should have a title (or caption), the axes should be labelled, and the scale, with units, should be shown.

Where the graph of displacement vs time is parallel to the time axis the object is at rest. If the graph rises to the right the object is moving forward. In regions

**Displacement vs time**



# Displacement vs time

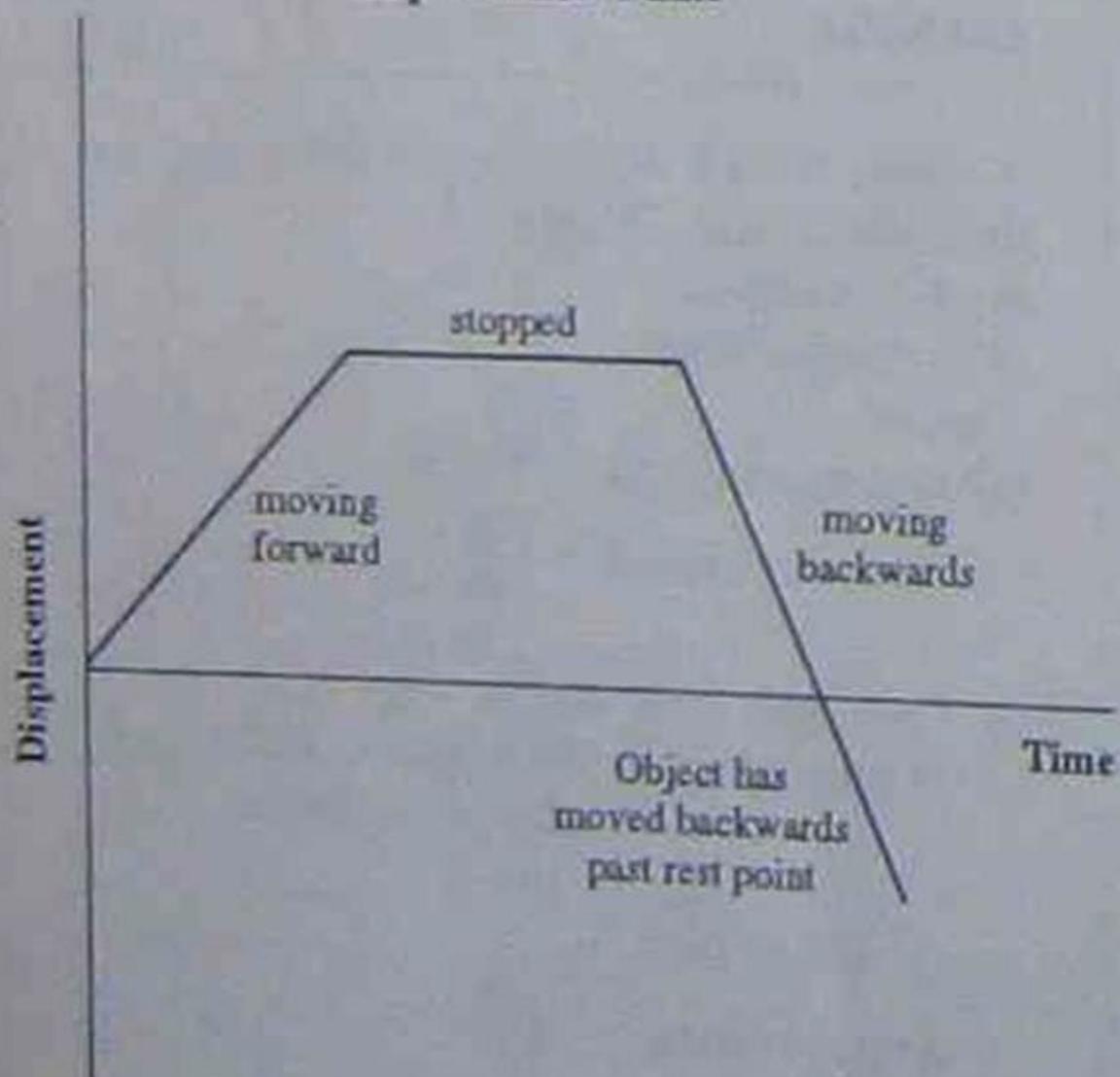


Fig. 1.1

where the graph falls to the right the object is moving backward. The steeper the graph the faster the object is moving. The direction of motion changes at maximum and minimum values of displacement, i.e. at peaks and troughs. When the graph moves below the time axis the object has moved backward past its starting point.

# Instantaneous velocity

Instantaneous velocity is the velocity of an object at a particular instant. It is defined as

$$\text{Limit}_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t}$$

The value of instantaneous velocity is the slope of the tangent to the graph of displacement vs time at a particular instant.

If the tangent slopes up to the right, i.e. slope is positive, the velocity is forward, but if the tangent slopes down to the right, i.e. the slope is negative, the object is moving backward. The steeper the slope of the tangent, the higher will be the instantaneous velocity.

If the graph of displacement vs time is a straight line we have *uniform motion* and the instantaneous velocity is the same at any time and always equal to the average velocity.

# Displacement vs time

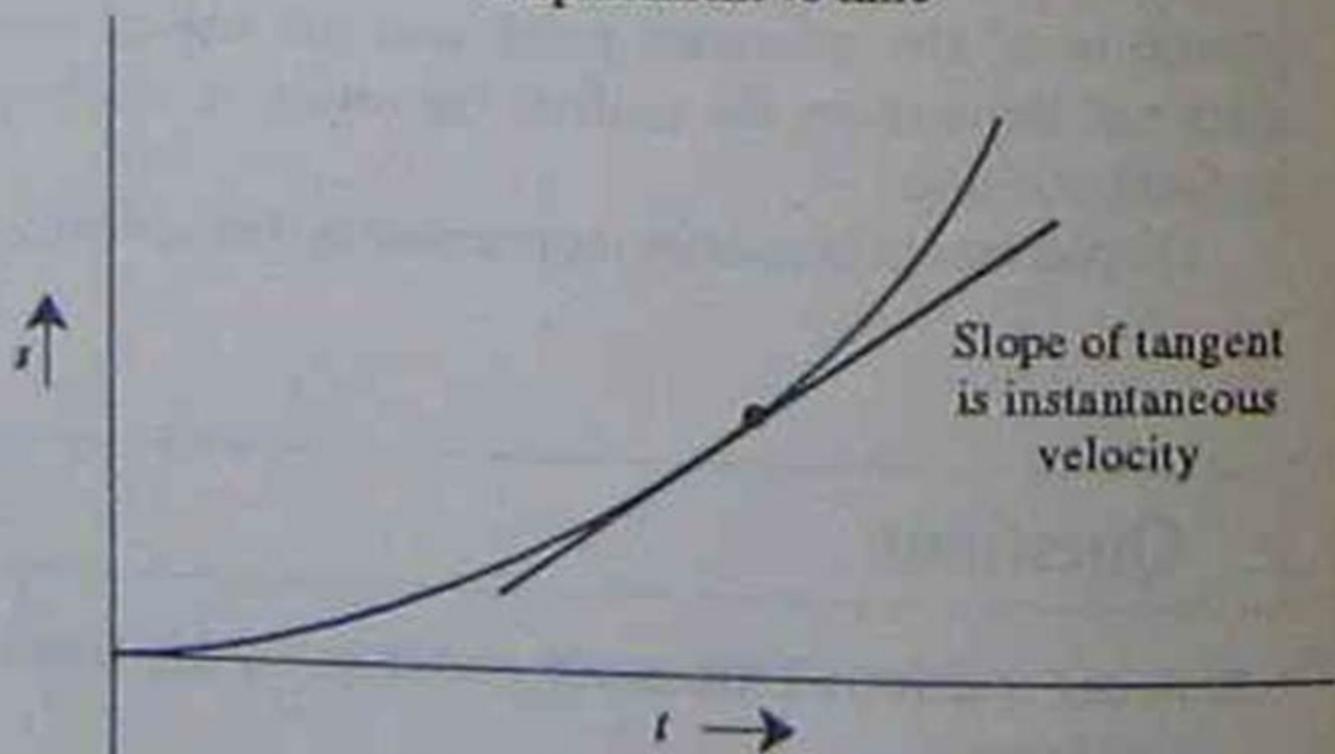


Fig. 1.2

# Average acceleration

When an object changes velocity it is accelerating. Acceleration is a vector quantity which has magnitude and a direction and in one-dimensional situations is positive when the object increases velocity and negative when it decreases velocity. Average acceleration is defined as follows:

$$\begin{aligned}\text{Average acceleration} &= \text{change in velocity} / \text{time} \\ &= (\text{final velocity} \\ &\quad - \text{initial velocity}) / \text{time}\end{aligned}$$

This can be written in symbols as

$$\text{Average } a = \frac{v - u}{t},$$

where  $a$  is acceleration,  $v$  is final velocity,  $u$  is initial velocity and  $t$  is time.

The unit of acceleration is the metre per second squared ( $\text{ms}^{-2}$ ), provided  $u$  and  $v$  are in metres per second and  $t$  is in seconds. These are SI units.

**EXAMPLE**

- (a) A car starts at rest and five seconds later it is travelling at  $23 \text{ m s}^{-1}$ . Calculate its average acceleration.
- (b) A car travelling at a steady  $20 \text{ m s}^{-1}$  takes four seconds to stop. Calculate its average acceleration.

*Answer*

$$(a) a = \frac{v - u}{t} = \frac{23 - 0}{5} = 4.6 \text{ m s}^{-2}$$

$$(b) a = \frac{v - u}{t} = \frac{0 - 20}{4} = -5 \text{ m s}^{-2}.$$

i.e. its acceleration is in the reverse direction and is  $5 \text{ m s}^{-2}$ .

# Graphing velocity 1

Velocity can be plotted against time. Time is always shown on the horizontal axis.

If the graph rises to the right the object is speeding up. If it falls to the right the object is slowing down.

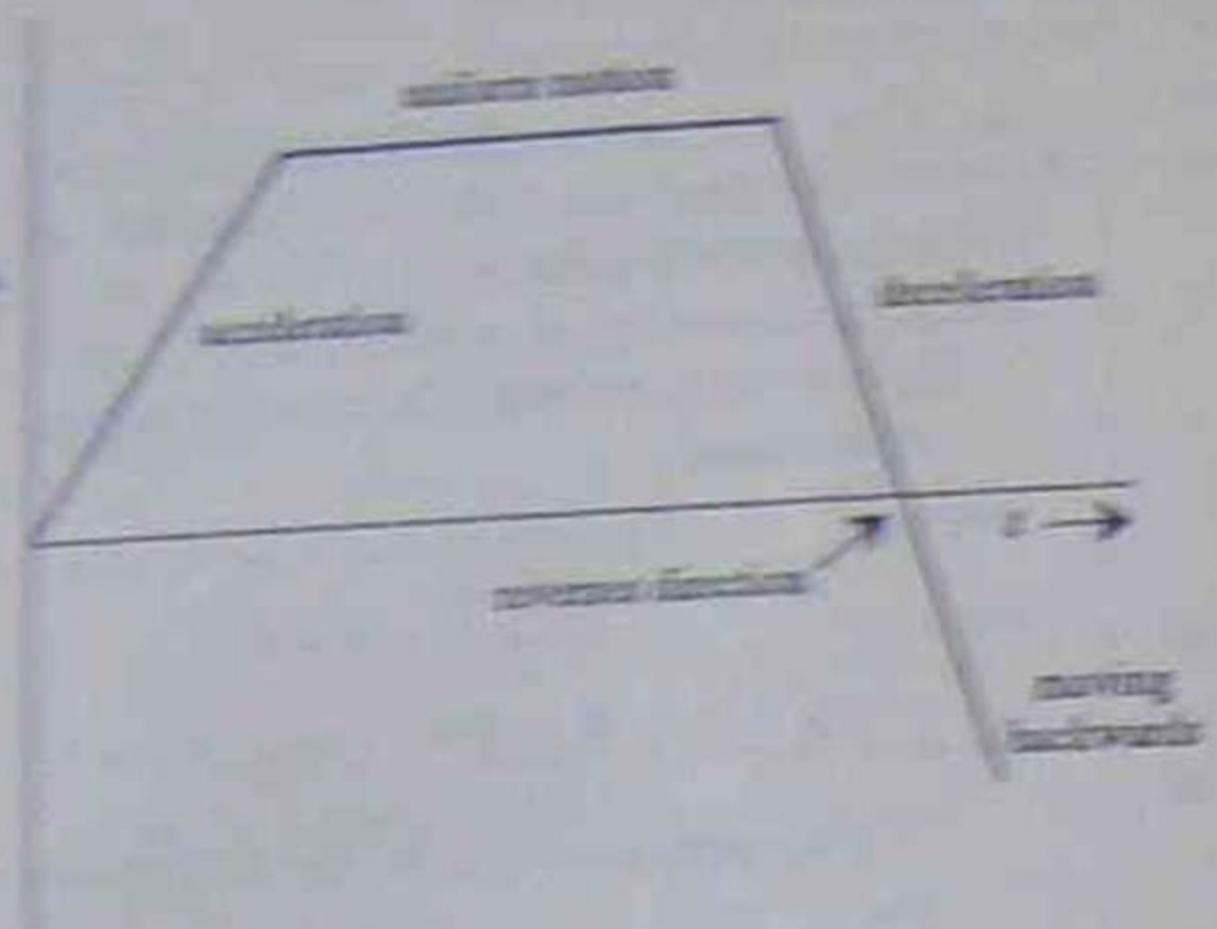
Where the graph is above the time axis the object is moving forward. If the graph drops below the time axis the object is moving backward.

If the graph is parallel to the time axis, the object has uniform motion.

Where the graph crosses the time axis the object is momentarily at rest as it changes direction.

# STUDY UNIT PHASES

Velocity versus time



## Graphing velocity 2

The slope of the graph of velocity vs time at a particular instant is the acceleration at that instant.

The area under the graph is the total displacement. Where the graph is below the time axis the area under the graph is between the graph and the time axis and is counted as negative for determining displacement.

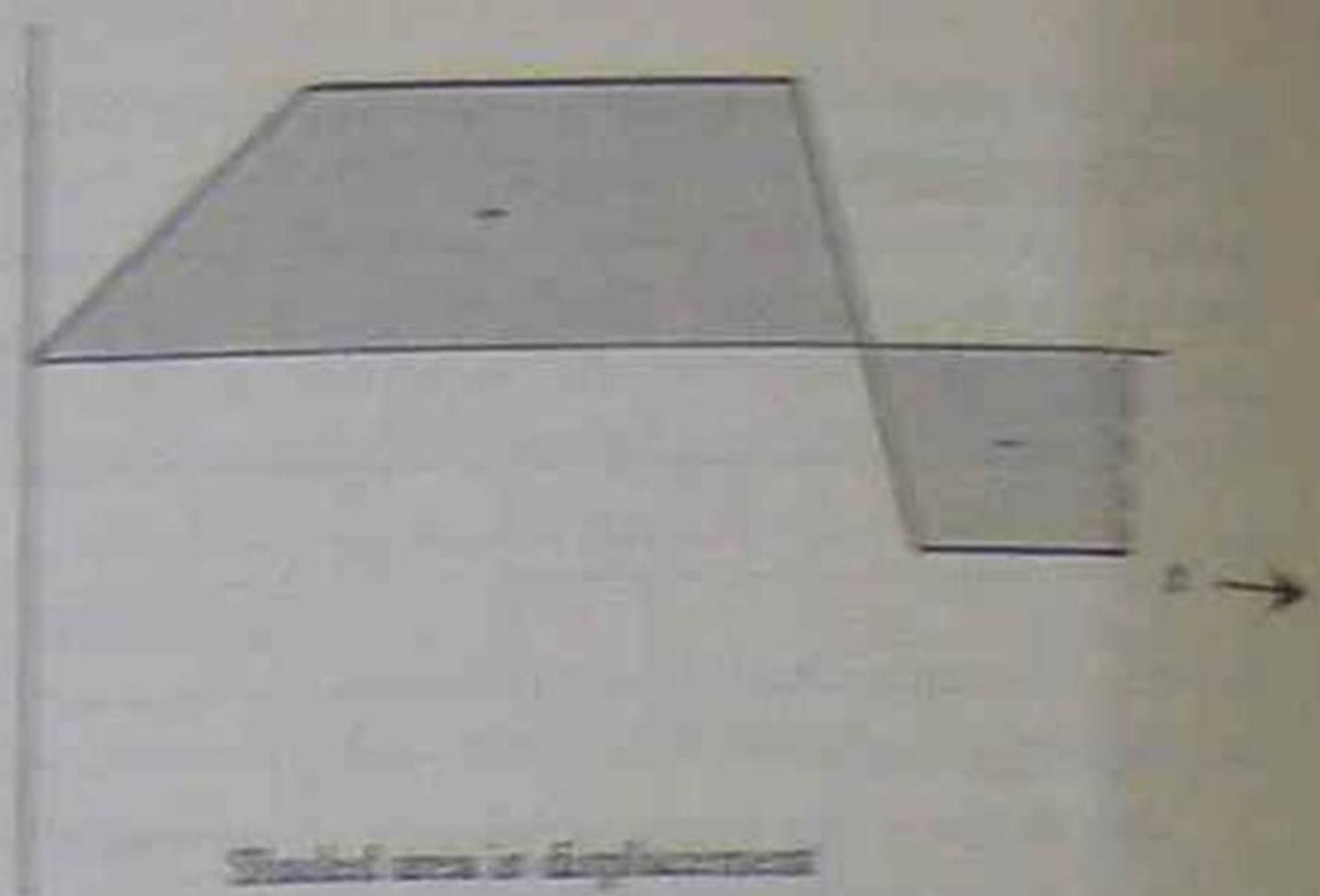


Fig. 1.4

## Equations of motion 1

We have already seen that acceleration is given by:

$$a = (v - u)/t$$

From this we can obtain the equation:

$$v = u + at$$

Remember that  $v$ ,  $u$ , and  $a$  are vectors. This means they have a direction associated with them. We show forward and backward motion with opposite signs.

## Equations of motion 2

When an object is accelerating uniformly its displacement after time  $t$  is given by:

$$s = ut + \frac{1}{2}at^2$$

Remember to use positive and negative signs to show opposite directions for  $s$ ,  $u$ , and  $a$ .

## Equations of motion 3

The equation  $s = ut + \frac{1}{2}at^2$  is a quadratic in  $t$ . This means that when we solve for  $t$  we can obtain two answers, one of which may be negative. A negative value for  $t$  means before the start. Most often we want the positive value for  $t$ .

## Equations of motion 4

If we eliminate  $t$  from the two equations of motion we have already dealt with, we obtain

$$v^2 = u^2 + 2as$$

Again, remember to use positive and negative signs to indicate direction.

## Key facts and equations

- Average velocity is total displacement divided by time.
- Average acceleration is the change in velocity divided by time.
- Opposite directions for displacement, velocity, and acceleration are shown by opposite signs.
- The slope of a displacement vs time graph is instantaneous velocity.
- The slope of a velocity vs time graph is acceleration.
- The area under a velocity vs time graph is displacement.

- Where initial velocity is zero and acceleration is constant, the slope of the graph of displacement vs time squared is half the acceleration.
- There are four equations of motion:

$$v_{\text{ave}} = \frac{(u + v)}{2} = \frac{s}{t}$$

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$