CHAPTER 13

Mechanical interactions II: Momentum in two dimensions

SYMBOL AND UNIT SUMMARY

Symbol	Quantity	Unit
$\underset{\sim}{p}_i$	initial momentum	N s or kg m s^{-1}
$\underset{\sim}{p}_f$	final momentum	N s or kg m s^{-1}
$\Delta\underset{\sim}{p}$	impulse or change of momentum	N s or kg m s^{-1}
$\underset{\sim}{F}$	force	N or kg m s^{-2}
KE	kinetic energy	J or N m or kg m^2 s^{-2}

Change in momentum at a boundary

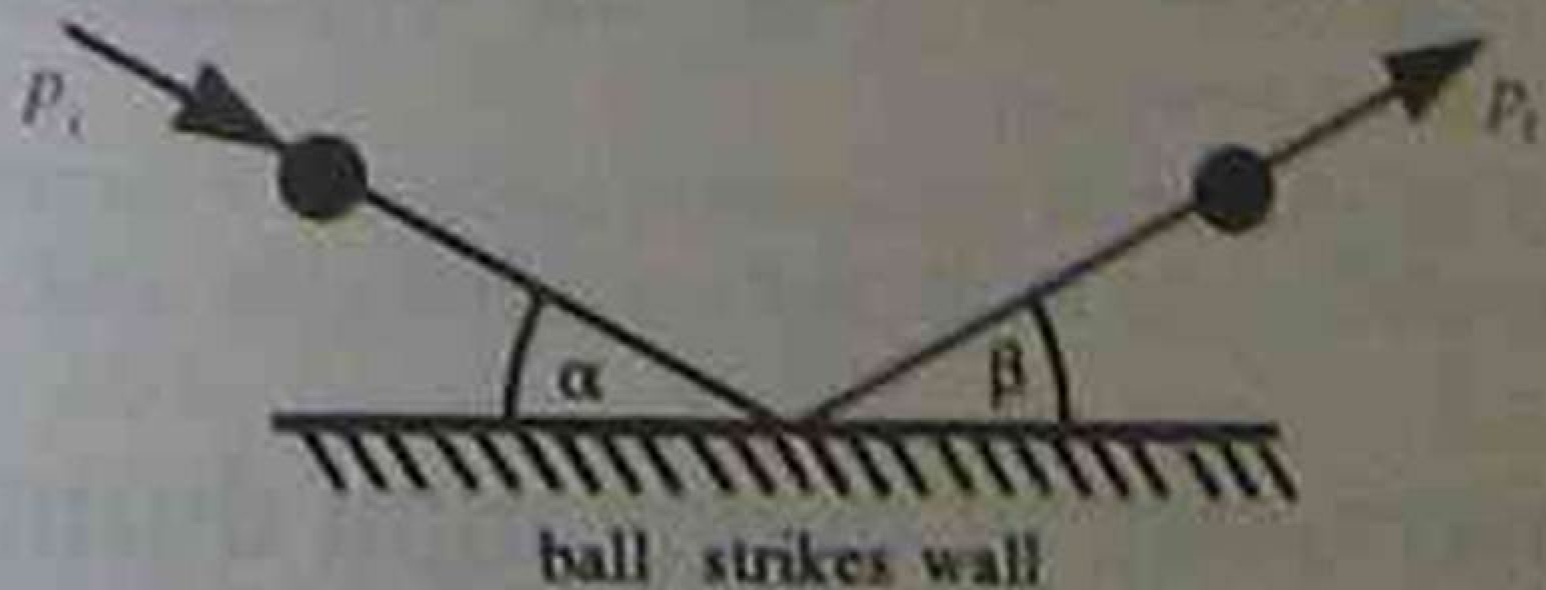


Fig. 13.1(a)

When a body collides with a boundary there is a change in momentum (impulse). This change in momentum, $\Delta \underline{p}$ can be determined vectorially by subtracting the initial momentum from the final momentum:

Change in momentum

$$\Delta \underline{p} = (\underline{p}_f - \underline{p}_i) = [\underline{p}_f + (-\underline{p}_i)]$$

The value of $\Delta \underline{p}$ is determined graphically by construction of a scaled vector diagram. Using the triangle of vector rule to add minus the initial momentum $(-\underline{p}_i)$ to the final momentum $(+\underline{p}_f)$:

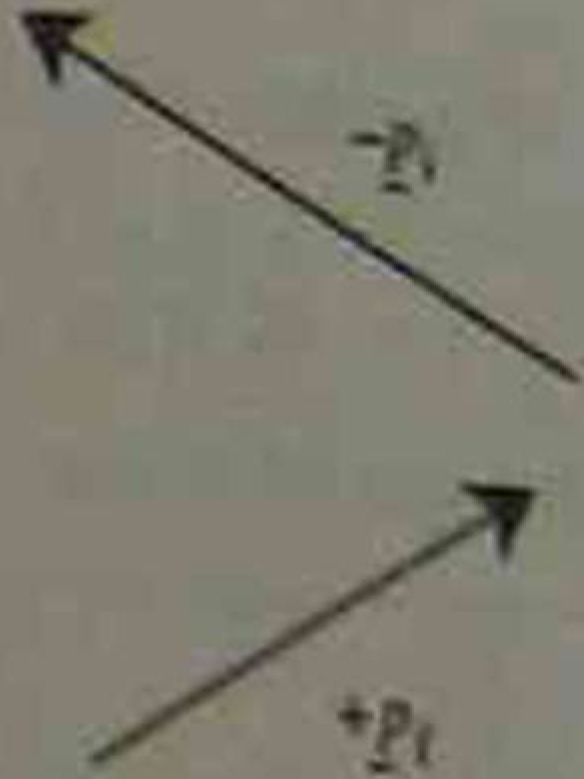


Fig. 13.1(b)

Change in momentum

$$\Delta p = [p_f + (-p_i)] = (p_f - p_i)$$

A vector diagram is used in Figure 13.1(c) to determine $\Delta \underline{p}$.

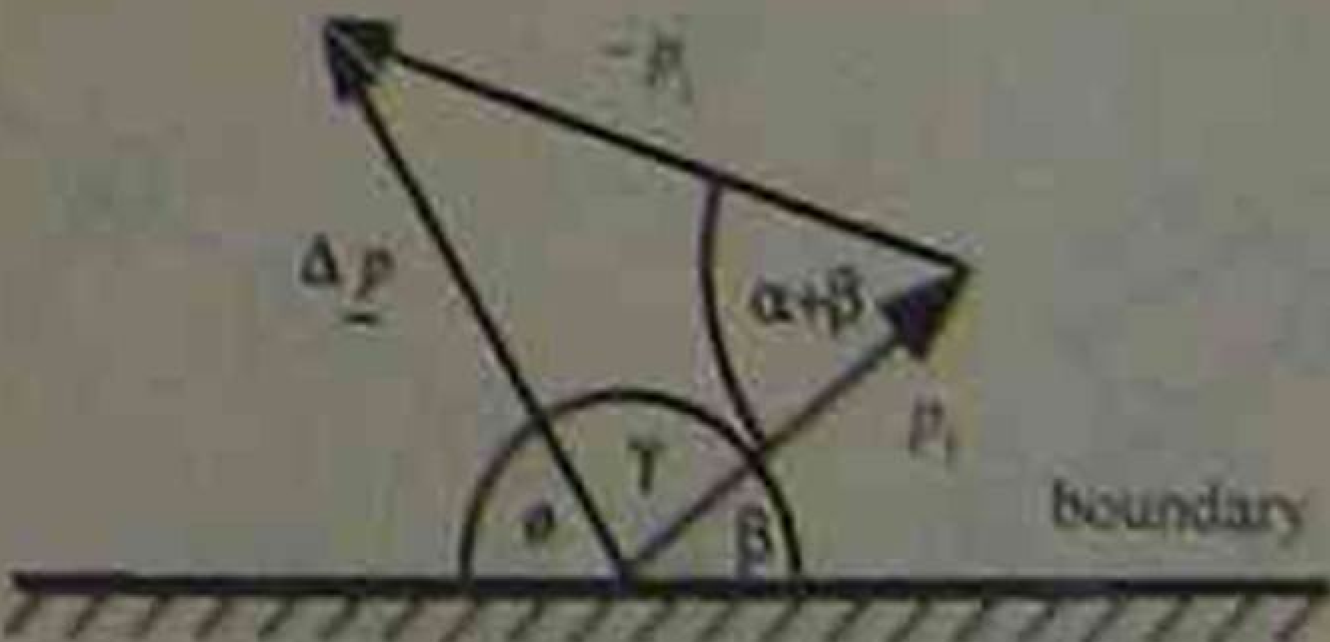


Fig. 13.1(c)

EXAMPLE

A ball of mass 2 kg with velocity 3 m s^{-1} strikes a boundary at 45° and rebounds at 2 m s^{-1} at 60° to the boundary.

Calculate:

(a) the ball's initial momentum;

(b) the ball's final momentum;

(c) the impulse of the ball graphically;

(d) the average force between the ball and the boundary if the time of contact of the ball with the boundary is 0.2 s.

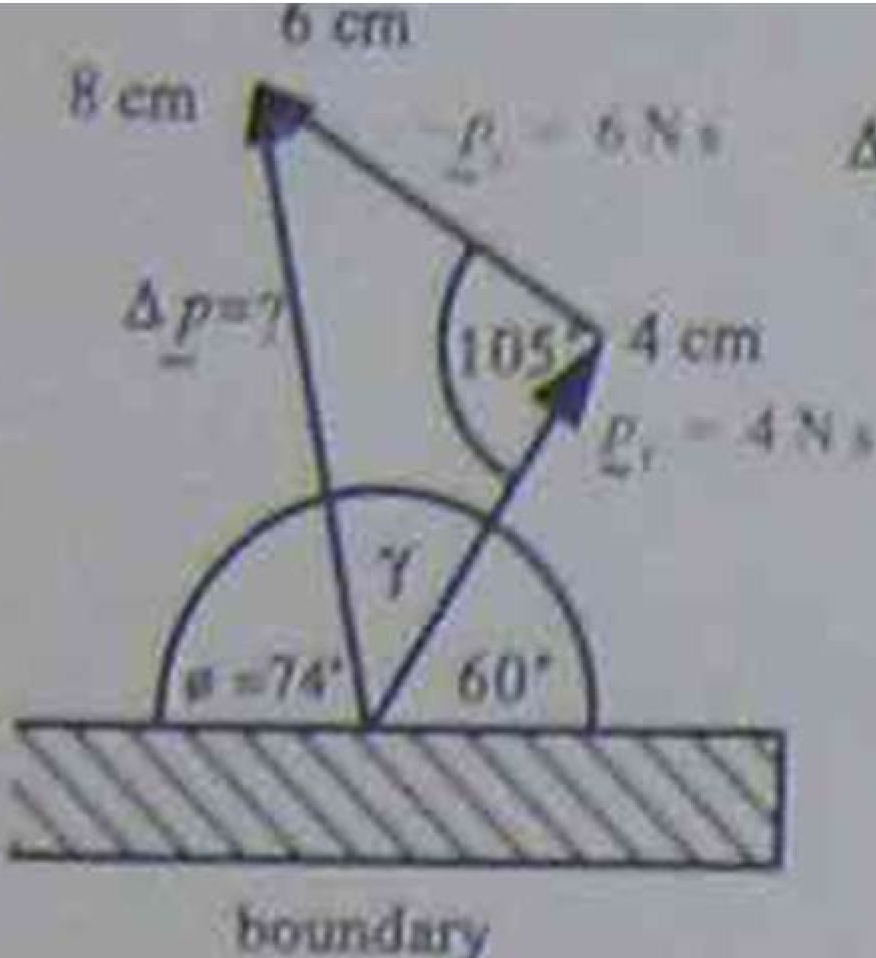
Answer

(a) $p_i = 2 \times 3 = 6$ N s at 45° to the boundary on the approach side.

(b) $p_r = 2 \times 2 = 4$ N s at 60° to the boundary on the rebound side.

(c) Graphically, scale 1 cm = 1 N s.

6 cm



$\Delta \underline{p} = 8.0$ N s at 74° to boundary on approach side

Fig. 13.2

(d) $\underline{F} = \dfrac{\Delta \underline{p}}{\Delta t} = \dfrac{8.0}{0.2}$ N $= 40$ N at 74° to the boundary on the approach side.

Conservation of momentum during glancing collisions

The Law of Conservation of Momentum applies to collisions in an isolated system, so:

$$\left.\begin{array}{c}\text{sum of momenta before}\\ \text{collision}\end{array}\right\} = \left\{\begin{array}{c}\text{sum of momenta after}\\ \text{collision}\end{array}\right.$$

i.e. $\sum \underline{p}(\text{before}) = \sum \underline{p}'(\text{after})$.

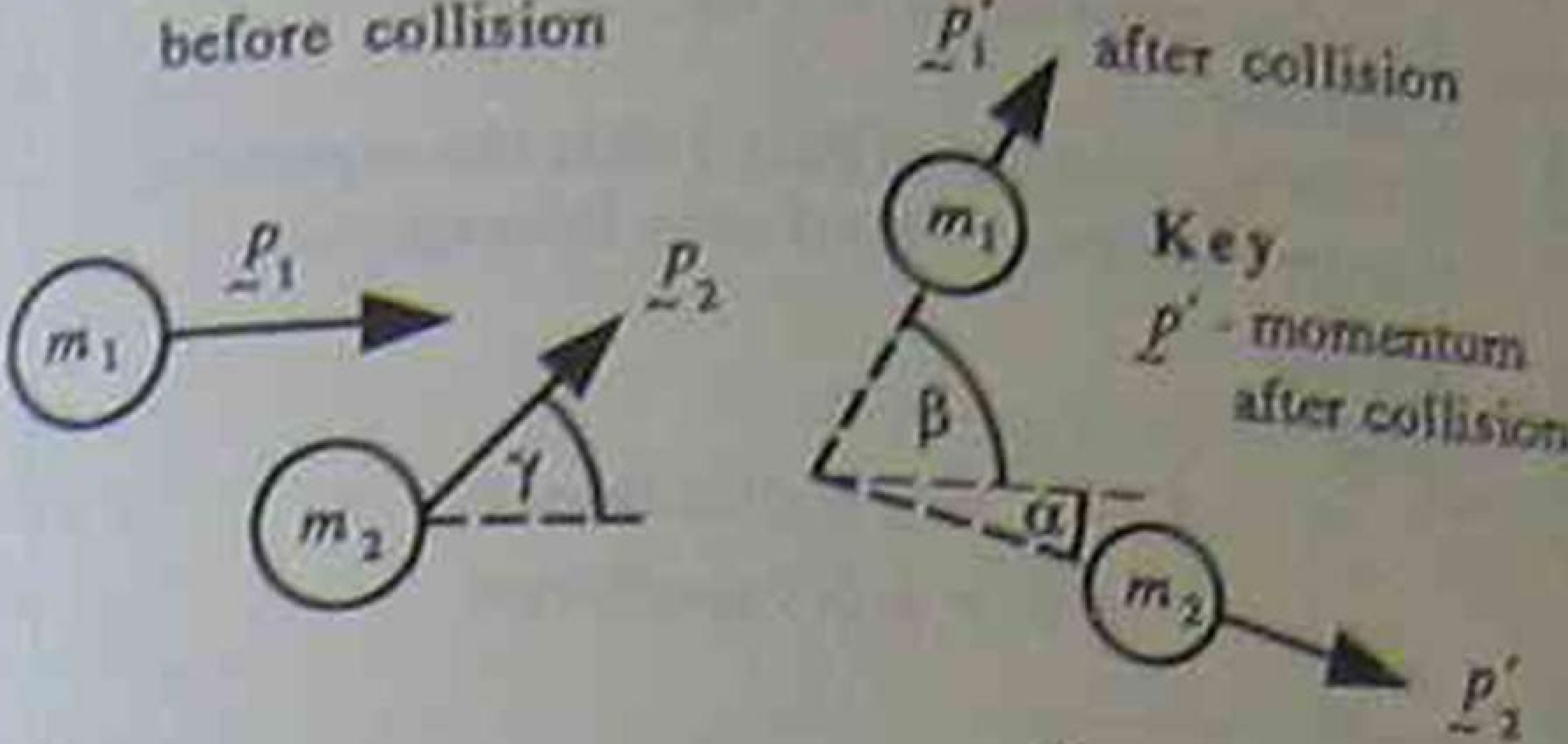
Σ, Greek sigma, means sum. The prime ($'$) means after collision.

For conservation of momentum, when two balls, say 1 and 2, collide, the impulse of ball 1 plus impulse of ball 2 is zero.

i.e. $\Delta \underline{p}_1 + \Delta \underline{p}_2 = 0$

For a two-dimensional, i.e. glancing, collision, we can equate the initial and final momenta graphically using vector diagrams.

Graphically



α =

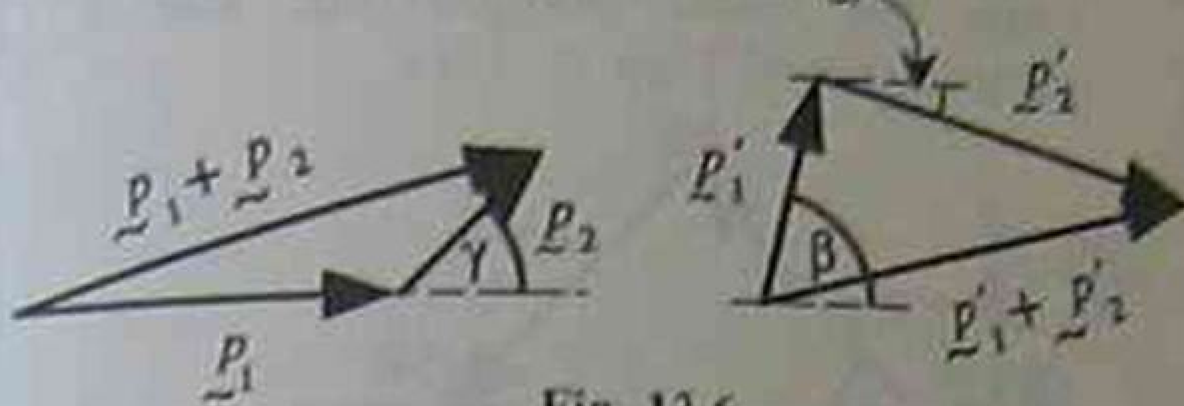


Fig. 13.6

For conservation of momentum:

$$\underset{\sim}{p}_1 + \underset{\sim}{p}_2 = \underset{\sim}{p}'_1 + \underset{\sim}{p}'_2$$

Also for momentum to be conserved, the change in momentum in the system is zero.

i.e. $\qquad \Delta\underset{\sim}{p}_1 + \Delta\underset{\sim}{p}_2 = 0 \qquad (1)$

$$\Delta\underset{\sim}{p}_1 = (\underset{\sim}{p}'_1 - \underset{\sim}{p}_1) \qquad \Delta\underset{\sim}{p}_2 = (\underset{\sim}{p}'_2 - \underset{\sim}{p}_2)$$

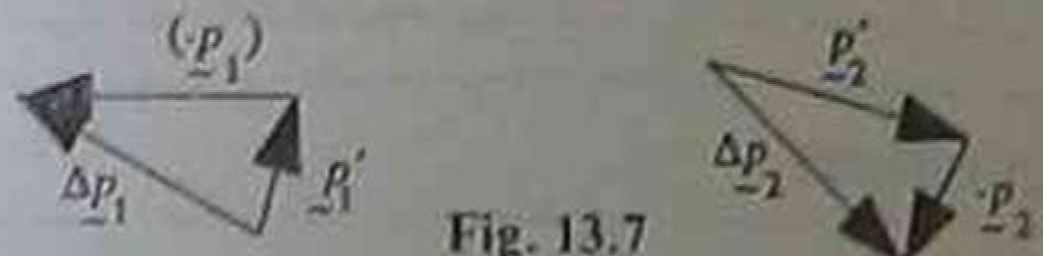


Fig. 13.7

As can be seen from Figure 13.7 and Equation 1:

$$\Delta\underset{\sim}{p}_1 = -\Delta\underset{\sim}{p}_2$$

EXAMPLE

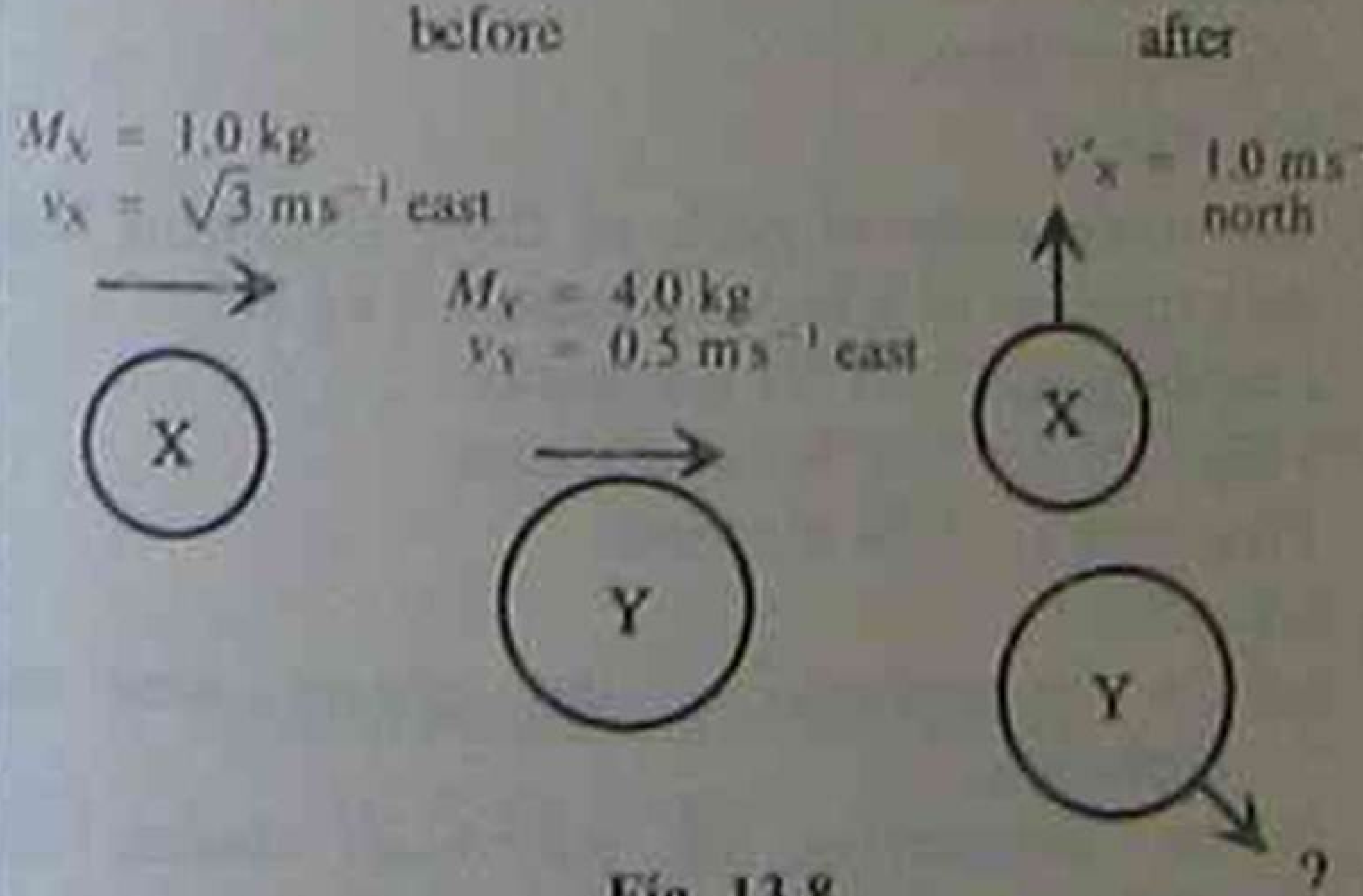


Fig. 13.8

Calculate:

(a) the initial momentum of ball X;

(b) the final momentum of ball X;

(c) the impulse of ball X;

(d) the initial momentum of ball Y.

(e) the impulse of ball Y.

(f) Sketch a vector diagram that shows how to calculate the final momentum of ball Y.

Answer

(a) $\underset{\sim}{p}_x = 1.0 \times \sqrt{3} = \sqrt{3}$ N s east

(b) $\underset{\sim}{p}'_x = 1.0 \times 1.0 = 1.0$ N s north

(c) $\Delta \underset{\sim}{p}_x = \underset{\sim}{p}'_x + (-\underset{\sim}{p}_x)$

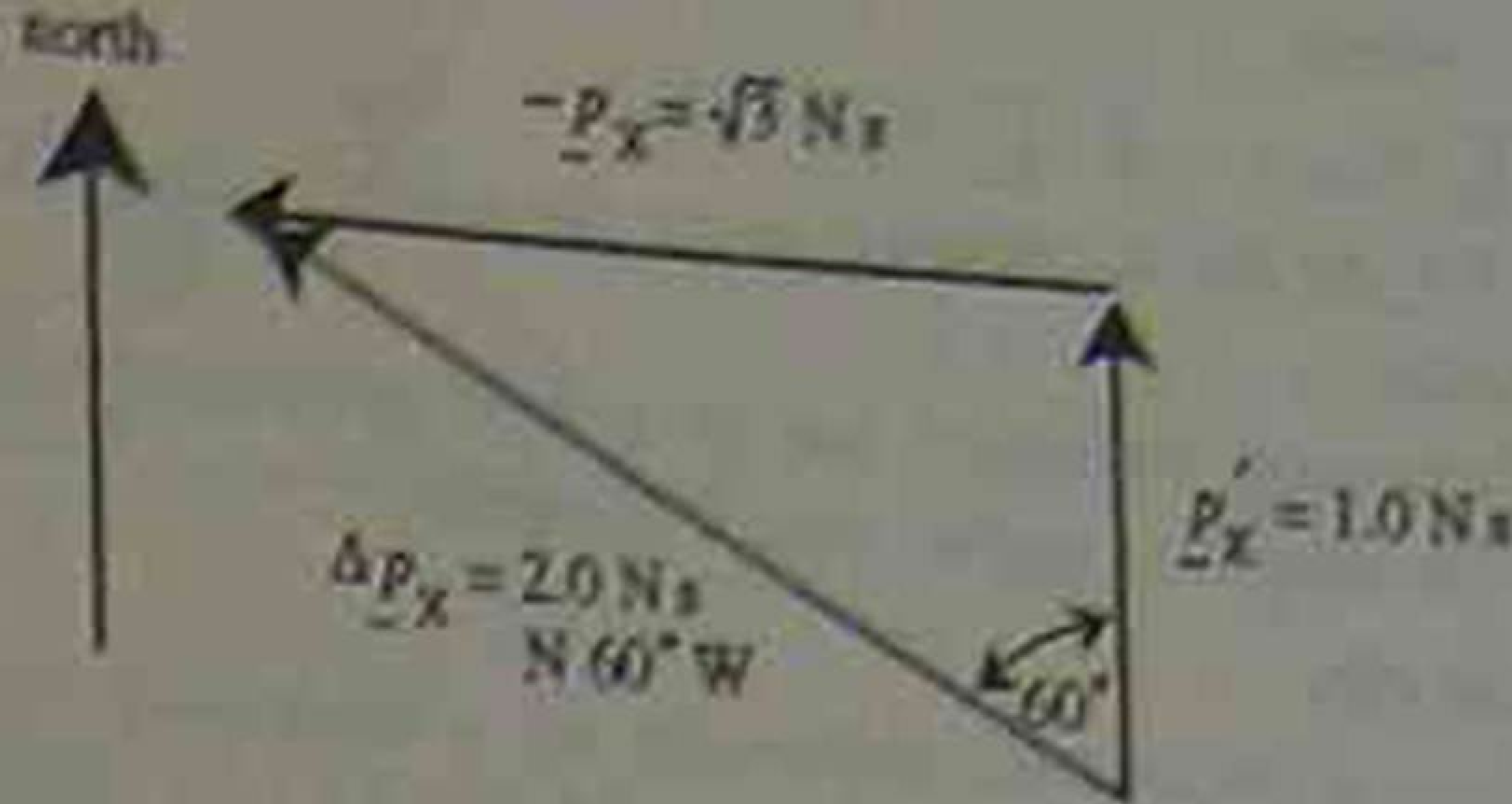


Fig. 13.9

(d) $\underline{p}_y = 4.0 \times 0.5$ N s east $= 2.0$ N s east

(e) $\Delta\underline{p}_y = -\Delta\underline{p}_x = 2.0$ N s south 60° east

(f) $\underline{p}'_y = \Delta\underline{p}_y + \underline{p}_y$

north

$\Delta \underline{p}_X = 2.0$ N s south 60° east

(f) $\underline{p}'_Y = \Delta \underline{p}_Y + \underline{p}_Y$

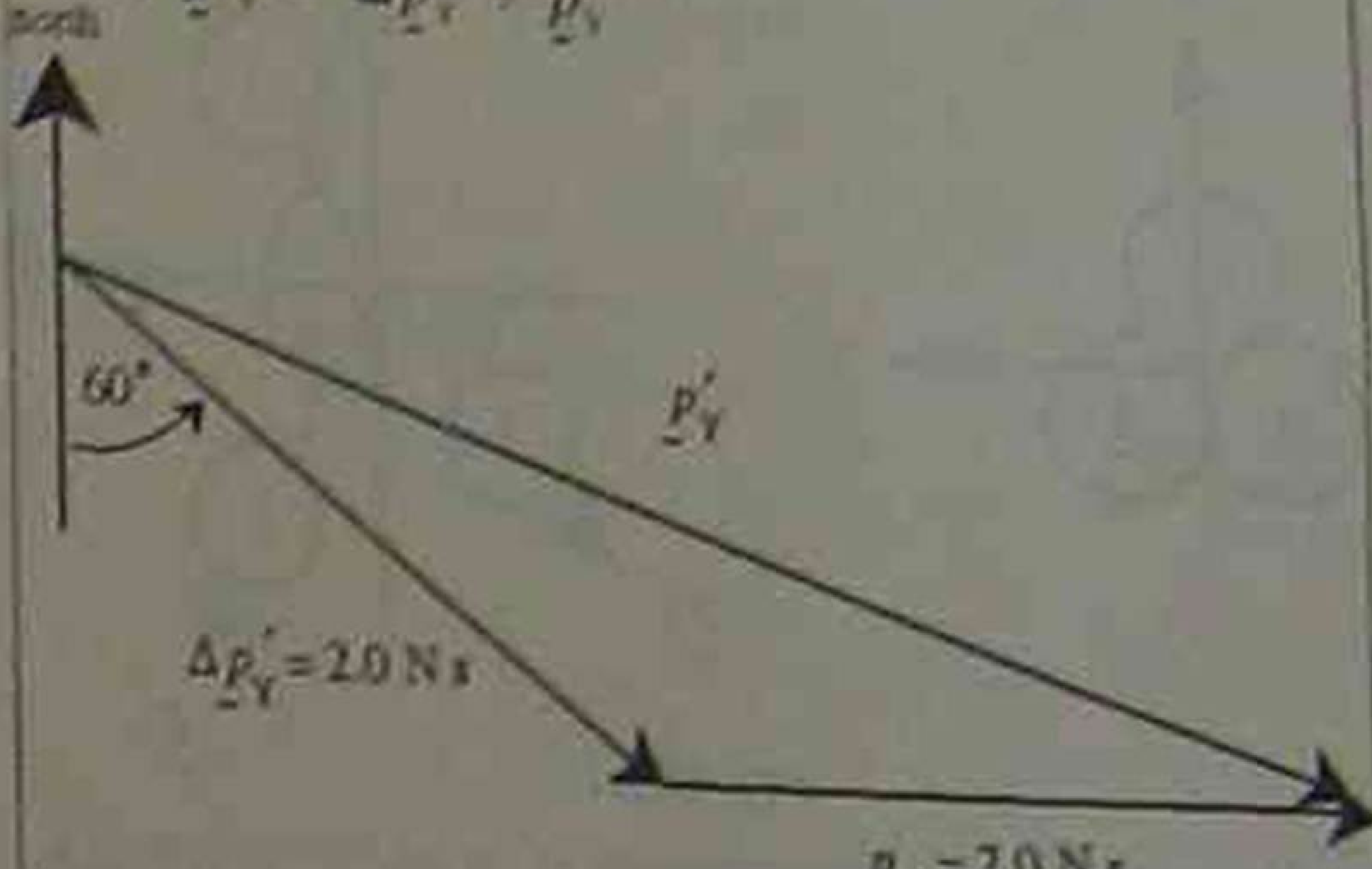


Fig. 13.10

Conservation of momentum during explosions

For a body in an isolated system exploding into separate parts, the total momentum of the parts after the explosion will be equal to the total momentum of the body before the explosion, which is usually zero.

Thus when

$$\sum p_{before} = 0, \sum p_{after} = 0$$

The KE of the exploding parts comes from the PE in the body.

Explosion problems can be solved graphically, that is by constructing scaled vector diagrams.

Graphically

In a two-dimensional explosion of a stationary body into three fragments as shown in Figure 13.11,

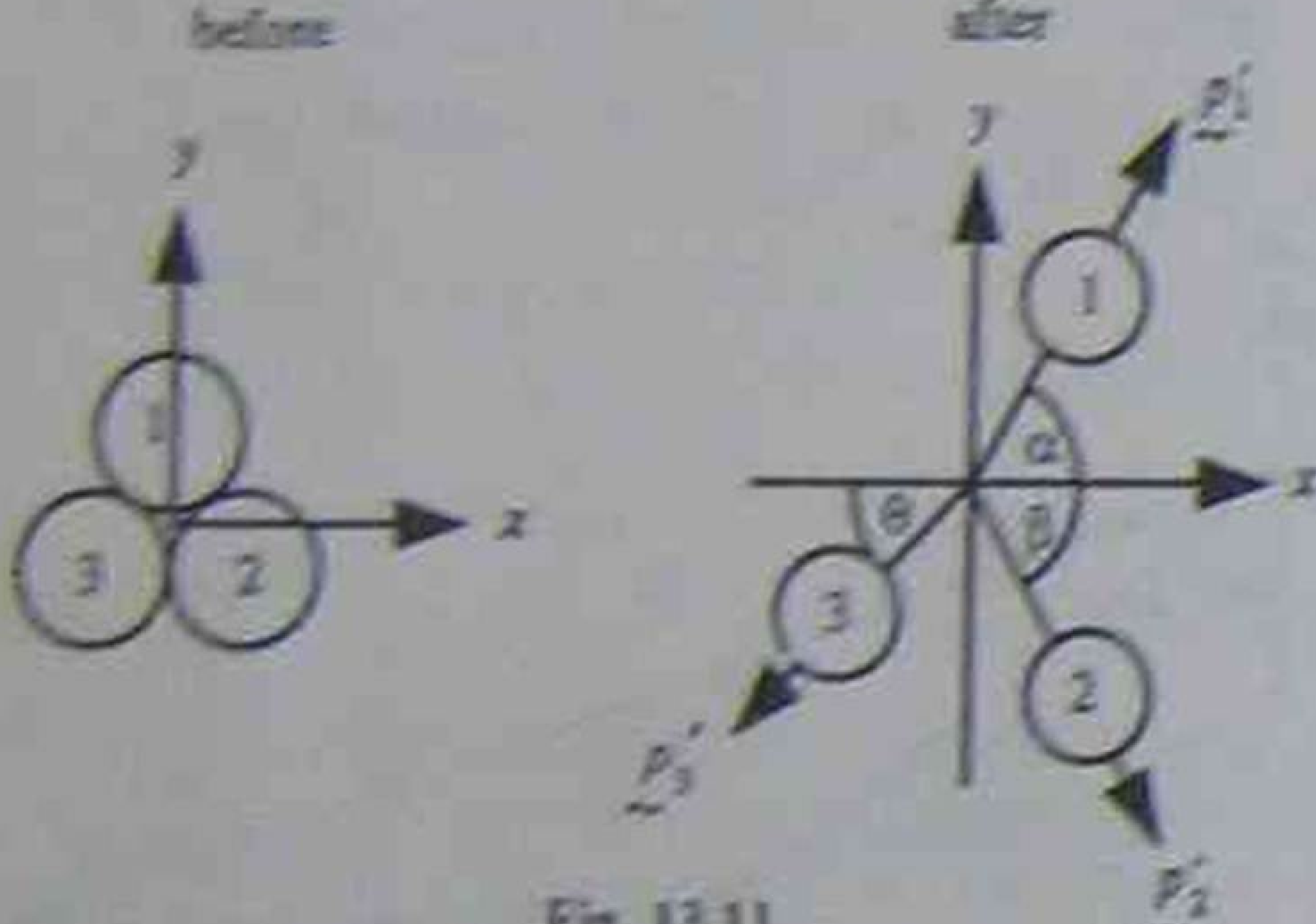


Fig. 13.11

for conservation of momentum $0 = \underline{p}'_1 + \underline{p}'_2 + \underline{p}'_3$,
so the vector polygon used to do this sum is a closed figure as shown in Figure 13.12.

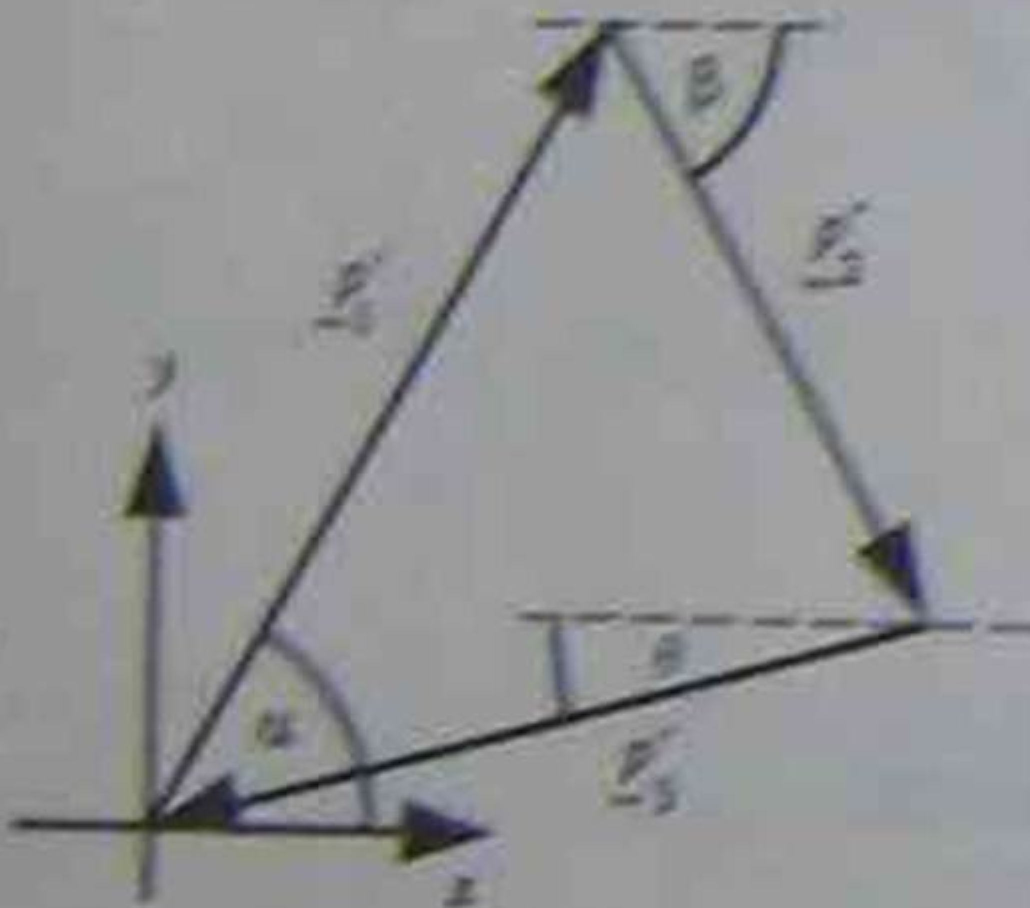


Fig. 13.12

EXAMPLE

A body at rest at the intersection of the x and y axes explodes into three parts:

Part 1 of mass $m_1 = 2$ kg with velocity
$v_1 = 4\ \text{m s}^{-1}$ north-east
Part 2 of mass $m_2 = 2$ kg with velocity
$v_2 = 3\ \text{m s}^{-1}$ south-east
Part 3 of mass $m_3 = 0.5$ kg.

Calculate:

(a) the momentum of Part 3 after the explosion;
(b) the velocity of Part 3 after the explosion.

Answer

(a) $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$

$\vec{p}_1 = 2 \times 4 = 8$ N s north-east

$\vec{p}_2 = 2 \times 3 = 6$ N s south-east

$\vec{p}_3 = ?$

Using the vector polygon rule and a suitable scale of 1 cm = 2 N s:

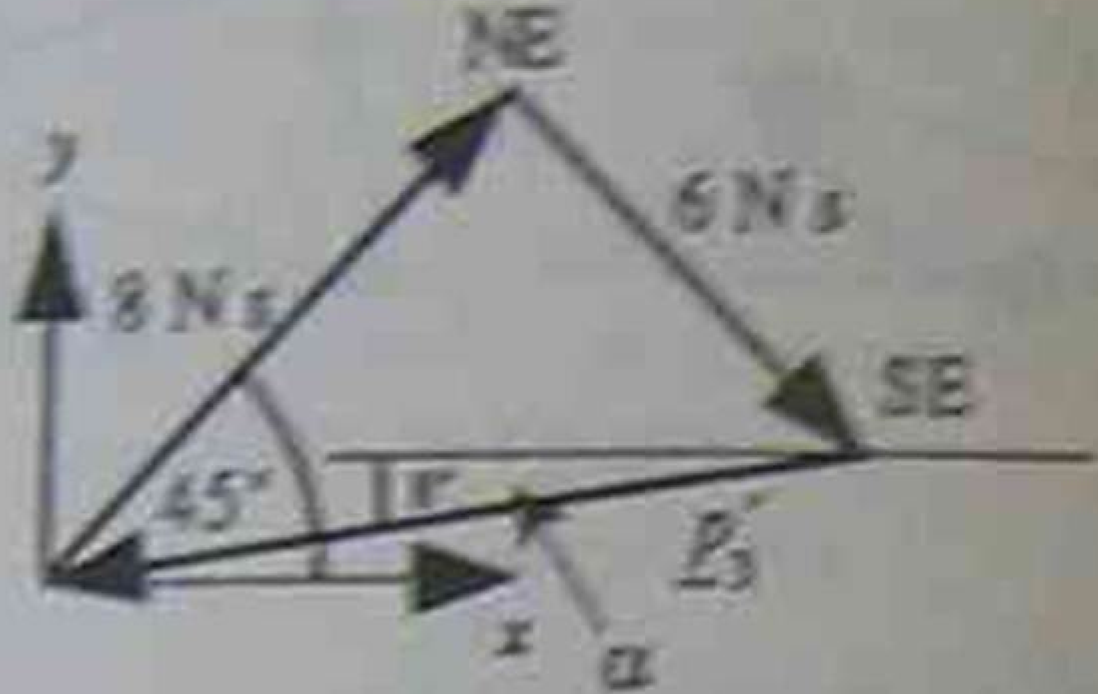


Fig. 13.13

$\underline{p}_3'$ is 5 cm long

therefore $\underline{p}_3' = 10$ N s west 8° south.

(b) $\underline{v}_3' = \underline{p}_3'/m = 10/0.5$

$= 20\ \mathrm{m\,s^{-1}}$ west 8° south.

ADDITIONAL WORKED EXAMPLE

A stationary rock explodes into four fragments as shown in Figure 13.14(a).

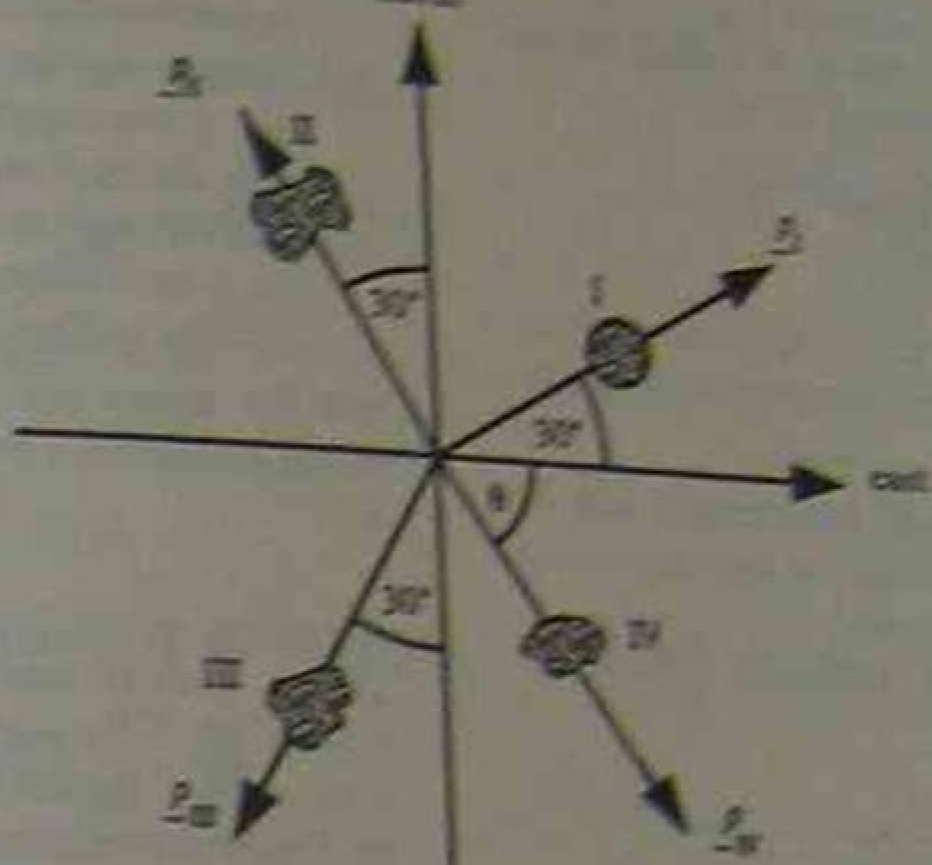


Fig. 13.14(a)

Fragment I $m_1 = 1.0$ kg $v_1 = 2.0$ m s^{-1} E 30° N

Fragment I $m_1 = 1.0\ \text{kg}$ $v_1 = 2.0\ \text{m s}^{-1}$ E 30° N

Fragment II $m_2 = 2.0\ \text{kg}$ $v_2 = 1.0\ \text{m s}^{-1}$ N 30° W

Fragment III $m_3 = 1.0\ \text{kg}$ $v_3 = 3.0\ \text{m s}^{-1}$ S 30° W

Fragment IV $m_4 = 0.2\ \text{kg}$ $v_4 = ?$ E ?° S

Graphically determine Fragment IV's:

(a) direction θ;

(b) momentum $\underline{p}_{IV}$;

(c) velocity $\underline{v}_4$

Answer

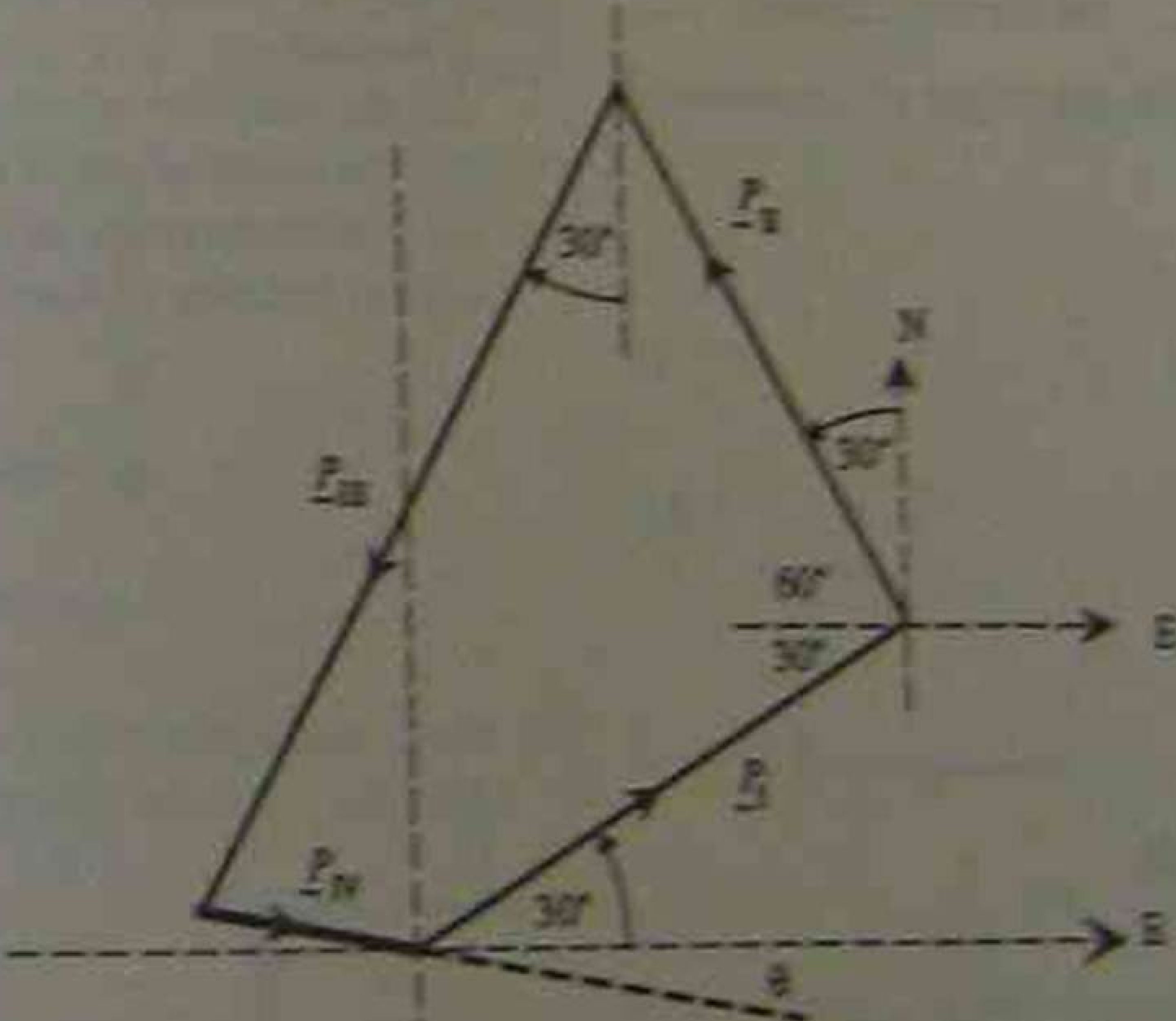


Fig. 13.14(b)

For conservation of momentum

$$\underline{p}_{i} + \underline{p}_{ii} + \underline{p}_{iii} + \underline{p}_{iv} = 0$$

$\underline{p}_{i} = 2.0\ \text{Ns E } 30^\circ \text{ N}$ $\qquad$ $\underline{p}_{iii} = 3.0\ \text{Ns S } 30^\circ \text{ W}$

$\underline{p}_{ii} = 2.0\ \text{Ns N } 30^\circ \text{ W}$ $\qquad$ $\underline{p}_{iv} = \ ? \quad \text{E } \theta^\circ \text{ S}$

Using the polygon of vectors rule and a suitable scale 1 cm = 0.2 Ns we get

(a) θ; E 10° S

(b) $\underline{p}_{iv} = 0.78$ Ns E 10° S

(c) $\underline{v}_{4} = 3.9\ \text{ms}^{-1}$ E 10° S

Key facts and equations

- Impulse or change of momentum

$$\Delta \underline{p} = (\underline{p} - \underline{p})$$

is determined graphically using a scaled vector diagram. See Figures 13.1(a) and 13.1(c).

- The Law of Conservation of Momentum is:

$$\sum \underline{p} = \sum \underline{p}'$$

Key: Greek sigma means sum ($\underline{p}'$ is often used to mean momenta after collision or explosion in many places in this text).

- The Law of Conservation of Momentum can be applied to two-dimensional collisions and explosions. Again this can be done graphically.
  For two-dimensional glancing collisions/explosions:
  A. Glancing collision
     See Figure 13.6.
  B. Explosion
     See Figures 13.11 and 13.12.
  For a body at rest before it explodes, $\sum \underset{\sim}{p} = 0$. If the body explodes to form three fragments, $\sum \underset{\sim}{p}' = 0$, therefore $\underset{\sim}{p}'_1 + \underset{\sim}{p}'_2 + \underset{\sim}{p}'_3 = 0$, which is represented by a vector triangle in which the end of $\underset{\sim}{p}'_3$ is at the start of $\underset{\sim}{p}'_1$ as shown in Figure 13.13.