

CHAPTER 12

Forces II: Force in two dimensions

SYMBOL AND UNIT SUMMARY

Symbol	Quantity	Unit
F	force	N
T	tension in string or cable	N
m	mass	kg
a	acceleration	m s^{-2}
g	acceleration due to gravity	m s^{-2}
θ, ϕ	angle	degree ($^{\circ}$)
R	normal reaction	N

Components of a force

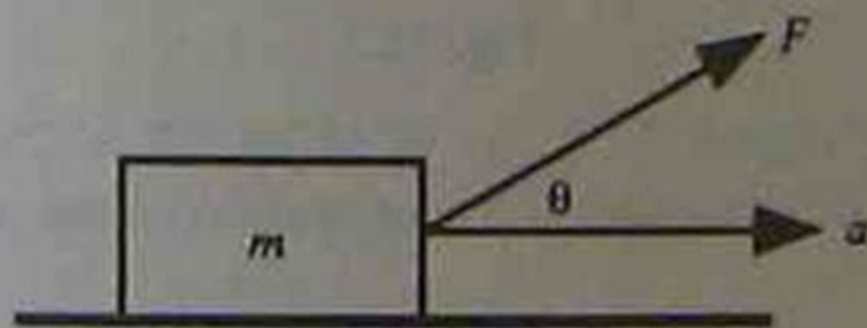


Fig. 12.1

In Figure 12.1 a force F is shown producing an acceleration a at an angle θ to the direction of F . The acceleration is produced by the horizontal component of the force F in the direction of the acceleration.

We can calculate the acceleration as follows:

$$F \cos \theta = ma$$

$$\text{i.e. } a = F \cos \theta / m$$

If the vertical component of F is greater than the weight of the mass, the mass will be lifted off the horizontal surface.

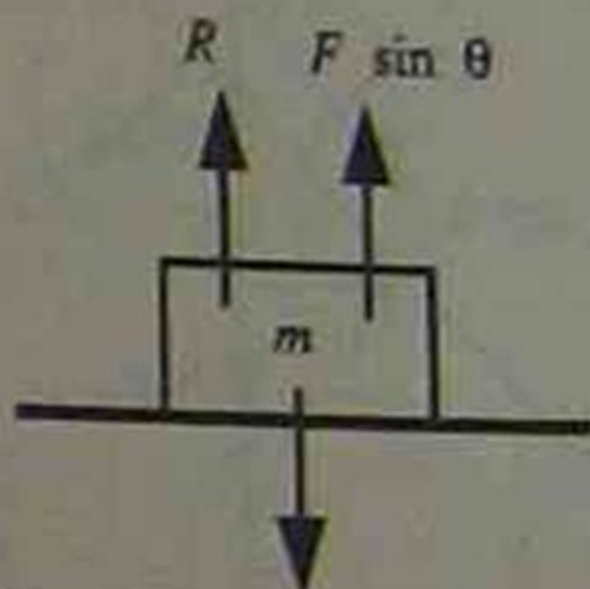


Fig. 12.2 mg

If the vertical component of F is less than mg the mass will remain on the surface. In this case the sum of the vertical forces acting on the mass is zero. The mass is pressing down on the surface and according to Newton's Third Law this means that the surface is applying an equal and opposite force on the mass. If we call this reaction force R , then we can see that

$$mg = F \sin \theta + R$$

i.e. $R = mg - F \sin \theta$

Acceleration down slopes

If a mass m is placed on a frictionless slope inclined at an angle θ to the horizontal, the weight of the object mg will have a component down the slope and also at right angles to the slope.

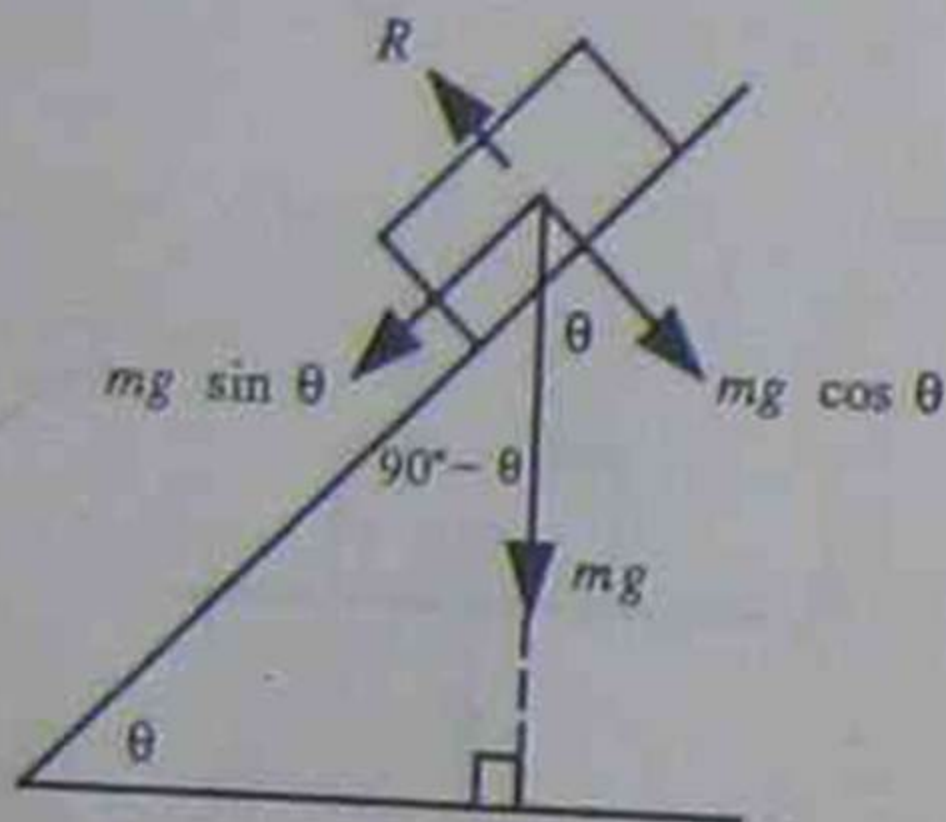


Fig. 12.4

Fig. 12.4

The acceleration a , down the slope is due to the component of gravity down the slope.

$$ma = mg \sin \theta$$

$$a = g \sin \theta$$

The component of gravity at right angles to the surface is $mg \cos \theta$. From Newton's Third Law this is equal and opposite to the normal reaction R which is the force applied by the surface of the slope to the object.

Acceleration up slopes

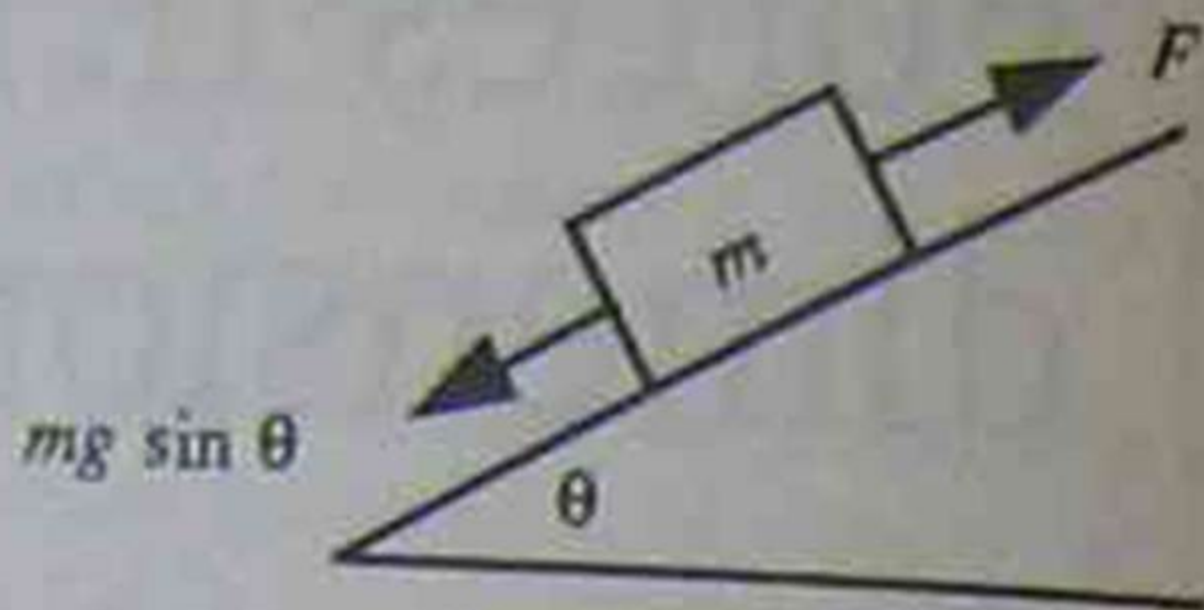


Fig. 12.6

Suppose a force F is applied so that it acts on a mass m on a frictionless slope inclined at an angle θ to the horizontal as shown in Figure 12.6. The component of weight mg acting down the slope is $mg \sin \theta$.

Force acting down the slope is $mg \sin \theta$.

If $F > mg \sin \theta$, the mass will accelerate up the slope.

$$ma = F - mg \sin \theta$$

$$a = (F - mg \sin \theta)/m$$

If $F = mg \sin \theta$, the mass will not accelerate. It will remain stationary or move either up or down the slope with uniform velocity.

If $F < mg \sin \theta$, the mass will accelerate down the slope.

$$ma = mg \sin \theta - F$$

$$a = (mg \sin \theta - F)/m$$

Slopes and pulleys

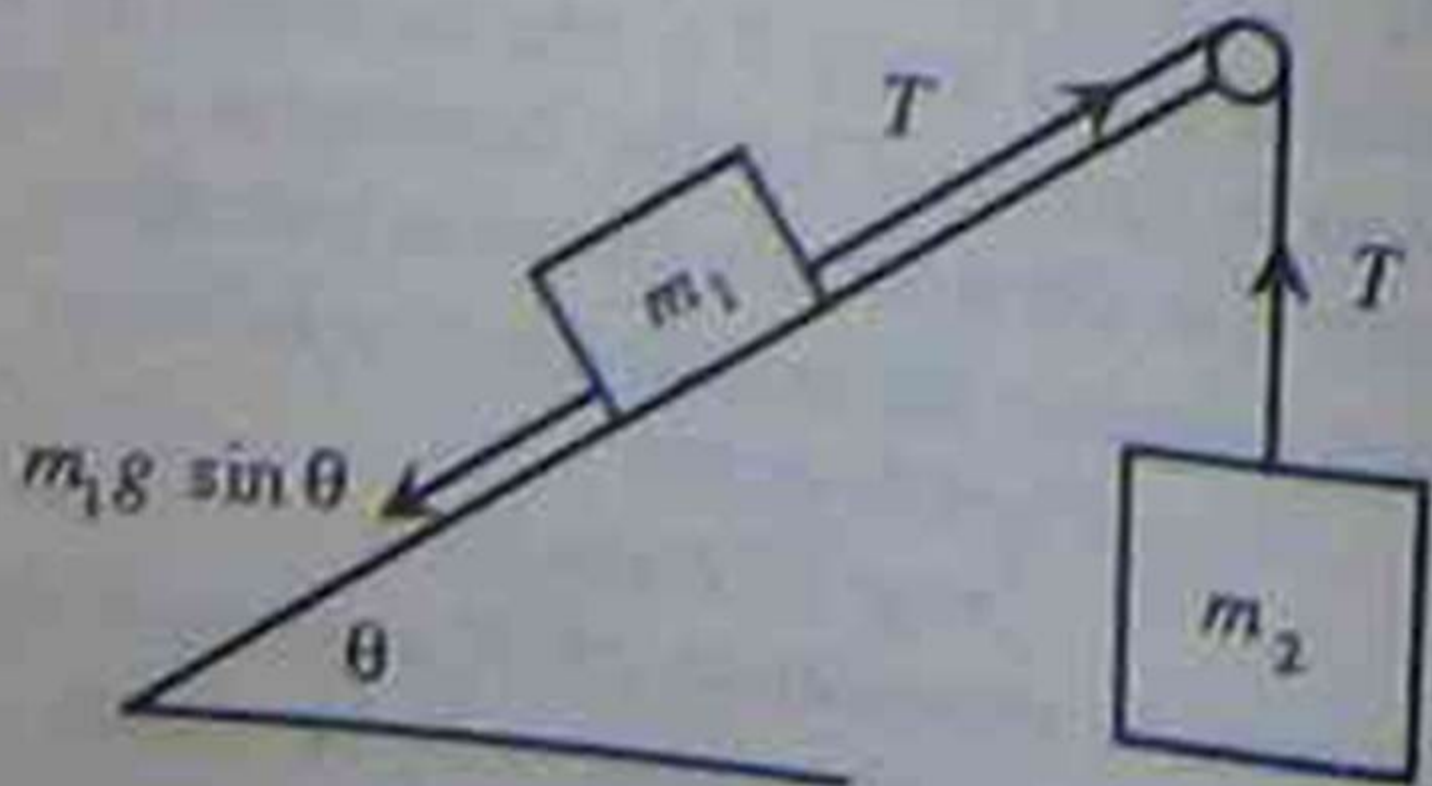


Fig. 12.8

Figure 12.8 shows that a mass m_1 is on a frictionless slope inclined at an angle θ to the horizontal. A light inextensible string connects m_1 to m_2 over a frictionless pulley as shown. There are two forces acting on m_1 , namely $m_1 g \sin \theta$ down the slope and the tension in the string T up the slope. There are two forces acting on m_2 , namely gravity ($m_2 g$) down, and T , the tension in the string, up.

If $m_1 g \sin \theta$ equals $m_2 g$ there is no resultant force on m_1 or m_2 and the system will either be at rest or m_1 and m_2 will each have uniform velocity. Then the value of T will be $m_2 g$.

If $m_1 g \sin \theta$ is greater than $m_2 g$, m_2 will accelerate up and m_1 will accelerate down the slope. Both will have the same acceleration, a . For m_1 we can write:

$$\begin{aligned} m_1 a &= m_1 g \sin \theta - T \\ \text{i.e. } T &= m_1 g \sin \theta - m_1 a \end{aligned} \quad (1)$$

For m_2 we can write:

$$\begin{aligned} m_2 a &= T - m_2 g \\ \text{i.e. } T &= m_2 a + m_2 g \end{aligned} \quad (2)$$

Hence from Equations 1 and 2 we can write:

Hence from equations 1 and 2 we can write:

$$m_1 g \sin \theta - m_1 a = m_2 a + m_2 g$$

$$\text{i.e. } a = \frac{m_1 g \sin \theta - m_2 g}{m_1 + m_2}$$

From this the value of T can be calculated.

If $m_1 g \sin \theta$ is less than $m_2 g$, m_1 will accelerate up the slope and m_2 will accelerate down. Each will have acceleration a . For m_1 we can write:

$$m_1 a = T - m_1 g \sin \theta$$

$$\text{i.e. } T = m_1 g \sin \theta + m_1 a \quad (3)$$

For m_2 we can write:

For m_2 we can write:

$$\begin{aligned} m_2 a &= m_2 g - T \\ \text{i.e. } T &= m_2 g - m_2 a \end{aligned} \quad (4)$$

From Equations 3 and 4 we can write:

$$m_1 a + m_1 g \sin \theta = m_2 g - m_2 a$$

This gives the following expression for a .

$$a = \frac{m_2 g - m_1 g \sin \theta}{m_1 + m_2}$$

From this, T can be calculated.

EXAMPLE

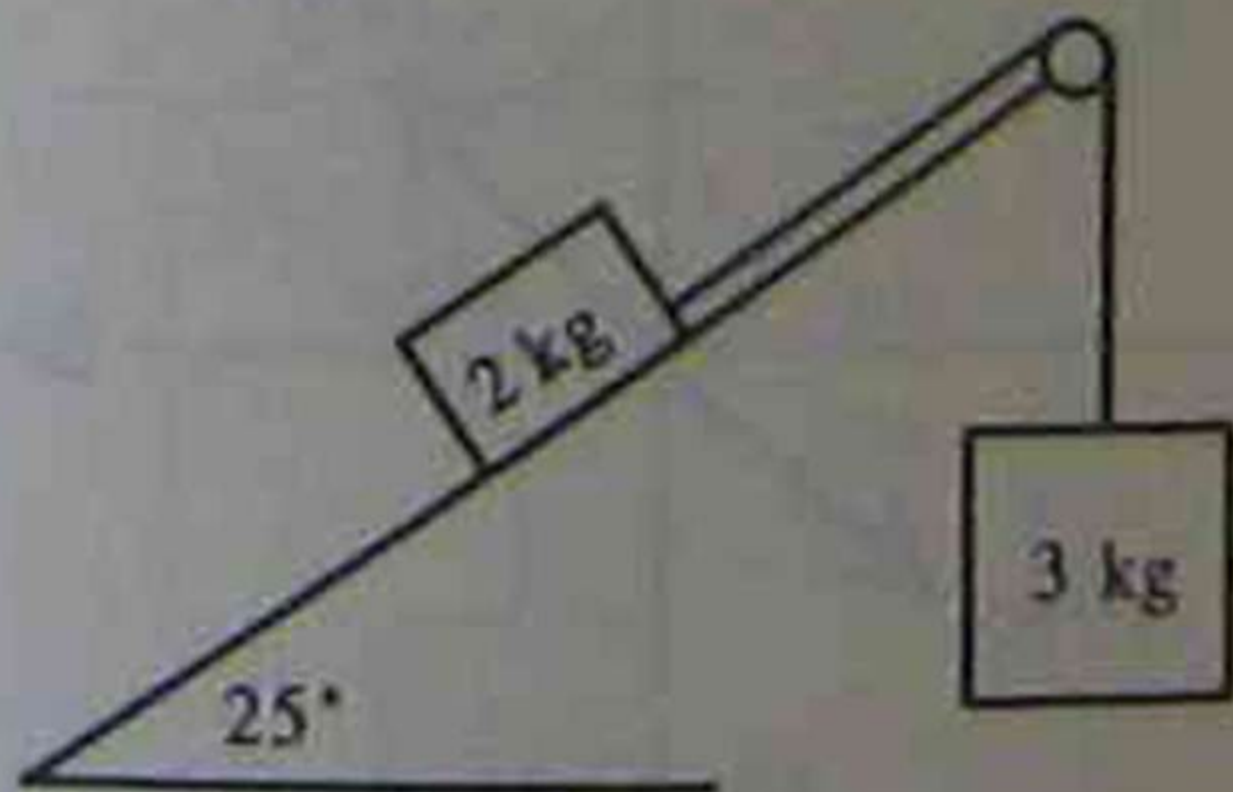


Fig. 12.9

A 2 kg mass is at rest on a frictionless slope as shown in Figure 12.9. This mass is connected by a light inextensible string over a frictionless pulley to a 3 kg mass. Calculate:

- the acceleration of the system;
- the tension in the string.

Answer

Answer

(a) Component weight of the 2 kg mass down slope = $2 \times 9.8 \times \sin 25^\circ$
 $= 8.3 \text{ N}$

Weight of the 3 kg mass = 3×9.8
 $= 29.4 \text{ N}$

Hence the 2 kg mass will accelerate up the slope and the 3 kg mass will accelerate down.
For the 2 kg mass:

$$2a = T - 2 \times 9.8 \times \sin 25^\circ$$
$$\text{i.e. } T = 2a + 2 \times 9.8 \times \sin 25^\circ$$
$$T = 2a + 8.3 \quad (1)$$

For the 3 kg mass:

$$T = 2a + 8.3$$

For the 3 kg mass:

$$3a = 3 \times 9.8 - T$$

$$\text{i.e. } T = 3 \times 9.8 - 3a$$

$$T = 29.4 - 3a$$

From Equations 1 and 2:

$$2a + 8.3 = 29.4 - 3a$$

$$a = \frac{29.4 - 8.3}{5}$$

$$\text{i.e. } a = 4.2 \text{ ms}^{-2}$$

(b) From Equation 2:

$$T = 29.4 - 3a$$

$$= 29.4 - 3 \times 4.2$$

$$= 16.8 \text{ N}$$

Mass suspended by a cable

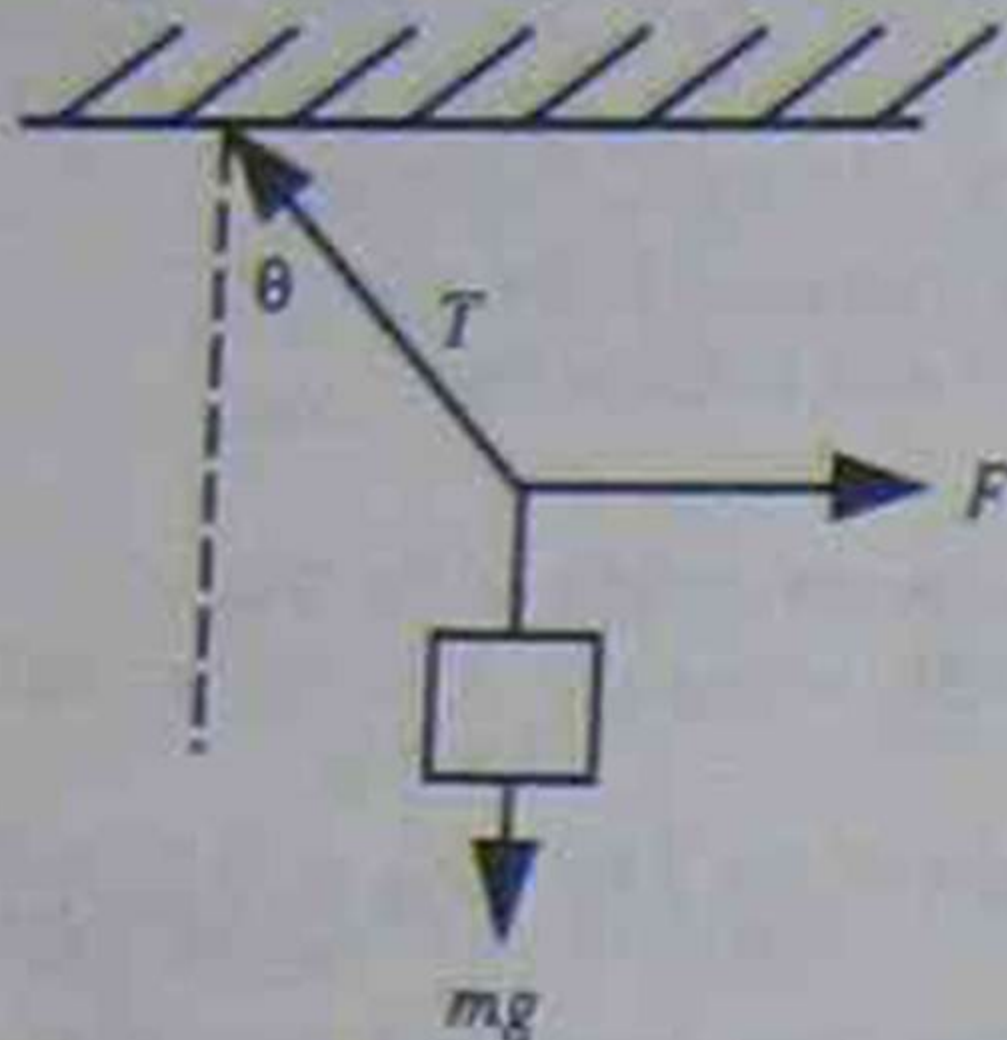


Fig. 12.11

Figure 12.11 shows a mass m hanging from a light cable and being pulled to one side by a horizontal force F . The mass is at rest and the cable suspending the mass is at an angle θ to the vertical.

Since the mass is at rest, the vertical component of the tension in the cable, T , must be equal and opposite to the weight of the mass. Hence:

$$T \cos \theta = mg$$

Also the horizontal component of T must be equal and opposite to F . If the cable is at an angle θ to the vertical it must be at an angle of $90^\circ - \theta$ to the horizontal. Hence:

$$T \cos (90^\circ - \theta) = F$$

$$\text{i.e. } T \sin \theta = F$$

EXAMPLE

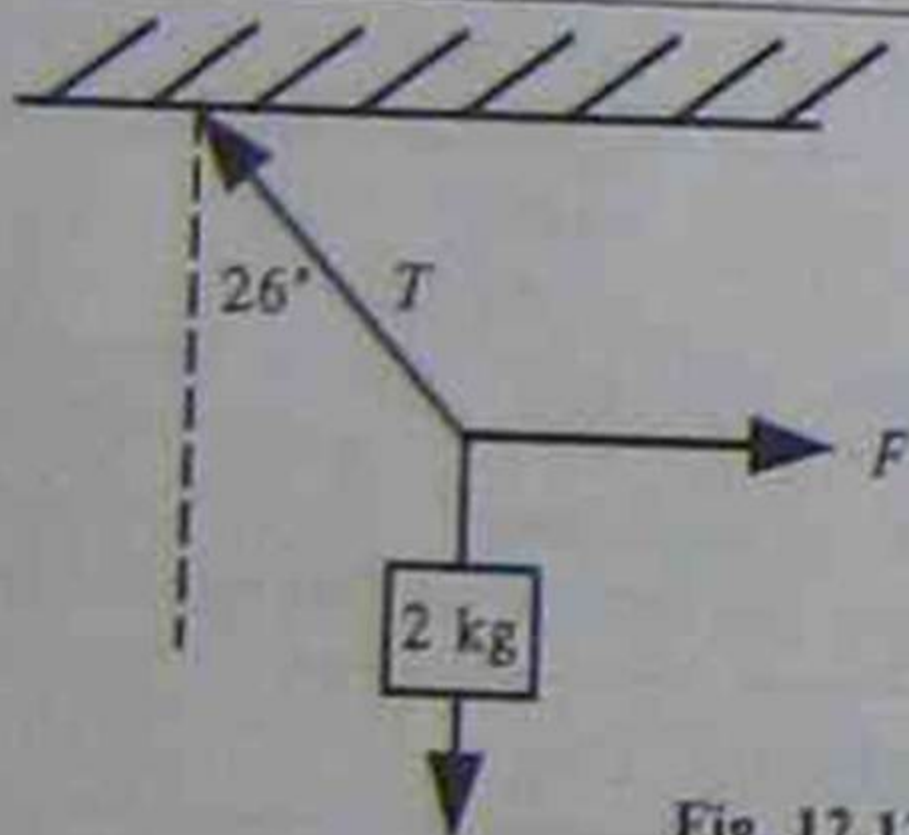


Fig. 12.12

A mass of 2 kg hangs by a light string and is pulled to one side by a force F . The string is at an angle of 26° to the vertical. Calculate T and F .

Answer

The vertical component of T is equal and opposite to the weight of the mass. Hence:

$$T \cos 26^\circ = 2 \times 9.8$$

$$T = \frac{2 \times 9.8}{\cos 26^\circ}$$
$$= 21.8 \text{ N}$$

The horizontal component of T is equal and opposite to F . Hence:

$$F = T \sin 26^\circ$$
$$= 9.6 \text{ N}$$

Resultant force

The resultant of two or more concurrent forces is the single force that could replace those forces and have the same effect. The resultant of several forces is the vector sum of those forces.

The equilibrant of two or more forces is that force which when added to those already acting gives a vector sum of zero. The equilibrant is equal and opposite to the resultant.

When concurrent forces are in equilibrium their vector sum is zero. Concurrent forces have lines of action that meet at a single point. In such cases we do not have to consider the turning effects of the forces present.

When adding two or more forces it is often useful to identify two directions at right angles to each other, e.g. vertical and horizontal or north and east. We can then take the components of the forces to be added in each of these directions and add them. The sum of the components in each direction is then the component of the resultant in that direction.

EXAMPLE

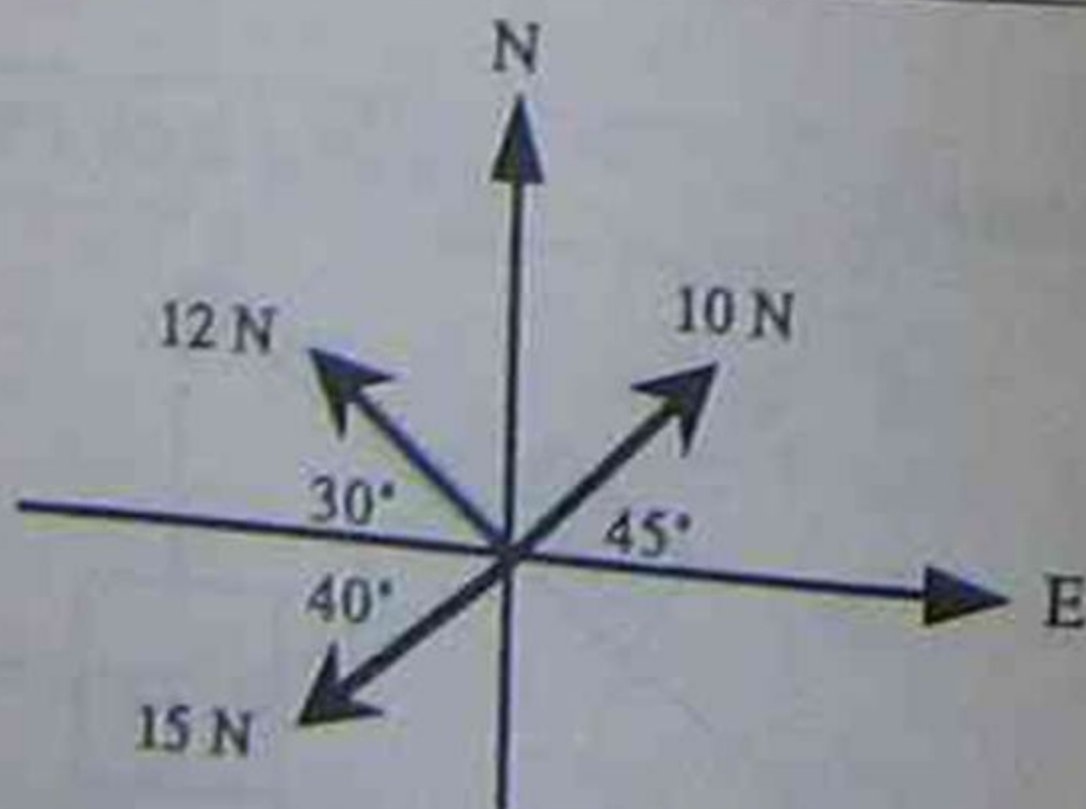


Fig. 12.14

Find the resultant of the forces in Figure 12.14.

Answer

Component of resultant in the east direction.

$$= 10 \cos 45^\circ - 12 \cos 30^\circ - 15 \cos 40^\circ$$

$$= -14.8 \text{ N}$$

Component of resultant in the north direction

$$= 10 \sin 45^\circ + 12 \sin 30^\circ - 15 \sin 40^\circ$$

$$= 3.43 \text{ N}$$

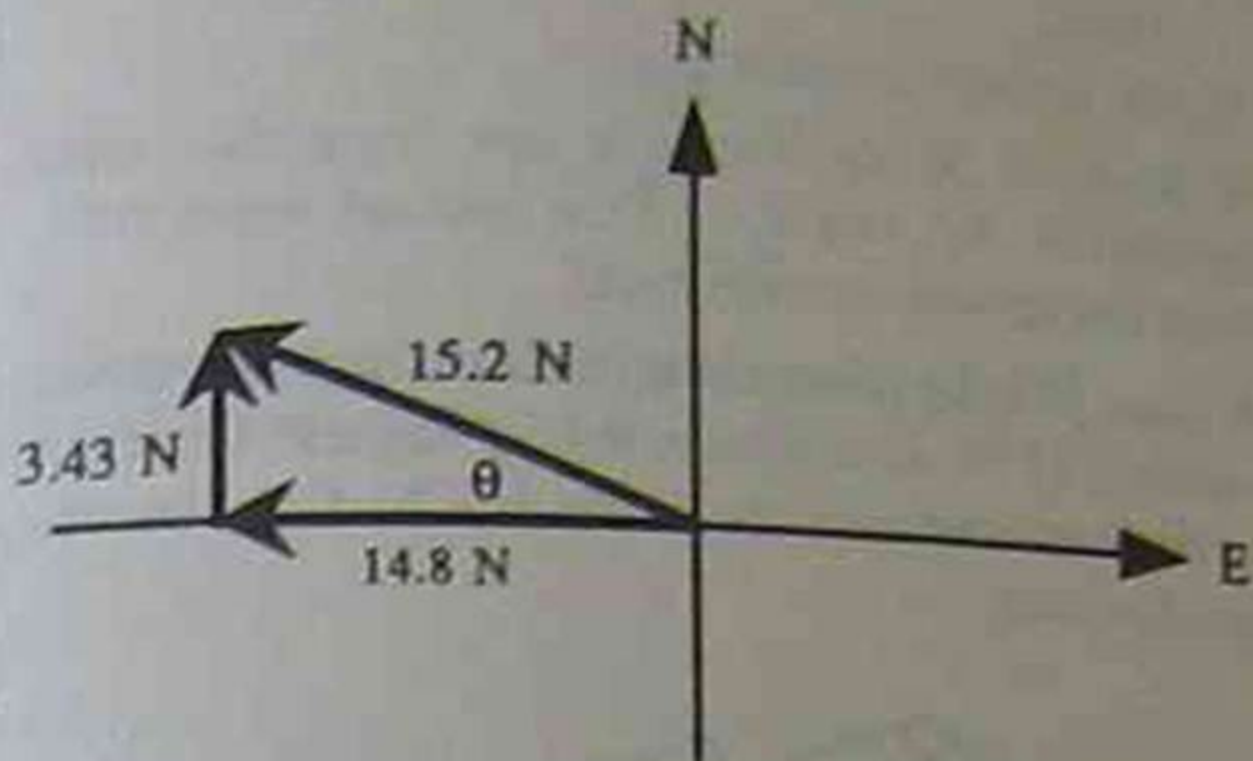


Fig. 12.15

$$\begin{aligned}\text{Resultant} &= \sqrt{3.43^2 + 14.8^2} \\ &= 15.2 \text{ N}\end{aligned}$$

$$\tan \theta = 3.43 / 14.8 = 0.2317$$

$$\text{Hence } \theta = 13^\circ$$

The resultant is 15.2 N in a direction 13° north of west.

Mass suspended by two cables

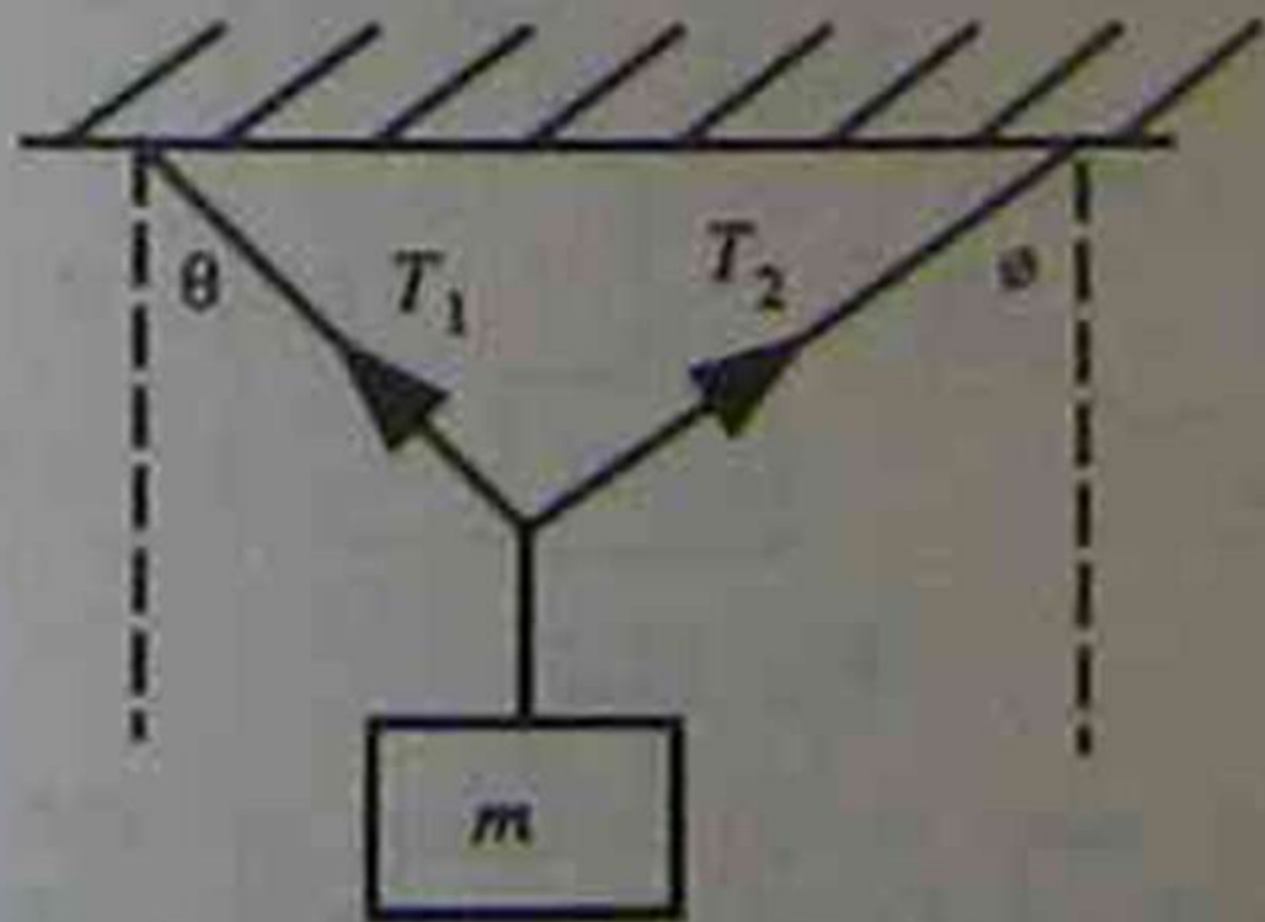


Fig. 12.16

Figure 12.16 shows a mass m suspended by two light cables. The cables are at angles θ and ϕ to the vertical as shown. The tensions in the cables are T_1 and T_2 and the system is in equilibrium.

The sum of the vertical components of T_1 and T_2 must be equal and opposite to the weight of the object. Hence:

$$mg = T_1 \cos \theta + T_2 \cos \phi \quad (1)$$

The horizontal component of T_1 must be equal and opposite to the horizontal component of T_2 . Hence:

$$T_1 \sin \theta = T_2 \sin \phi \quad (2)$$

We can find T_1 and T_2 from Equations 1 and 2. From Equation 2:

From Equation 2:

$$T_1 = \frac{T_2 \sin \phi}{\sin \theta} \quad (3)$$

If we substitute from Equation 3 into Equation 1 we obtain:

$$mg = T_2 \frac{\sin \phi}{\sin \theta} \cos \theta + T_2 \cos \phi$$

$$T_2 = \frac{mg}{\left(\frac{\sin \phi}{\sin \theta} \cos \theta + \cos \phi \right)} \quad (4)$$

If m , θ and ϕ are known, T_1 can be found from Equation 3.

Key facts and equations

- Force is a vector quantity. Any acceleration has the same direction as the net force, and is found by dividing the net force by mass.
- The component of a force F in a direction θ degrees to the direction of that force is $F \cos \theta$.
- The component of weight down a slope inclined at θ degrees to the horizontal is $mg \sin \theta$.

- The normal reaction for an object on a frictionless slope inclined at θ degrees to the horizontal is $mg \cos \theta$.
- The resultant of several forces is the single force that could replace those forces present and produce the same effect. The resultant is the vector sum of the forces present.
- The equilibrant of several forces is that force which when added to the forces already present gives a vector sum of zero. The equilibrant is equal and opposite to the resultant.

- When an object with several concurrent forces acting on it is in equilibrium the vector sum of the forces acting on it is zero.
- Concurrent forces have lines of action that meet at a single point. When concurrent forces act on a body we need not consider the turning effect of the forces.
- Forces can be added in two dimensions by resolving them in two directions at right angles to each other and totalling the components in each of these directions. These totals represent the components of the resultant in the respective directions. The magnitude and direction of the resultant may be determined from these components.