

Electrical interactions II:
Electrodynamics

SYMBOL AND UNIT SUMMARY

Symbol	Quantity	Unit
q	electric charge	C
i	electric current	A
V	potential difference	V
R	resistance	Ω
r	internal resistance	Ω
ϵ	emf of battery	V
α	resistivity	$\Omega \cdot m$
β	temperature coefficient of resistance	$^{\circ}C^{-1}$
P	power	W
W	work, energy	J

Electric current

An electric current consists of moving electric charges. The unit of current is the ampere (A). A current of one ampere is flowing if one coulomb of charge passes a given point in one second.

$$q = It$$

In this equation q is the charge in coulombs (C), I is the current in amperes, and t is the time in seconds.

In solids the charges that move are electrons. The protons in a solid are locked up in the nuclei of atoms and are not free to move about.

and are not free to move about.

Although current in solids is due to the movement of electrons, we often think in terms of conventional current which is imagined to be a flow of positive charge. Electrons flow from negative to positive but we imagine the current to be positive charge flowing from positive to negative. The origin of this potentially confusing situation lies in the way in which our understanding of electricity developed.

Drift velocity

In a solid conductor, current flow is due to electrons moving from the negative terminal of a battery to the positive terminal. However, conventionally, current flows from positive to negative. The average velocity of the conduction electrons is called the drift velocity:

$$I = nev,$$

where I is current, e the charge on one electron, n the number of conduction electrons per metre length of conductor, and v is the drift velocity.

Alternatively:

$$I = nevA,$$

where, in this case, n is the number of free electrons per cubic metre of the conducting material, and A is the cross-sectional area of the conductor in square metres.

Ohm's Law

The current in a conductor is proportional to the potential difference applied:

$$V \propto I$$

We can insert a constant, R , the resistance, to obtain

$$V = IR$$

$$V = IR$$

The unit of resistance is the ohm (Ω). A conductor has a resistance of one ohm if a current of one ampere flows in it when the potential difference is one volt.

If V is plotted against I so that V is the vertical axis, the graph is a straight line of slope R .

In electronics it is not unusual to come across conductors that do not obey Ohm's Law, i.e. non-ohmic conductors, but this is beyond the scope of this topic.

Resistivity

The resistance of a uniform wire is proportional to its length and inversely proportional to its cross-sectional area:

$$R \propto \frac{l}{a},$$

where l is the length of the wire in metres and a is its cross-sectional area in square metres. We can insert a constant and obtain:

$$R = \frac{\rho l}{a},$$

where ρ is the resistivity of the material of the wire. Resistivity is the resistance of a one metre cube of the material.

Temperature and resistance

As the temperature of a wire increases, so does its resistance. The resistance R_t at temperature $t^\circ\text{C}$ is given by:

$$R_t = R_0(1 + \alpha t),$$

where R_0 is the resistance at 0°C and α is the temperature coefficient of resistance for the material composing the wire.

composing the wire.

Most metals become perfect conductors, i.e. have zero resistance, at temperatures near absolute zero (i.e. near -273°C). Some materials have recently been shown to become superconductors at much higher temperatures.

If R_t is plotted against t so that R_t is the vertical axis, the graph cuts the R_0 axis at R_0 and is a straight line with slope $R_0 \alpha$.

Resistors in series

Two resistors R_1 and R_2 in series can be replaced with a single resistance R which has the same effect where:

$$R = R_1 + R_2$$

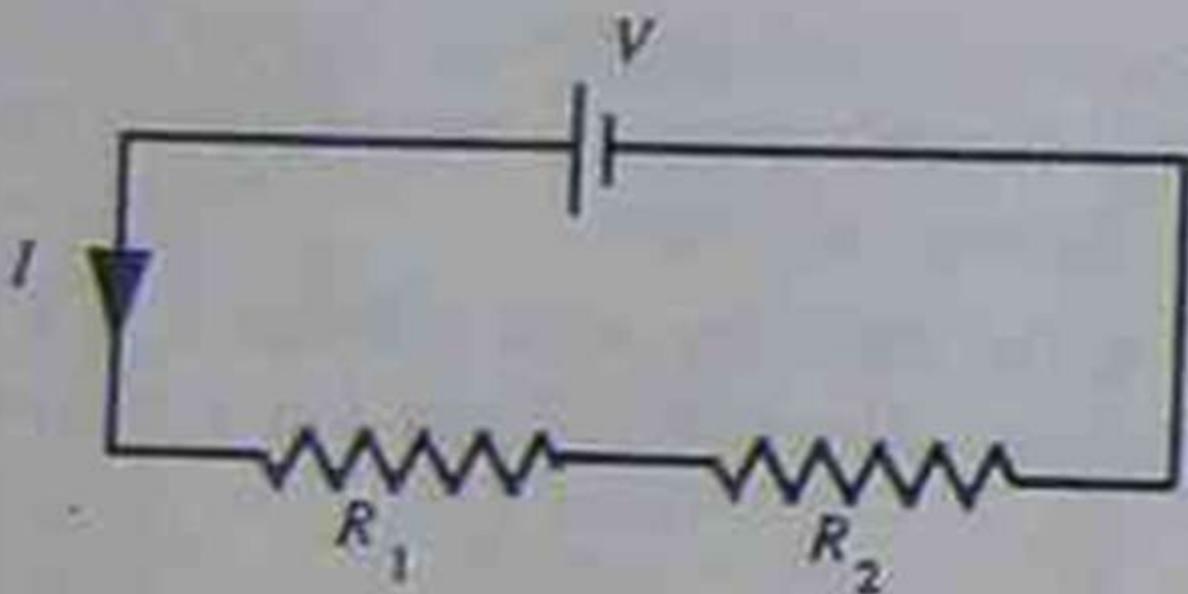


Fig. 14.1

The same current I flows in each resistor. The potential difference V_1 across R_1 is given by IR_1 . The potential difference V_2 across R_2 is given by IR_2 .

$$V = V_1 + V_2$$

$$IR = IR_1 + IR_2$$

There is no potential difference across the connecting wires in the circuit.

EXAMPLE

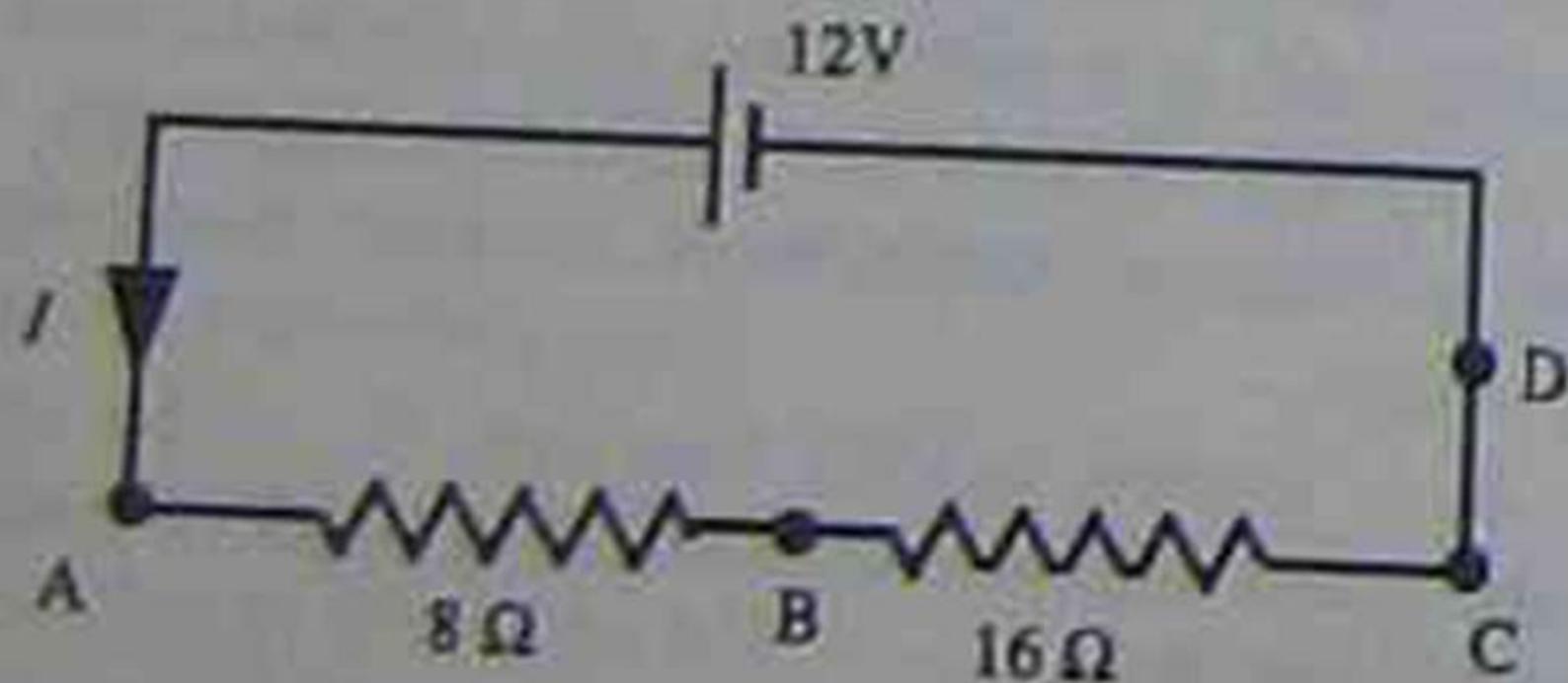


Fig. 14.2

Fig. 14.2

For the circuit shown in the diagram calculate:

- (a) the total resistance of the circuit;
- (b) the current in the circuit;
- (c) the potential difference between:
 - (i) A and B
 - (ii) B and C
 - (iii) A and C
 - (iv) C and D
 - (v) B and D.

Answer

$$(a) R = R_1 + R_2 = 8 + 16 \\ = 24 \Omega$$

$$(b) V = IR$$

$$\text{Hence } I = \frac{V}{R} = \frac{12}{24} \\ = 0.5 \text{ A}$$

$$(c) (i) V_{AB} = IR_1 = 0.5 \times 8 \\ = 4 \text{ V}$$

$$(ii) V_{BC} = IR_2 = 0.5 \times 16 \\ = 8 \text{ V}$$

$$(iii) V_{AC} = V_{AB} + V_{BC} \\ = 4 + 8 \\ = 12 \text{ V}$$

(iv) zero

(v) 8 V

Resistors in parallel

Two resistors R_1 and R_2 in parallel can be replaced by an equivalent resistance R which has the same effect on the circuit, where:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

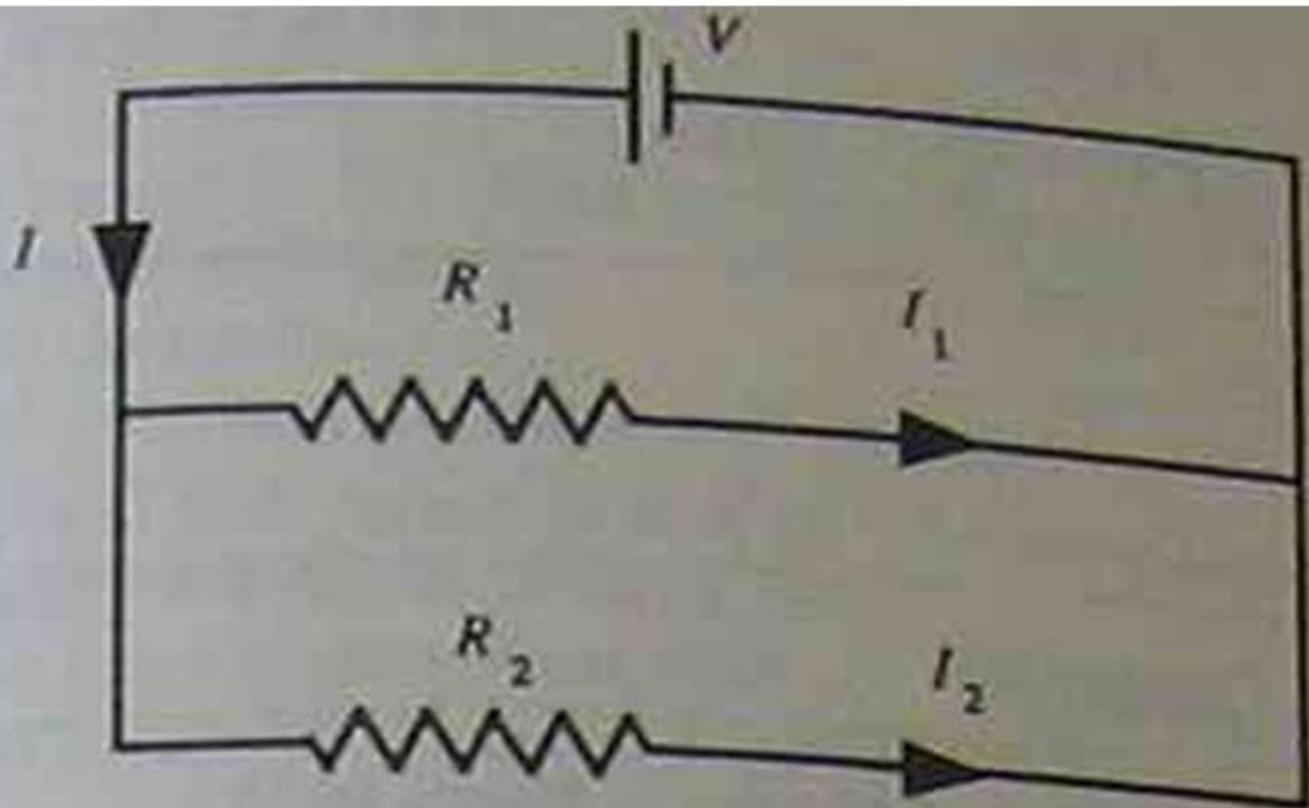


Fig. 14.4

The voltage across $R_1 = V$ and this equals the voltage across R_2 :

$$V = I_1 R_1 = I_2 R_2$$

Also

$$I = I_1 + I_2$$

EXAMPLE

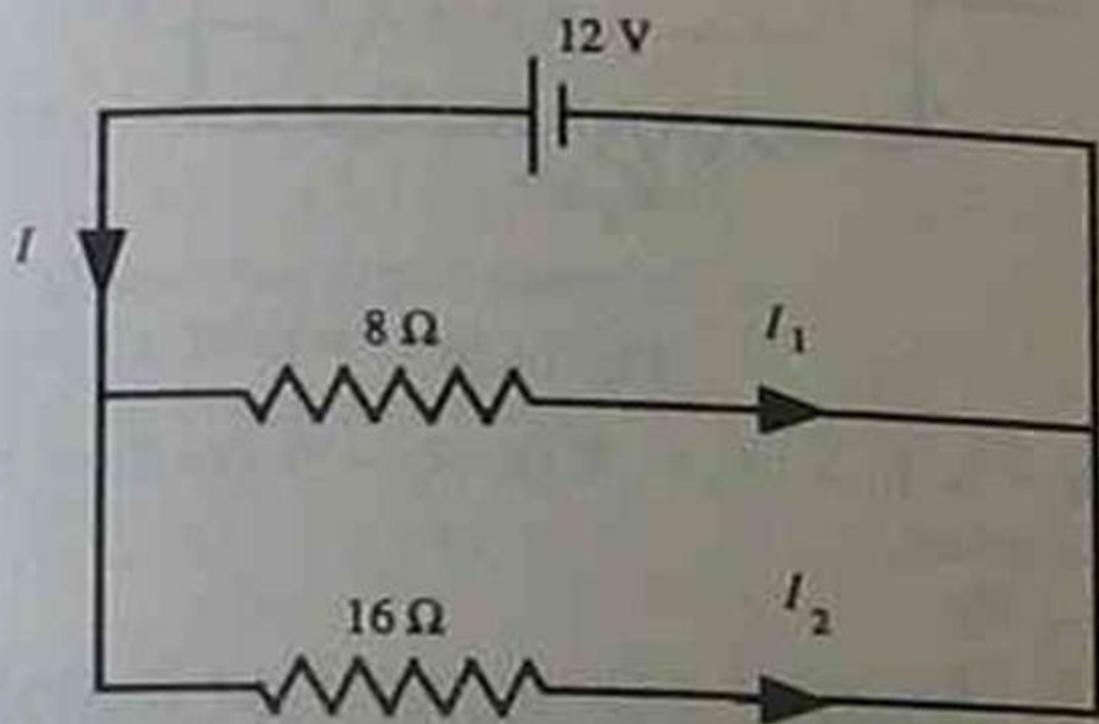


Fig. 14.5

For the circuit shown in the diagram, calculate:

- the total resistance of the circuit;
- the current I ;
- the current I_1 ;
- the current I_2 .

Answer

$$\begin{aligned} \text{(a)} \quad \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{1}{8} + \frac{1}{16} = \frac{3}{16} \end{aligned}$$

$$R = \frac{16}{3} = 5.3$$

$$\begin{aligned} \text{(b)} \quad I &= \frac{V}{R} = \frac{12}{5.3} \\ &= 2.3 \text{ A.} \end{aligned}$$

(c) The potential difference across the 8Ω resistor is 12 V.

$$I_1 = \frac{12}{8} = 1.5 \text{ A}$$

(d) The potential difference across the 16Ω resistor is 12 V.

$$I_2 = \frac{12}{16} = 0.75 \text{ A}$$

(Also $I_2 = I - I_1 = 2.3 - 1.5 = 0.8$ A.
The difference between 0.75 A and 0.8 A is
due to rounding off decimals.)

Simple networks

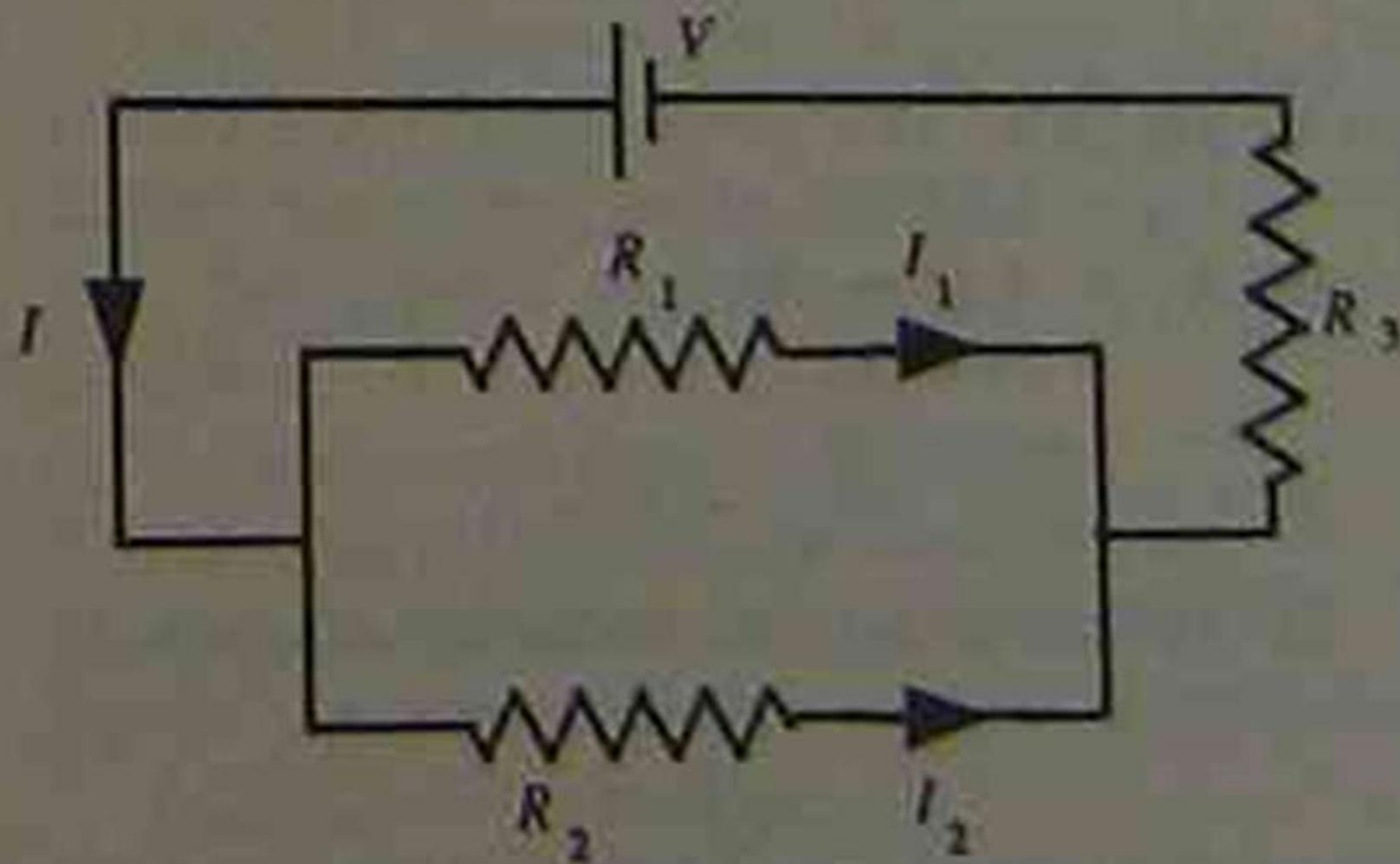


Fig. 14.8

To calculate the total resistance of the circuit in Figure 14.9, first calculate the parallel resistance R_p :

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

Then calculate the total resistance R_T :

$$R_T = R_p + R_3$$

The current I can then be found:

$$I = \frac{V}{R_T}$$

The potential difference across R_1 can then be found:

$$V_2 = IR_2$$

We can now calculate the potential difference across the parallel part of the circuit in two possible ways:

$$V_2 = IR_2$$

$$V_2 = V - V_1$$

The value of I can now be found:

$$I = \frac{V_2}{R_2}$$

The value of E can be found by two methods:

$$E = \frac{V_1}{R_1} \quad \text{or} \quad E = I - I_1$$

EXAMPLE

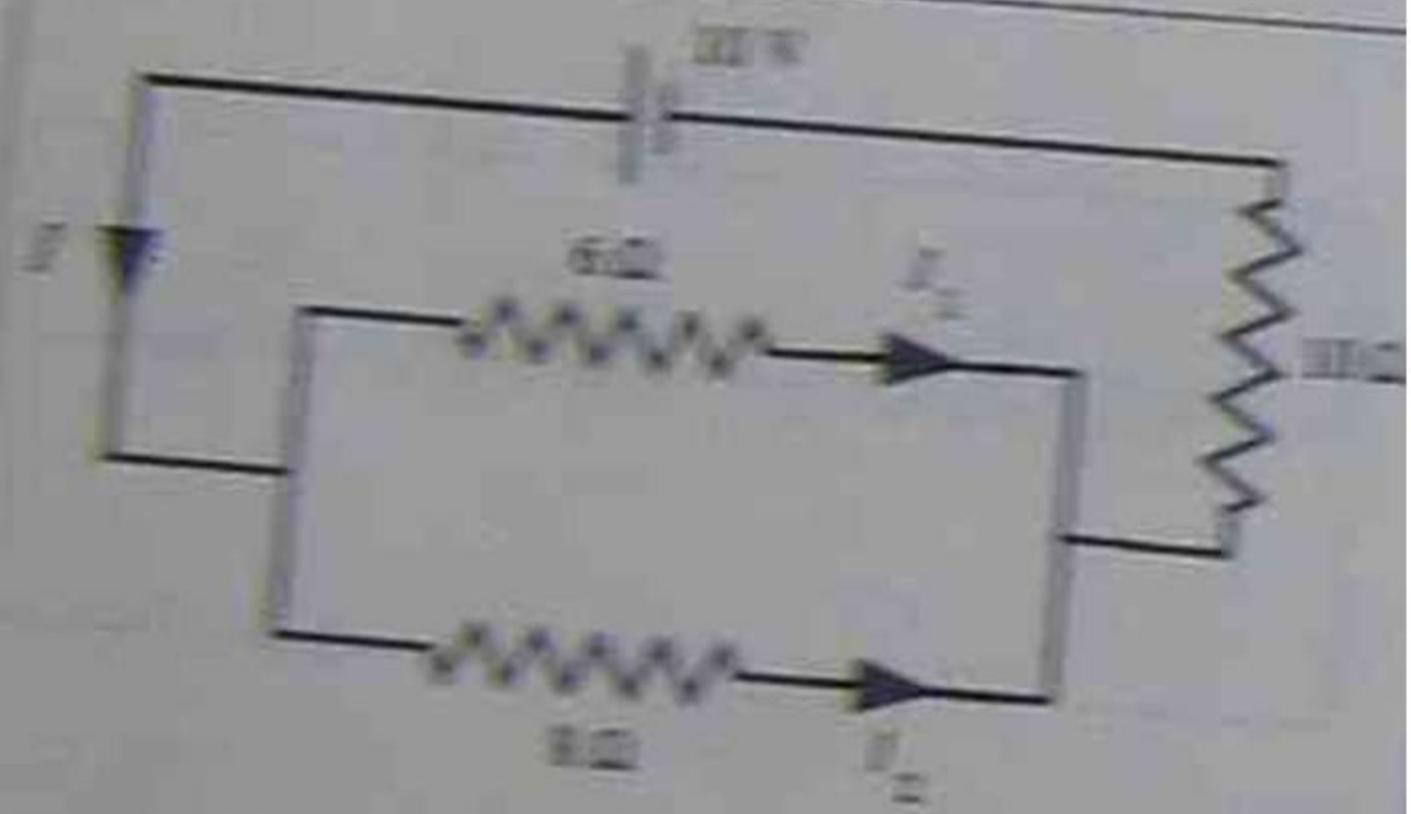


Fig. 14.9

For the circuit shown in Figure 14.9 calculate:
(a) the resistance of the parallel portion of the circuit.

(b) the total resistance

(c) I

(d) V_1

(e) V_2

Solve

$$(a) \frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{8}$$

$$R_{eq} = 2.67 \Omega$$

$$(b) R_{12} = 2.67 + 10 \\ = 12.67 \Omega$$

$$(c) I = \frac{V}{R_{12}} = \frac{10}{12.67} = 0.79 \text{ A}$$

(d) Voltage across the parallel part of circuit V_1 is

$$V_1 = IR_1 \\ = 2.4 \text{ V}$$

Power

When a current I flows through a resistor R , energy is used up and appears as heat. The rate at which energy is dissipated by a resistor is given by:

$$P = I^2 R$$

where P is power in watts.

Since $V = IR$, we can also write:

$$P = IV \text{ or } P = \frac{V^2}{R}$$

Over a period of time t seconds the energy dissipated is Pt .

Internal resistance of a battery

The components of a battery through which current flows have resistance. The total resistance of a circuit includes this internal resistance of the battery, R .

When a voltmeter is connected across the terminals of a battery on its own there is virtually no current because of the high resistance of the voltmeter. Hence there is no potential difference across R since IR is zero. The reading on the voltmeter is E , the electromotive force (emf) of the battery.

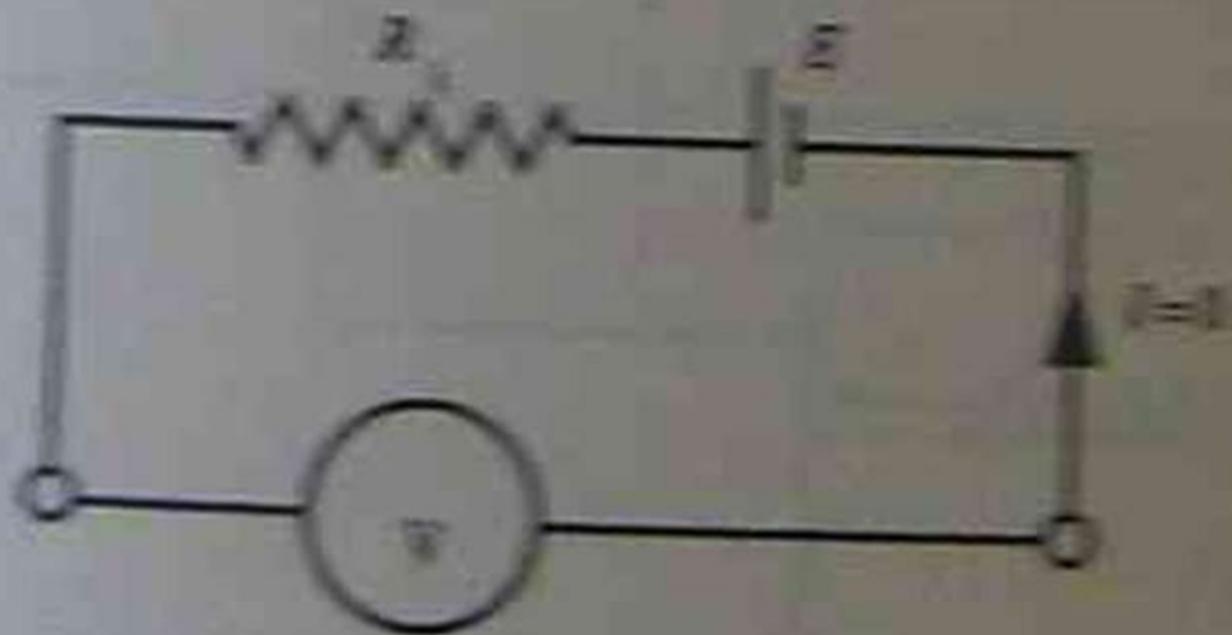


Fig. 14.12

If a resistor R is connected across the terminals of the battery in parallel with the voltmeter, the voltmeter reads the potential difference V across R . This will be IR , and will be less than \mathcal{E} , since some of the emf will be used in R .

We can consider R and R to be in series as shown in Figure 14.13.

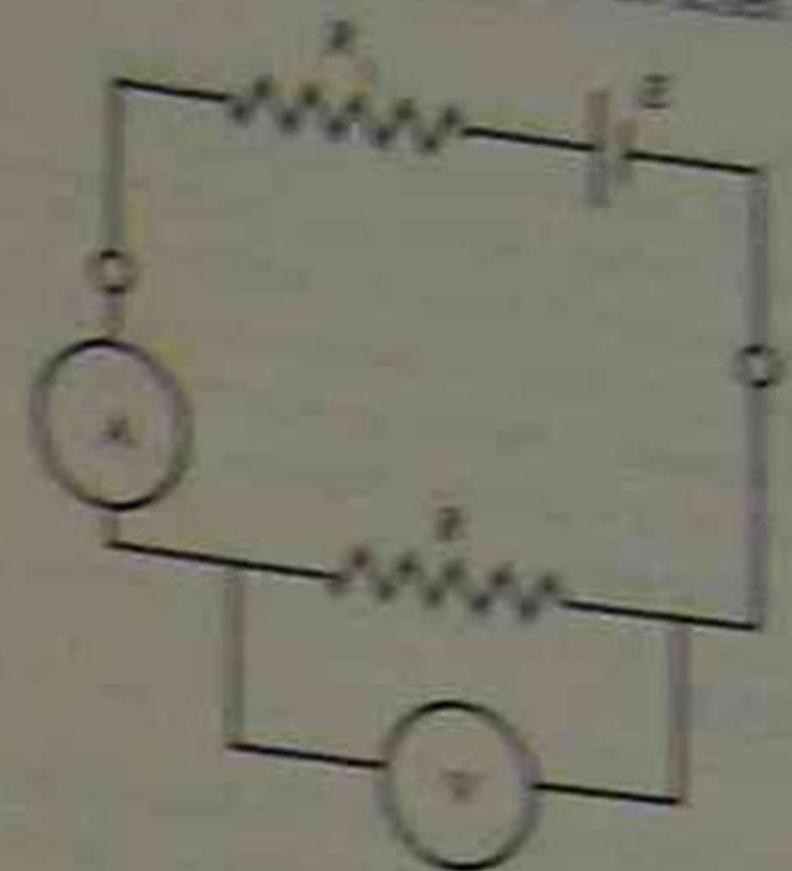


FIG. 14.13

For
 R

$$\mathcal{E} = IR + IR$$

$$IR = V$$

$$\mathcal{E} = IR + V$$

$$R = \frac{\mathcal{E} - V}{I}$$

EXAMPLE

A battery has a voltmeter connected across it and it reads 12 volts. A resistor is now connected across the battery and the reading on the voltmeter becomes 10 V. The current in the resistor is 0.5 A. Calculate the internal resistance of the battery.

Answer

$$\begin{aligned} R &= \frac{E - V}{I} \\ &= \frac{12 - 10}{0.5} \\ &= 4 \Omega \end{aligned}$$

Converting meters

An ammeter or a voltmeter is characterised by its resistance and also the current necessary for it to show a maximum reading. Meters can be interconverted by connecting resistors in series or parallel.

To convert a meter to an ammeter of a particular range you connect a resistor in parallel with the meter. Conversion to a voltmeter requires connection of a resistor in series with the meter.

EXAMPLE

A meter has a resistance of $200\ \Omega$ and shows a maximum scale deflection when a current of $5\ \text{mA}$ flows through it. How would you convert this meter to read:

- (a) 0 to $5\ \text{A}$?
- (b) 0 to $20\ \text{V}$?

Answer

- (a) We connect a resistor in parallel with the meter so that when $5\ \text{mA}$ flows through the meter, $4.995\ \text{A}$ flows through this resistance.

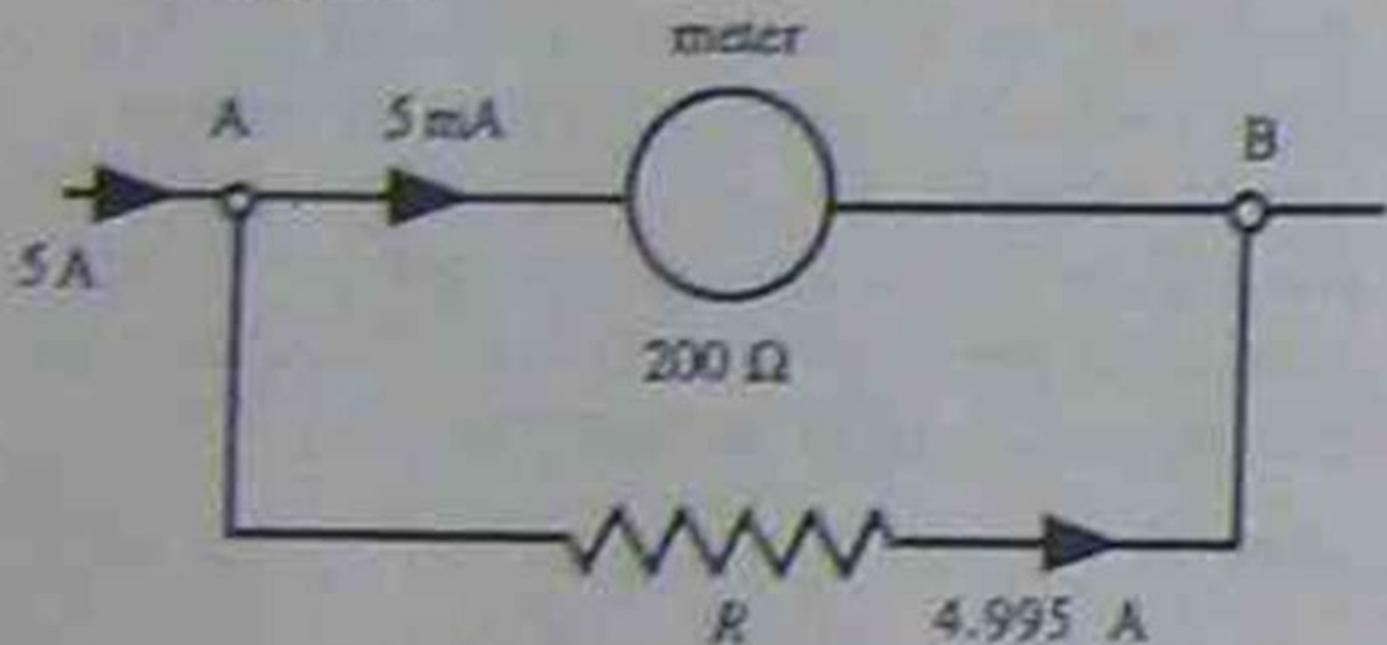


Fig. 14.14

The voltage from A to B is the potential difference across R and also across the meter. Hence

$$4.995R = 0.005 \times 200$$

$$R = \frac{0.005 \times 200}{4.995}$$

$$= 0.2 \Omega$$

- (b) We connect a resistor in series with the meter so that when 5 mA flows through it the potential difference across the meter plus the potential difference across the resistor is 20 V.

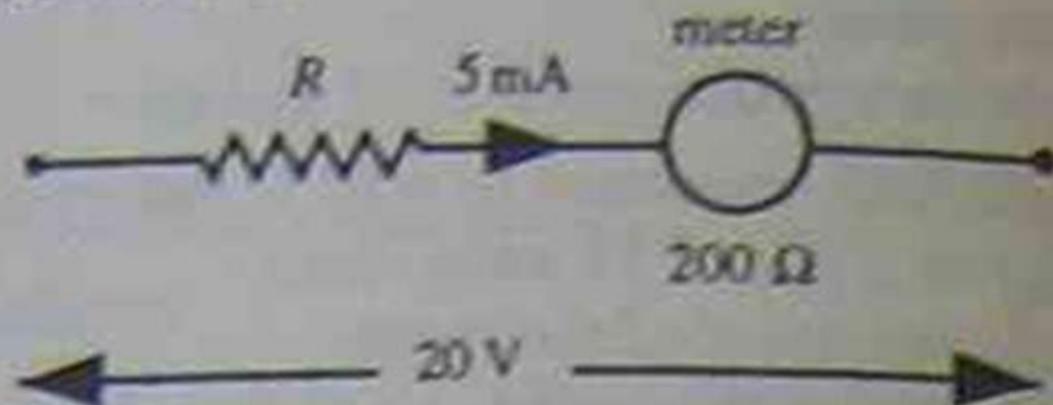


Fig. 14.15

$$\begin{aligned} \text{Then } 20 &= 0.005 \times R + 0.005 \times 200 \\ &= 0.005R + 1 \end{aligned}$$

$$0.005R = 19$$

$$\begin{aligned} R &= \frac{19}{0.005} \\ &= 3800 \Omega \end{aligned}$$

ADDITIONAL WORKED EXAMPLES

1.

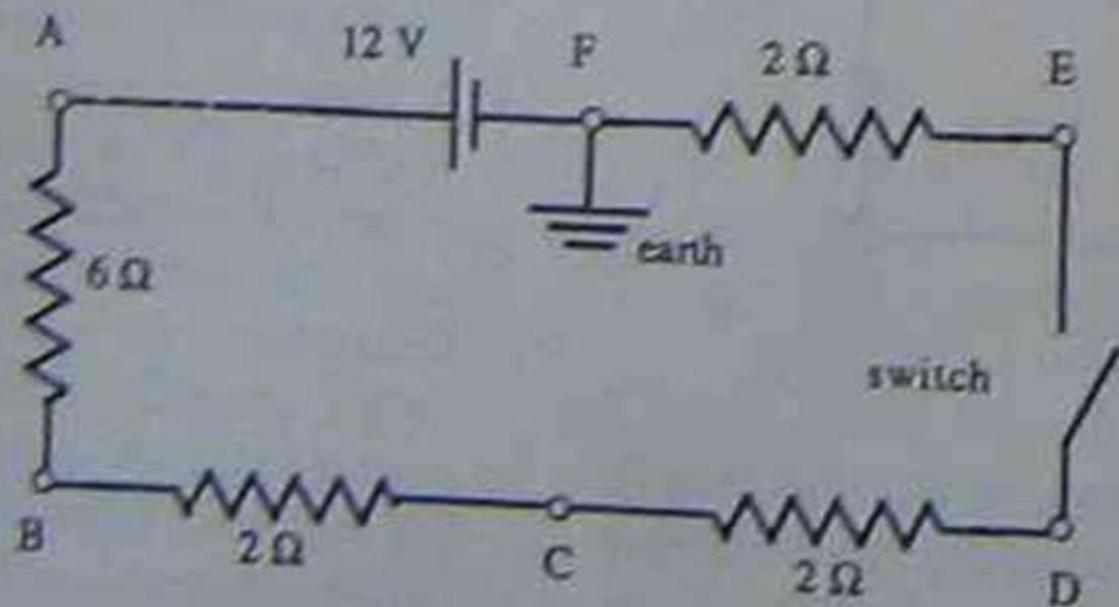


Fig. 14.16

- (a) When the switch in Figure 14.16 is left open, find the potential difference between:
- (i) A & B
 - (ii) B & C
 - (iii) D & E
 - (iv) F & E
- (b) When the switch is closed, find the potential at these points:
- (i) A
 - (ii) B
 - (iii) C
 - (iv) D
 - (v) E
 - (vi) F

Answer

(a) No current flows, so no potential difference exists across any resistor.

(i) zero;

(ii) zero;

(iii) 12 V, since D is at the same potential as A and E is at the same potential as F;

(iv) zero.

(b) Resistance of the circuit = $6 + 2 + 2 + 2$
= 12Ω

Current = 1 A

Potential at A is 12 V and at F is zero. Potential drops as you go around the circuit.

(i) 12 V.

(ii) Potential difference from A to B is $IV = 6$ V. Since A is at a potential of 12 V and B is 6 V lower, the potential at B is 6 V.

(iii) Potential difference from B to C is 2 V. Since B is at a potential of 6 V, C is at a potential of 4 V.

(iv) 2 V.

(v) 2 V since D and E are at the same potential.

(vi) zero.

2.

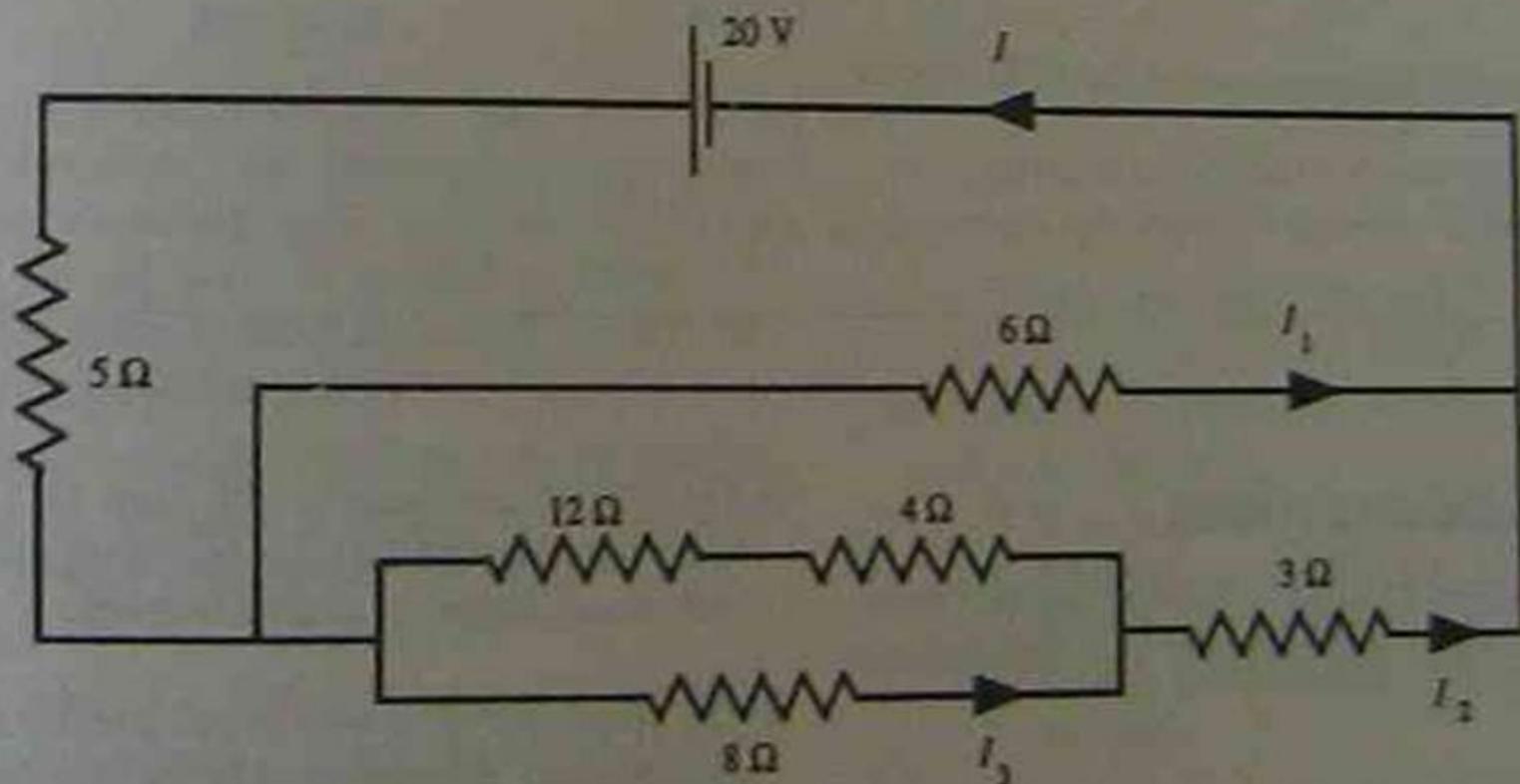


Fig. 14.17

For the circuit shown in the diagram calculate:

- (a) I
- (b) I_1
- (c) I_2
- (d) I_3
- (e) the potential difference across the $12\ \Omega$ resistor.

Answer

(a) Calculate the combined resistance of $8\ \Omega$ in parallel with the $12\ \Omega$ and $4\ \Omega$ resistors:

$$\frac{1}{R} = \frac{1}{8} + \frac{1}{16}$$
$$R = 5.3\ \Omega$$

Calculate the combined resistance of the $6\ \Omega$ resistor in parallel with the $12\ \Omega$, $4\ \Omega$, $8\ \Omega$ and $3\ \Omega$ resistors:

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{8.3}$$
$$R = 3.48\ \Omega$$

Hence the total resistance = $8.48\ \Omega$.

$$I = \frac{V}{R_1} = \frac{20}{8.48}$$
$$= 2.36\ \text{A}$$

$$= 2.36 \text{ A}$$

(b) Potential difference across the 5Ω resistor:

$$\begin{aligned} V &= IR \\ &= 2.36 \times 5 \\ &= 11.79 \text{ V} \end{aligned}$$

Hence the potential difference across the 6Ω resistor is:

$$20 - 11.79 = 8.2 \text{ V}$$

Hence

$$8.2 = I_1 \times 6$$

$$I_1 = 1.37 \text{ A}$$

(c) Potential difference across the 12Ω , 4Ω , 8Ω and 3Ω branch of the circuit is also 8.2 V . Also the resistance of this branch is 8.3Ω .

Hence

$$8.2 = I_2 \times 8.3$$

$$I_2 = 0.99 \text{ A}$$

(d) Potential diff

$$I_3 = 0.99 \text{ A}$$

(d) Potential difference across the 3Ω resistor is $3 \times 0.99 = 2.96 \text{ V}$.

Hence the potential difference across the 12Ω and 4Ω resistors in parallel with 8Ω is:

$$8.2 - 2.96 = 5.24 \text{ V}$$

For the 8Ω resistor,

$$5.24 \text{ V} = 8 \times I_3$$

$$I_3 = 0.65 \text{ A}$$

(e) Potential difference across the 12Ω and 4Ω branch of the circuit is 5.24 V .

Hence the current

$$= \frac{5.24}{16}$$

$$= 0.33 \text{ A}$$

Potential difference across the 12Ω resistor:

$$V = IR$$

$$= 0.33 \times 12$$

$$= 4 \text{ V}$$

Key facts and equations

- An electric current consists of moving electric charge. The unit of current is the ampere (A), one coulomb per second, $q = It$.

- The current in a wire is related to the drift velocity of electrons:

$$I = nevA \text{ or } I = nev$$

Note that the meaning of n is different in each of these two expressions.

- Ohm's Law states that current is proportional to potential difference.

- The unit of resistance is the ohm (Ω). A conductor has a resistance of 1Ω if a potential difference of one volt causes a current of one ampere. $V = IR$.
- The resistance of a wire is proportional to its length and inversely proportional to its cross-sectional area:

$$R = \frac{\rho l}{a}, \text{ where } \rho \text{ is the resistivity.}$$

- The resistance of a wire varies with temperature:

$$R_t = R_0 (1 + \alpha t)$$

where α is the temperature coefficient of resistance, R_0 is the resistance at 0°C , t is the temperature ($^\circ\text{C}$), and R_t is the resistance at temperature t .

- The equivalent of two resistors in series is given by
 $R = R_1 + R_2$

- The equivalent of two resistors in parallel is given by:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

- The power dissipated in a resistor is given by
 $P = IV$, or $P = I^2 R$

or

$$P = \frac{V^2}{R}$$

- The internal resistance of a battery is given by:

$$R_i = \frac{E - V}{I}$$

where E is the emf, V is the potential difference across a load and I is the current.

- To convert ammeters, connect resistors in parallel.
To convert voltmeters, connect resistors in series.