

SYMBOL AND UNIT SUMMARY

Symbol	Quantity	Unit
a	wave — amplitude	m
T	— period	s
I	— intensity	$\text{W m}^{-2} \text{s}^{-1}$
f	— frequency	s^{-1} or Hz
λ	— length	m
v or c	— velocity	m s^{-1}
θ	— phase angle	°
i	angle of incidence	°
r	angle of reflection	°
C	critical angle	°
n_1, n_2	refractive index — medium 1 to medium 2	
f_B	beat frequency	Hz

Reflection and refraction of wave fronts

Reflection

A wave front is a line which may be along the crest or trough of a wave and is at right angles to the wave's direction of travel, identified as a ray

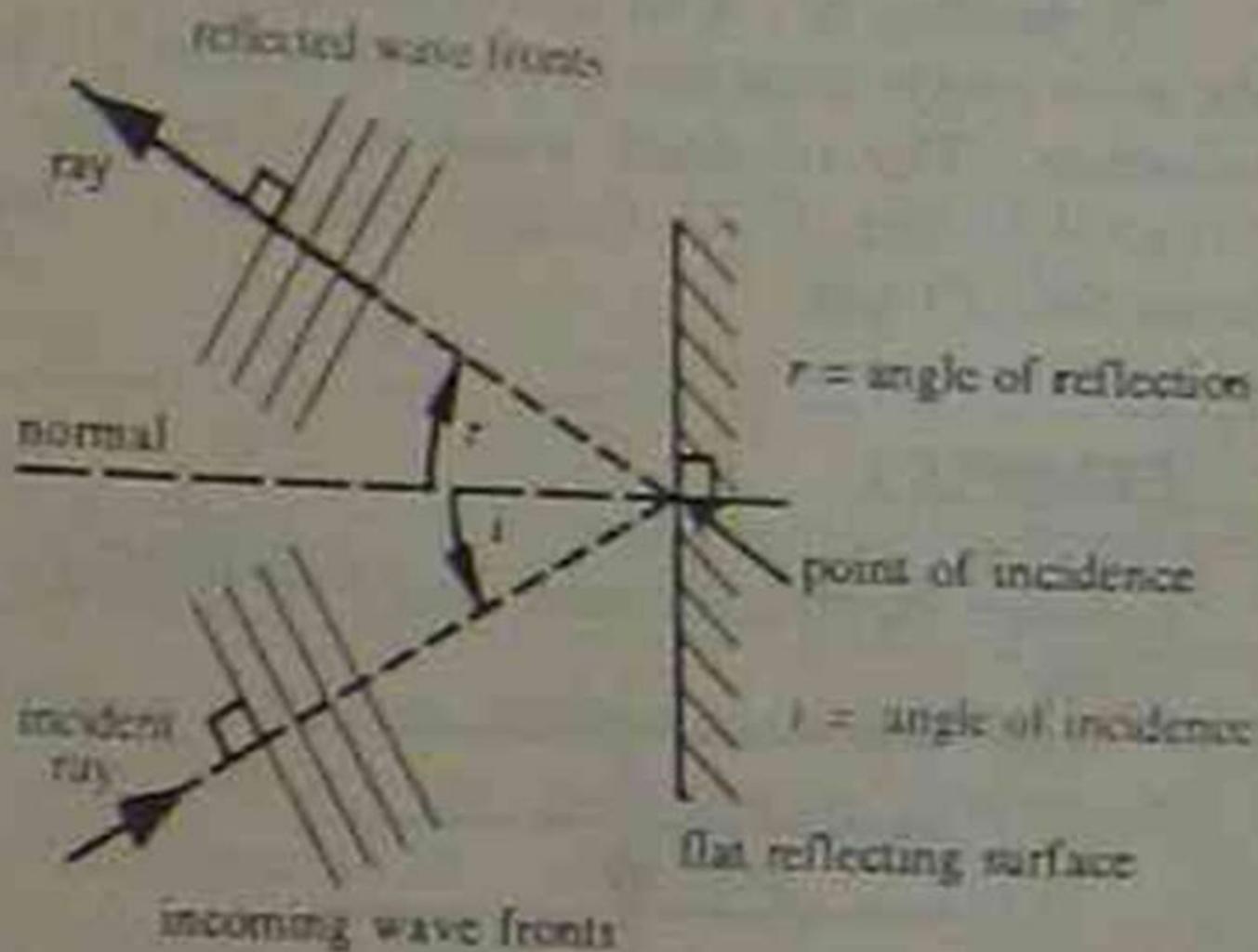


Fig. 16.1

An uneven surface scatters rays.

Reflected waves obey the laws of reflection:

1. Angle of incidence = angle of reflection.
2. The incident ray, normal and reflected ray are all in the same plane at the point of incidence.

Refraction

Fast long waves travelling towards a beach from a medium of deep water into a medium of shallow water on a sand bar slow down and bunch up.

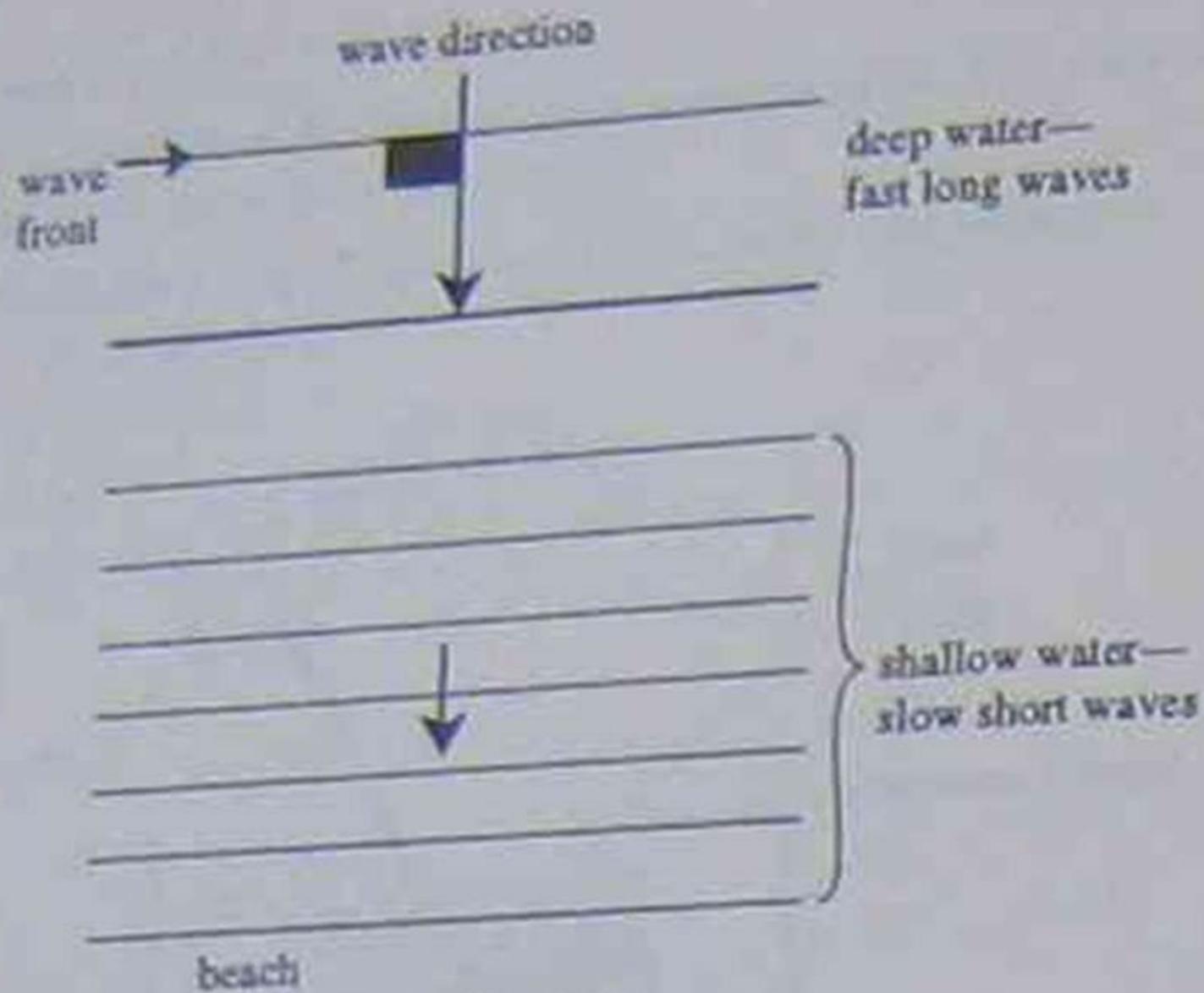


Fig. 16.2

The bending of a wave due to change of velocity as the wave moves from one medium to another is called refraction. The refracted waves obey the laws of refraction. See HSC Course Elective 2, Wave Properties of Light.

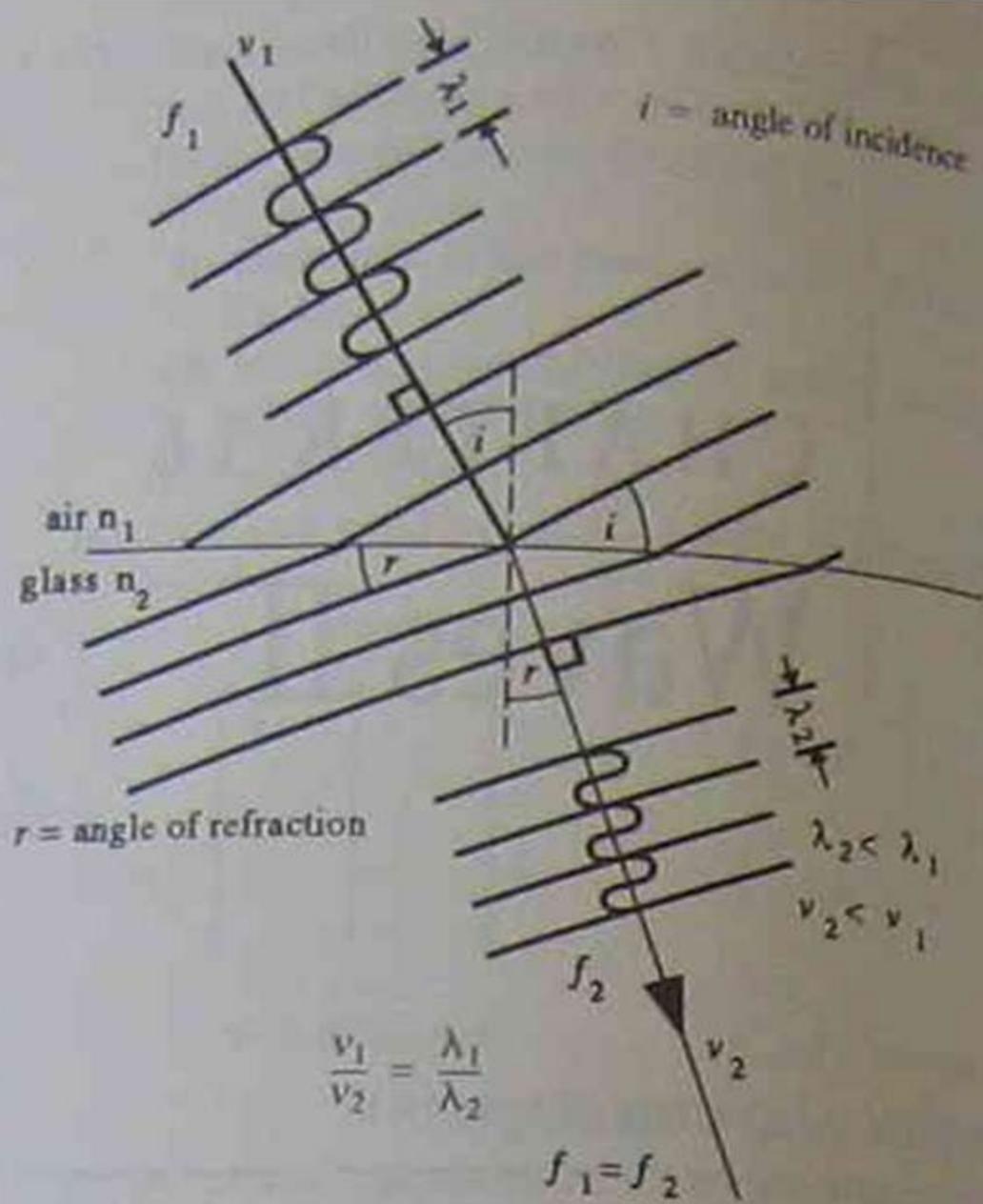
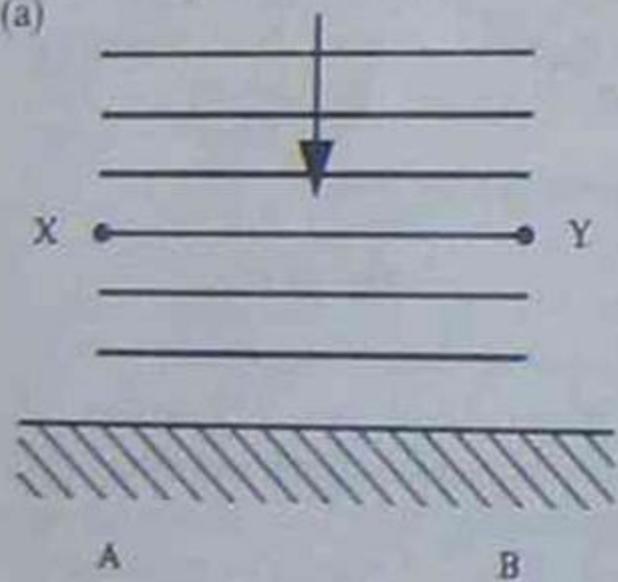


Fig. 16.3

EXAMPLES

i. Draw the reflected wave front from boundary AB in Figure 16.4(a) and (b).

(a)



(b)

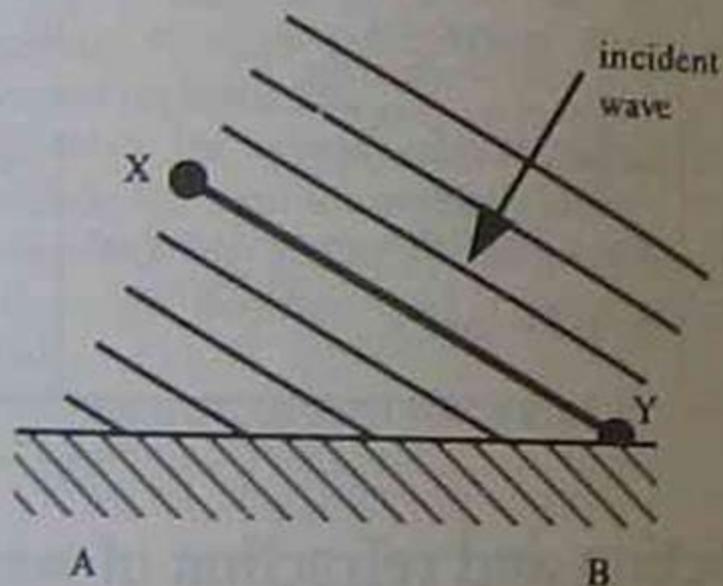
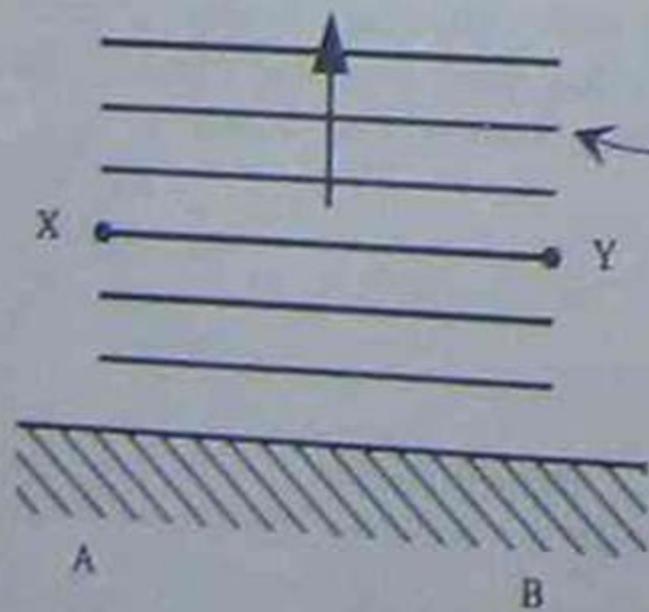


Fig. 16.4

Answer

(a)



reflected
wave
fronts

(b)

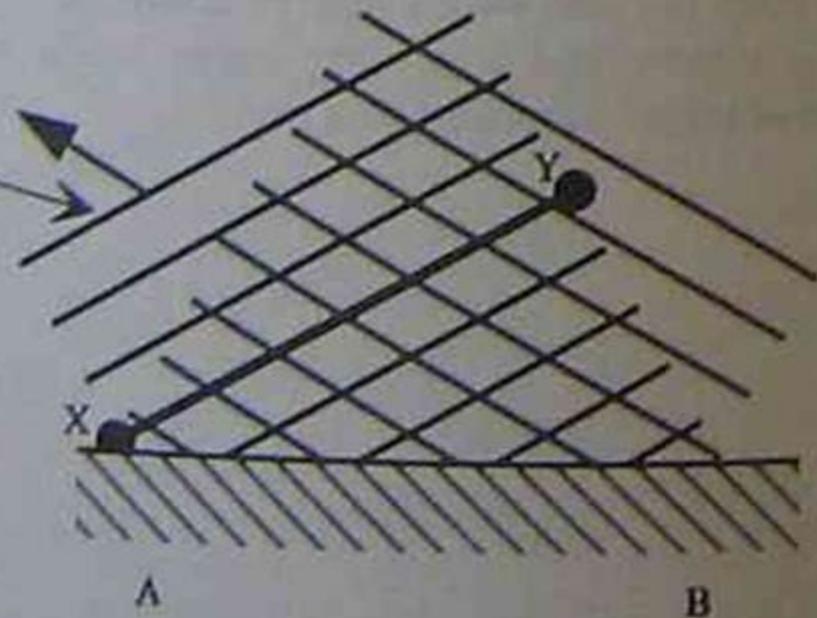


Fig. 16.5

2. Draw the refraction of plane wave fronts approaching a deep to shallow water boundary at an angle to the normal.

Answer

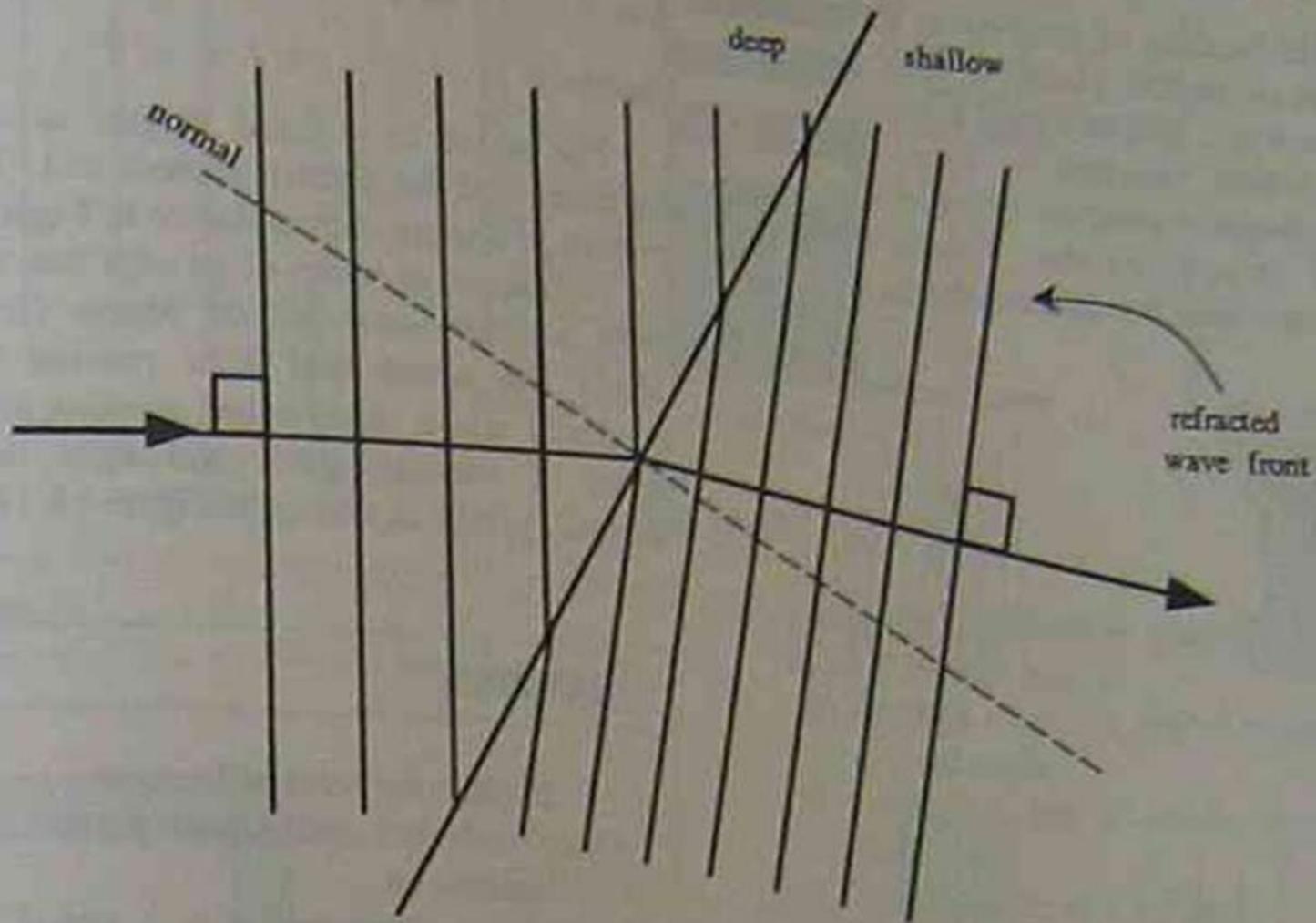


Fig. 16.6

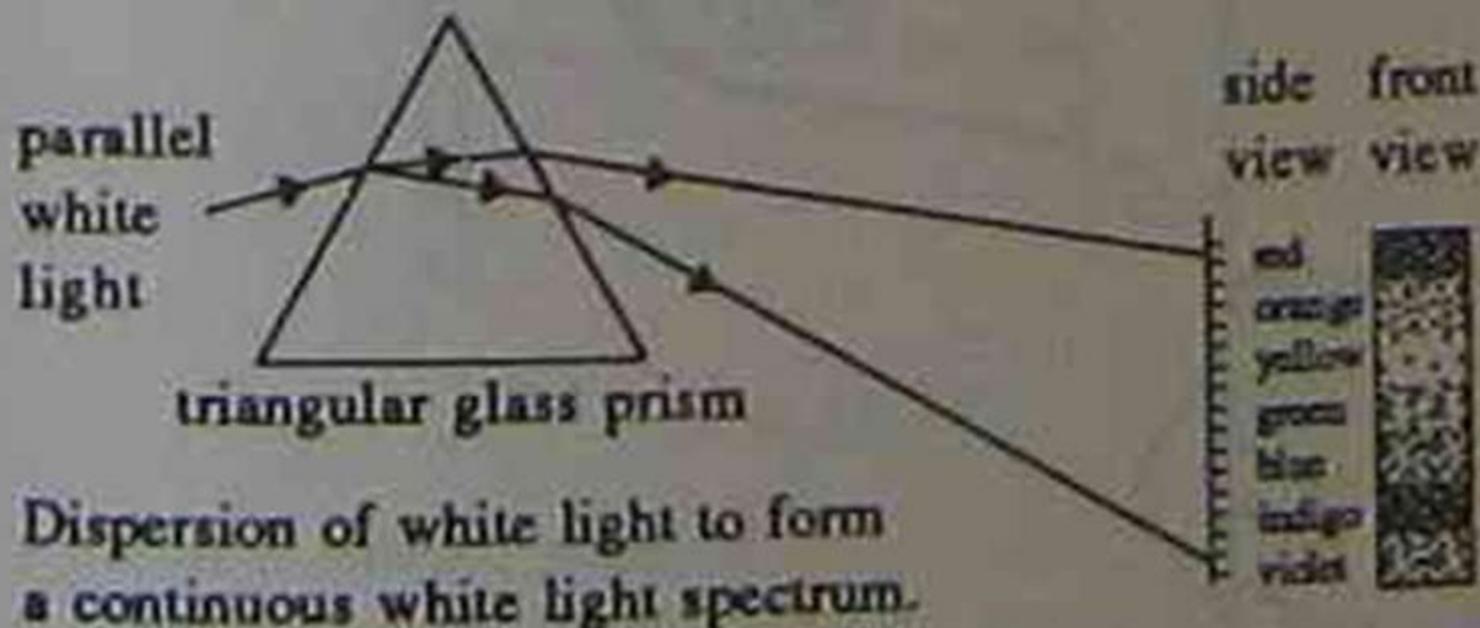
Dispersion of waves

Dispersion is the separation of waves of different frequencies, initially travelling together when they enter a new medium where velocity depends on frequency. Glass, diamond, gemstones, water and oil disperse white light; paraffin wax disperses microwaves, and calcium fluoride disperses infra-red waves.

EXAMPLE

Sketch the continuous spectrum of white light formed when light passes obliquely through a glass prism.

Answer



Dispersion of white light to form a continuous white light spectrum.

Fig. 16.7

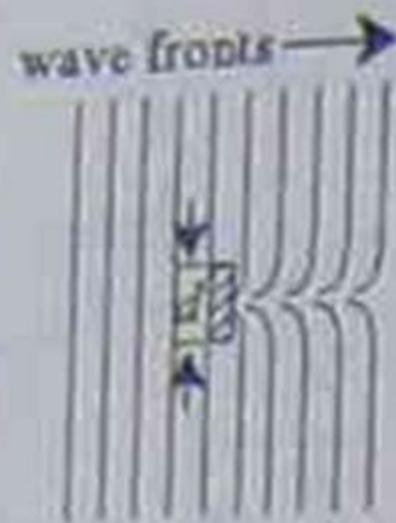
Diffraction of waves

Diffraction is the bending of progressive waves round edges, apertures or objects placed in their path, e.g. sound and water waves diffract round corners.

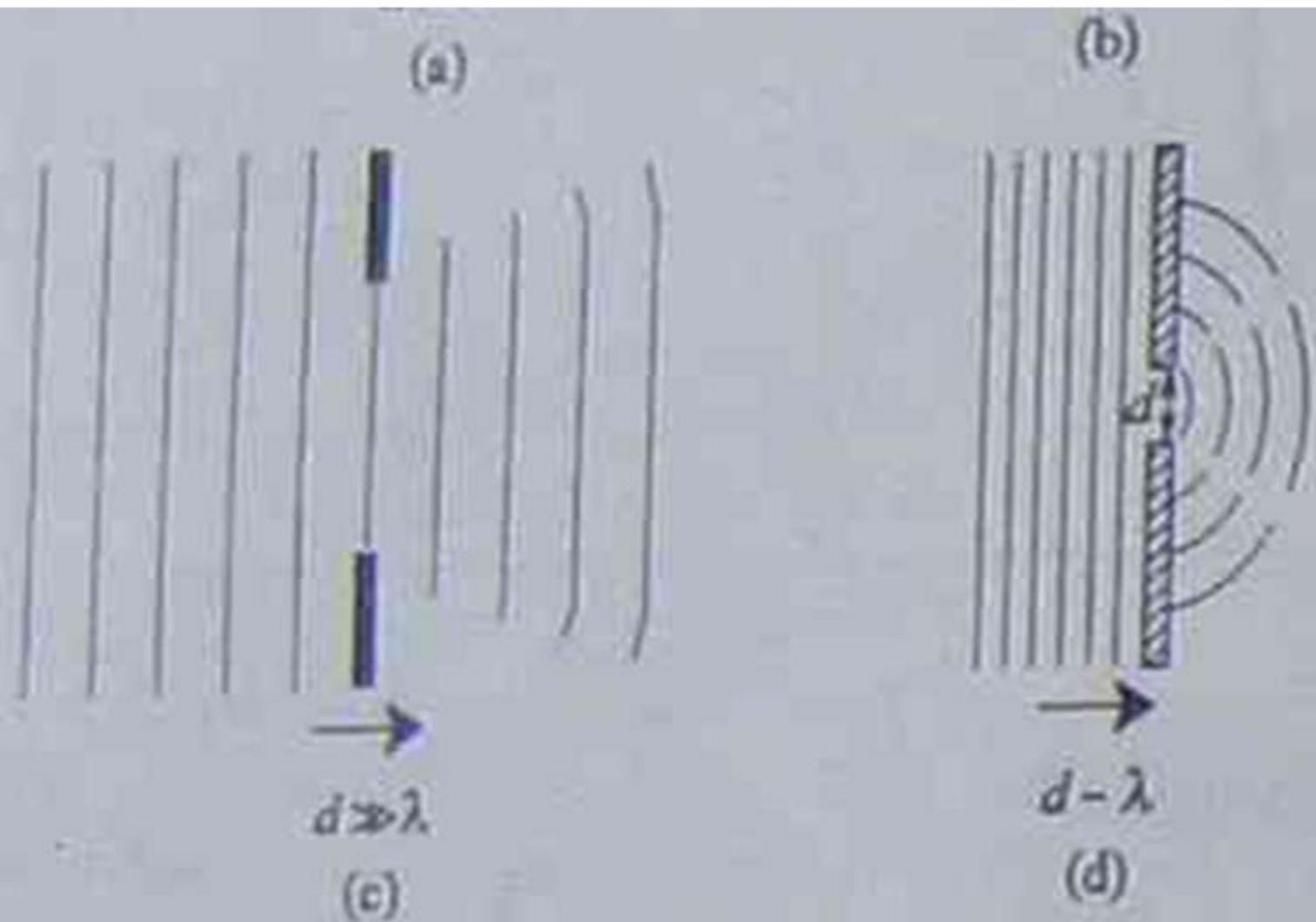
Diffracted waves interfere to form diffraction patterns whose shapes depend on the ratio λ/d , where d is the object or aperture size. Typical diffraction patterns for a plane wave incident on objects are shown in Figure 16.9.



$$d \gg \lambda$$



$$d \sim \lambda$$



diffraction pattern produced
by a single slit

Fig. 16.9

Fig. 16.9

The maxima and minima in the diffraction pattern are easily identified when the size of the object or aperture is close to or less than the wavelength, that is, $d \sim \lambda$ (see Fig. 16.9(d)).

The intensity profile of the diffraction pattern of a single slit can be deduced assuming that the slit is acting as a line of point sources in phase (Fig. 16.10).

For first minima:

$$\sin \theta = \lambda / d$$

For $\theta < 5^\circ$, $\sin \theta \doteq \tan \theta = \frac{x}{L}$,

therefore $x/L = \lambda/d$

For waves of a fixed length, as d decreases, θ increases and the pattern spreads out. The diffraction pattern of a sharp edge is shown in Figure 16.11.

Diffraction of waves at an edge was recorded in the 1665 posthumous work of Maria Grimaldi (1618-1663). He noted that light passing a sharp edge produced not a sharp-edged shadow but one with a series of alternate dark and light bands with an intensity profile as shown in Figure 16.11.

EXAMPLE

- (a) Explain the term diffraction.
- (b) Sketch the diffraction pattern for waves incident on
- (i) an object with $d \gg \lambda$ and $d \sim \lambda$;
 - (ii) a single slit with $d \gg \lambda$ and $d \sim \lambda$;
 - (iii) a straight edge.
- (c) At what angle would you expect the first minimum in the diffraction pattern of a single slit of width 0.01 mm, with waves of length 400 nm?

Answer

- (a) Spreading out of waves at a barrier.
- (b) (i), (ii) and (iii), see text above.
- (c) $\sin \theta = \lambda/d = 4.00 \times 10^{-7} / 10^{-5} = 0.04$,
therefore $\theta = 2.29^\circ$.

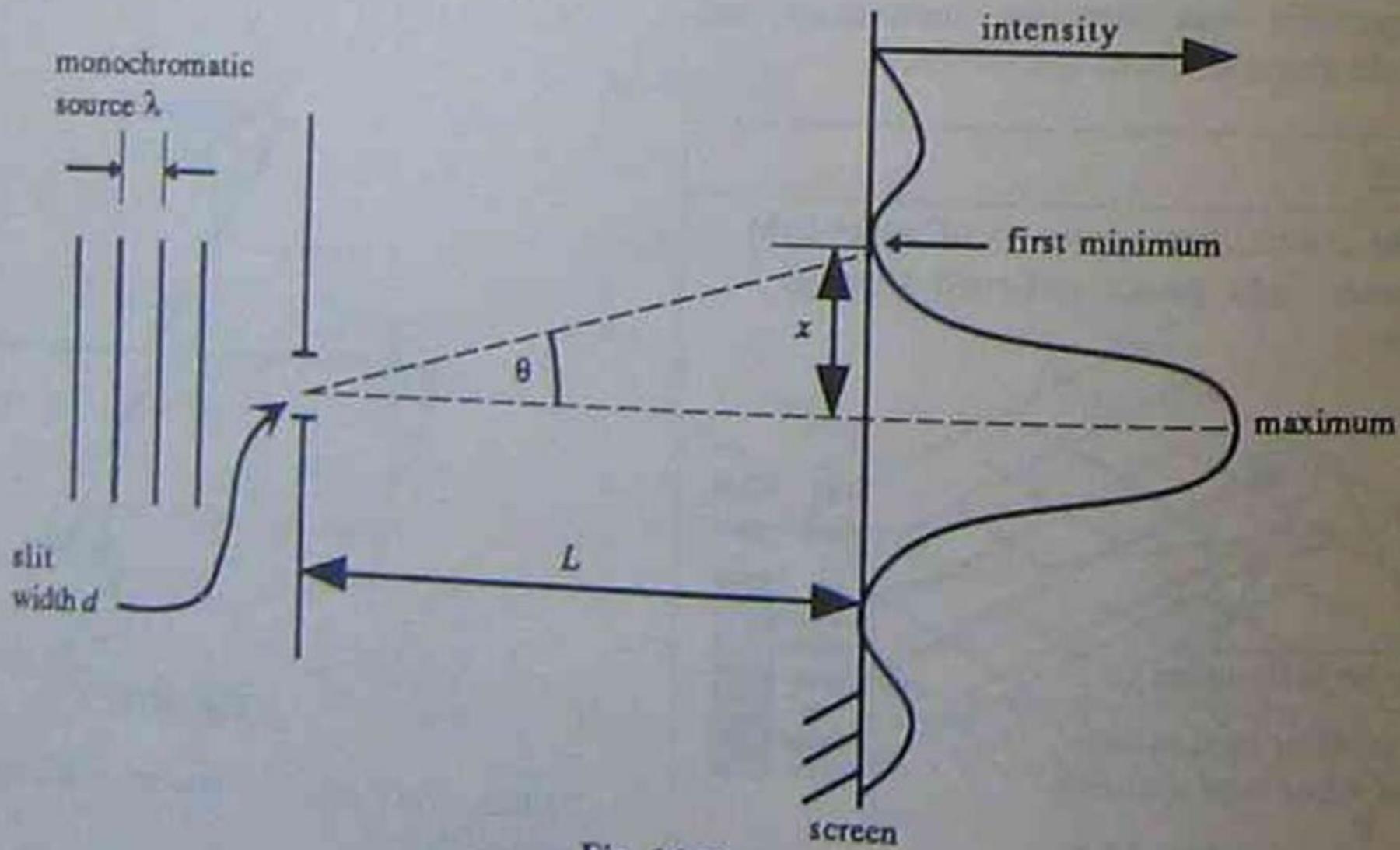
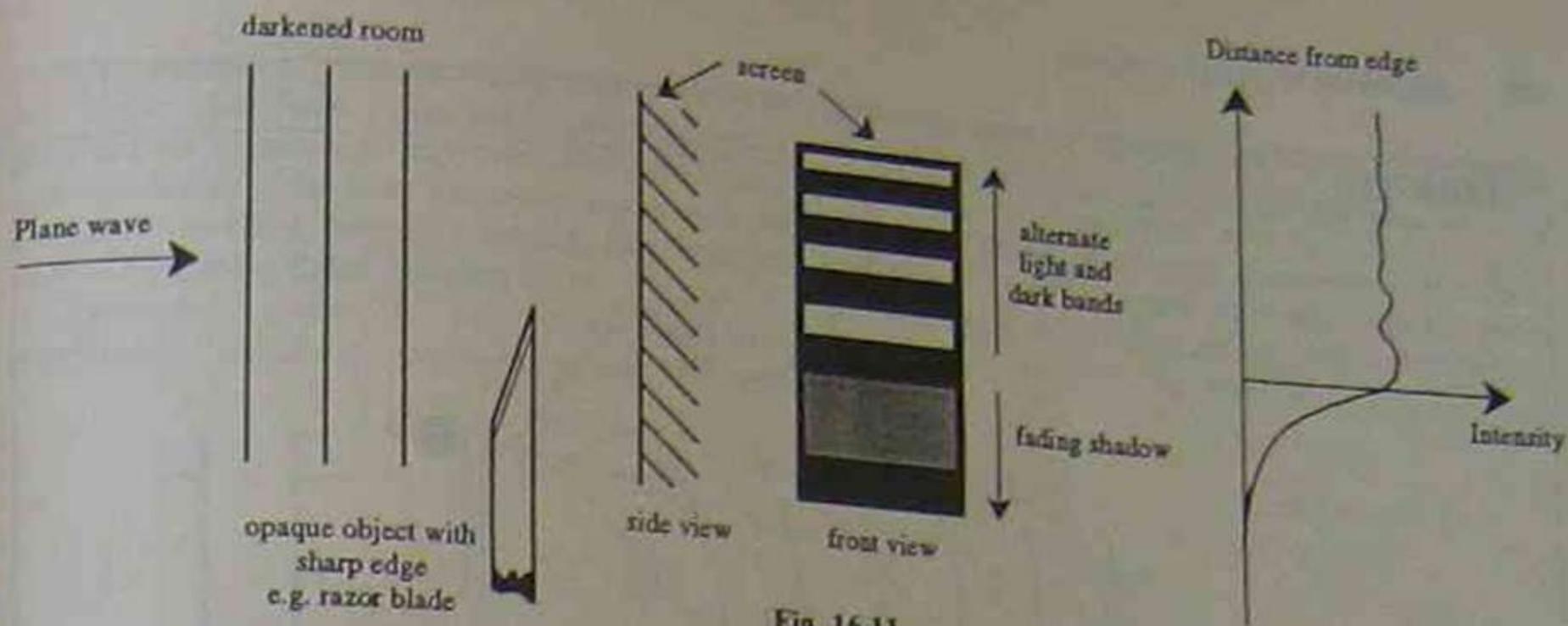


Fig. 16.10



Superposition and interference of pulses and waves

When pulses and waves come together or are superimposed a new wave is formed with its own amplitude, intensity, wavelength, frequency and velocity.

Points in a wave moving in the same direction at the same time are said to be in phase.

Pulses and waves are instantaneously changed by the process of interference. At a particular instant when two pulses or waves travelling towards each other in the same medium meet, their displacements add, in accordance with the Principle of Superposition which is:

'At any instant the displacement Y of a particle in the medium is the sum of the displacements ($y_1 + y_2$) that each pulse or wave would cause on its own, i.e. $Y = y_1 + y_2$ where y_1 is the particle's displacement in pulse 1 and y_2 is the particle's displacement in pulse 2.'

At an instant when y_1 and y_2 are superimposed in phase, constructive interference yields a large pulse with amplitude $y_1 + y_2$. When pulses with the same amplitude meet out-of-phase, $y_1 = -y_2$, destructive interference occurs.

Simple waves interfere and usually form a more complex wave.

Two out of phase pulses of the same shape travelling towards each other interfere as shown in Chapter 6, in the section on periodic waves, reflection and transmission.

EXAMPLES

1.

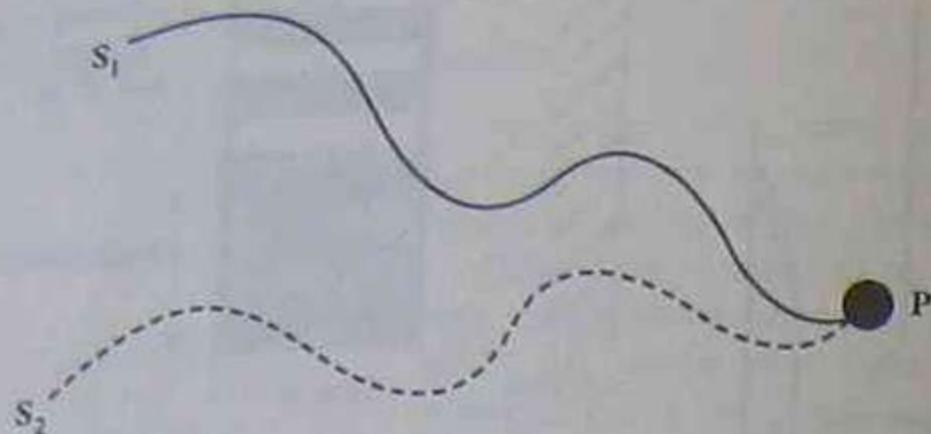
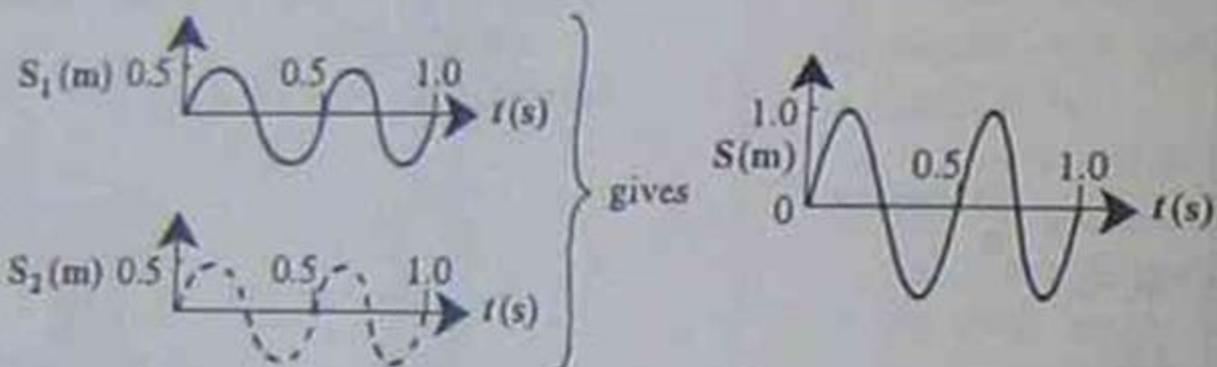


Fig. 16.14

Waves from sources S_1 and S_2 have length 1 m and amplitude 0.5 m and speed 2 m s^{-1} . Sketch the displacement versus time graph for a particle at P when (a) S_1 and S_2 are in phase and (b) when S_1 and S_2 are out of phase.

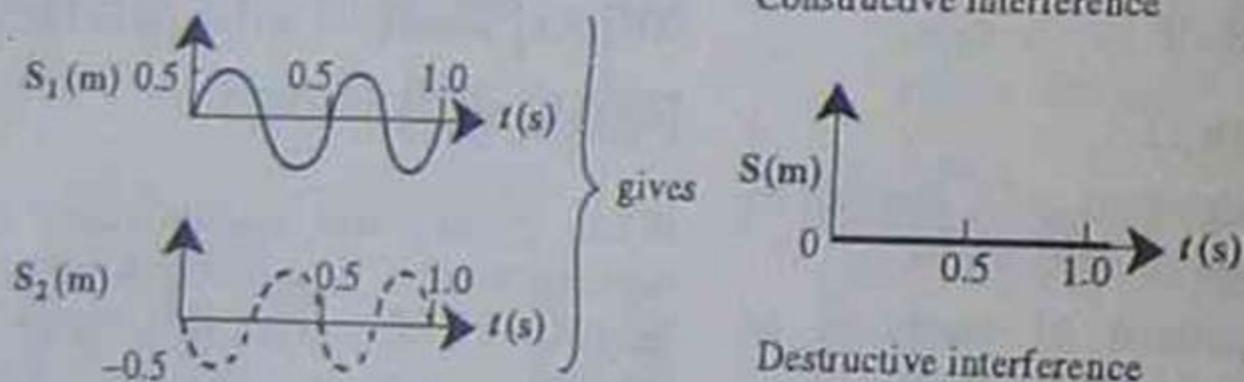
Answer

(a)



Constructive interference

(b)



Destructive interference

Fig. 16.15

Fig. 16.15

2. Starting at the same point on a piece of graph paper, draw a transverse wave of length 8 cm and amplitude 4 cm and a second wave in phase with the first of the same amplitude but of length 16 cm. Apply the Principle of Superposition to determine the resultant wave form.

Answer

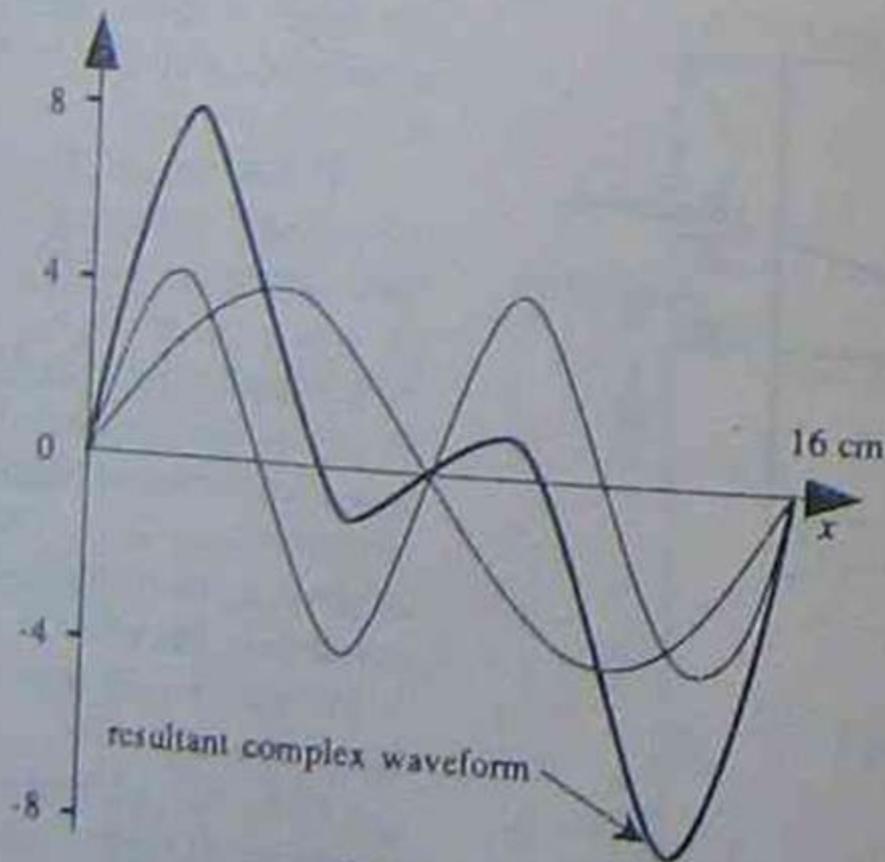


Fig. 16.16

Superposition of coherent waves, that is, waves at any instant that have a constant phase relationship, gives rise to constructive and destructive interference. Consider waves of the same frequency, wavelength and amplitude travelling through a medium from coherent sources S_1 and S_2 . Refer to Figure 16.17.

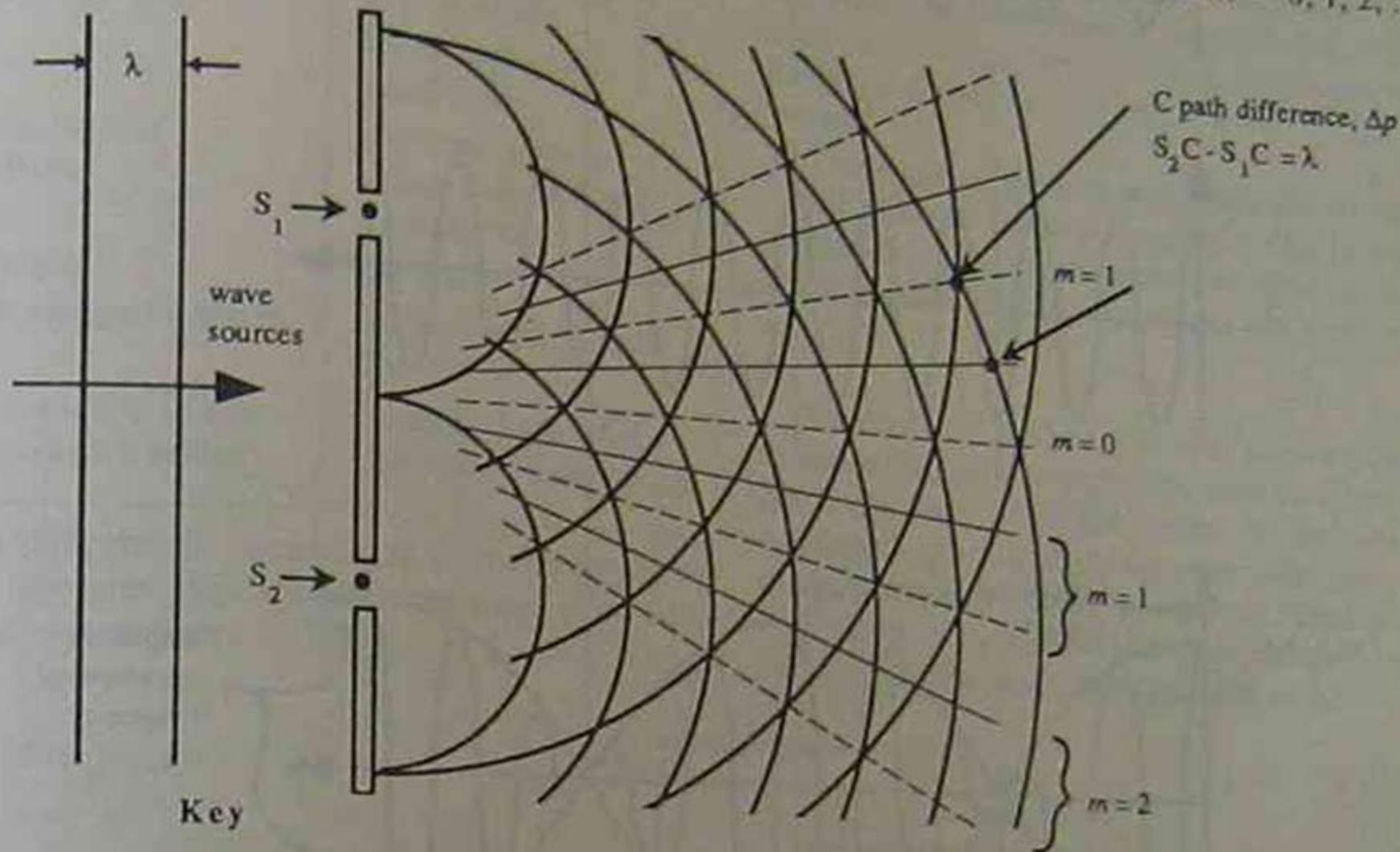
Antinodes, points of maximum constructive interference, occur at positions where the path

difference between superimposed waves from sources S_1 and S_2 is a whole number of wavelengths,

$$\text{i.e. } \Delta p = m\lambda \text{ where } m = 0, 1, 2, \dots$$

Nodes, points of complete destructive interference, are found when the path difference between waves from S_1 and S_2 is an odd number of half-wavelengths,

$$\text{i.e. } \Delta p = (m + 1/2)\lambda \text{ where } m = 0, 1, 2, \dots$$



Key

- lines of destructive interference, called nodal lines, are dark fringes in the case of light
- - - lines of constructive interference, called anti-nodal lines, are bright fringes in the case of light

Fig. 16.17

Beats

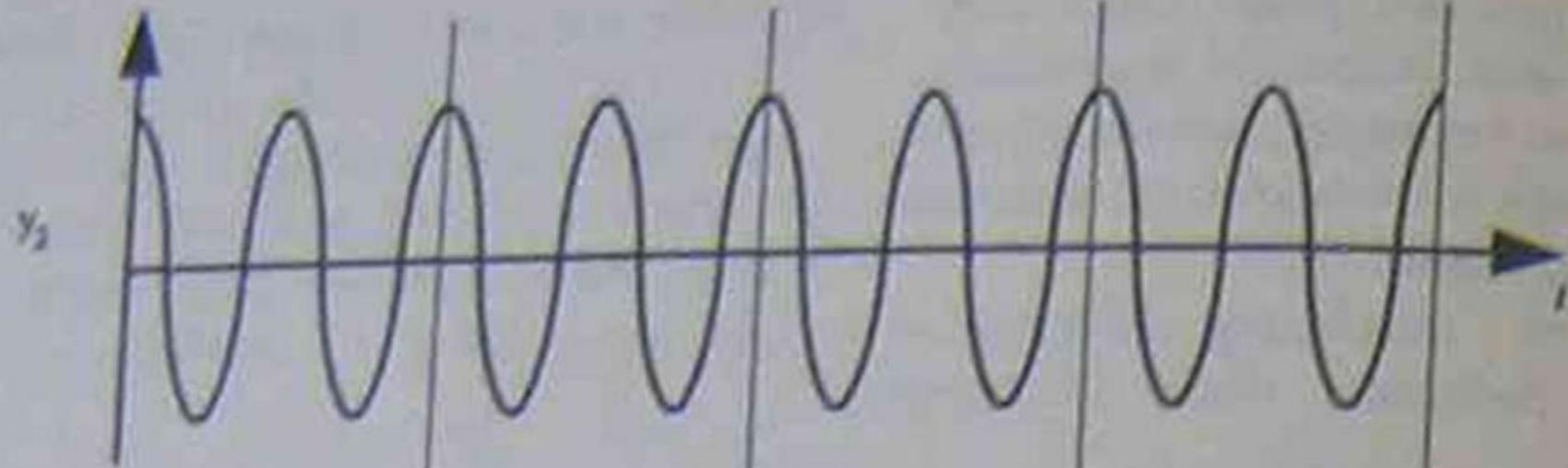
Two waves travelling in the same direction with equal speed and about the same amplitude but slightly different frequency f_1 and f_2 interfere to produce a low-frequency (long-wavelength) wave called a beat. (See Fig. 16.18 on page 172 and the section on superposition of waves.) The amplitude envelope of the beat has the following frequency:

$$f_b = (f_1 - f_2)/2 \text{ where } f_1 > f_2.$$

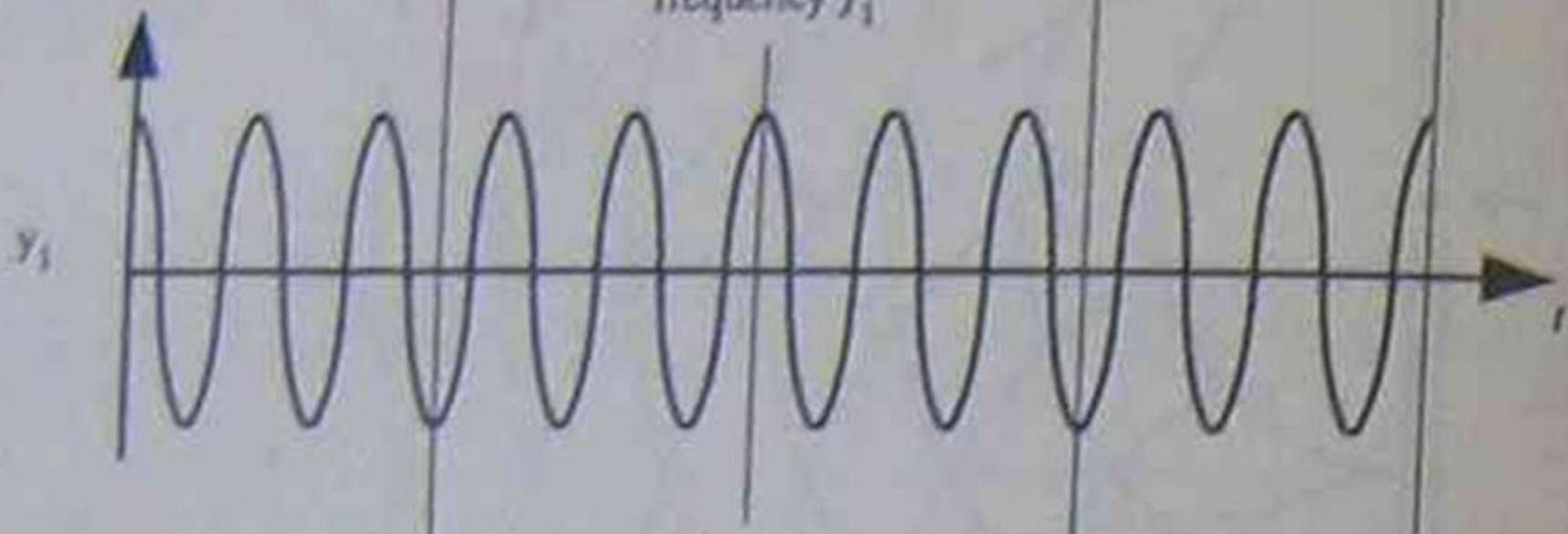
But the beat is detected when its amplitude is maximum or minimum, therefore observed beat frequency, called the pulse frequency f_p , is:

$$f_p = (f_1 - f_2) = 2f_b.$$

frequency f_2



frequency f_1



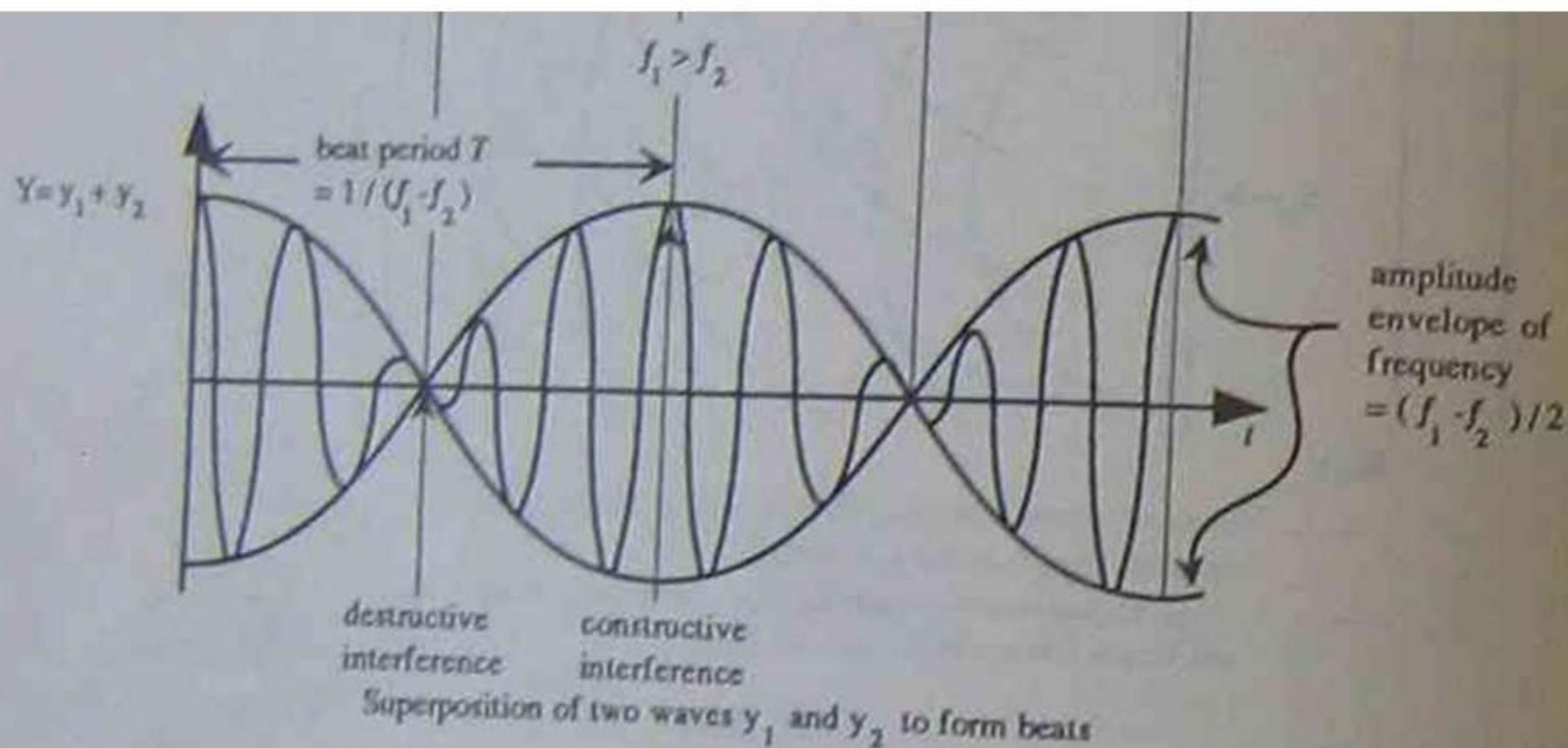


Fig. 16.18

ance or sympathetic vibrations

Resonance or sympathetic vibrations

Resonance occurs when an object is set vibrating by being subjected to the energy of impulses at a frequency close to the natural frequency of the object. Large-amplitude (high-energy) waves are produced in the object, which is said to vibrate in sympathy with the impulses.

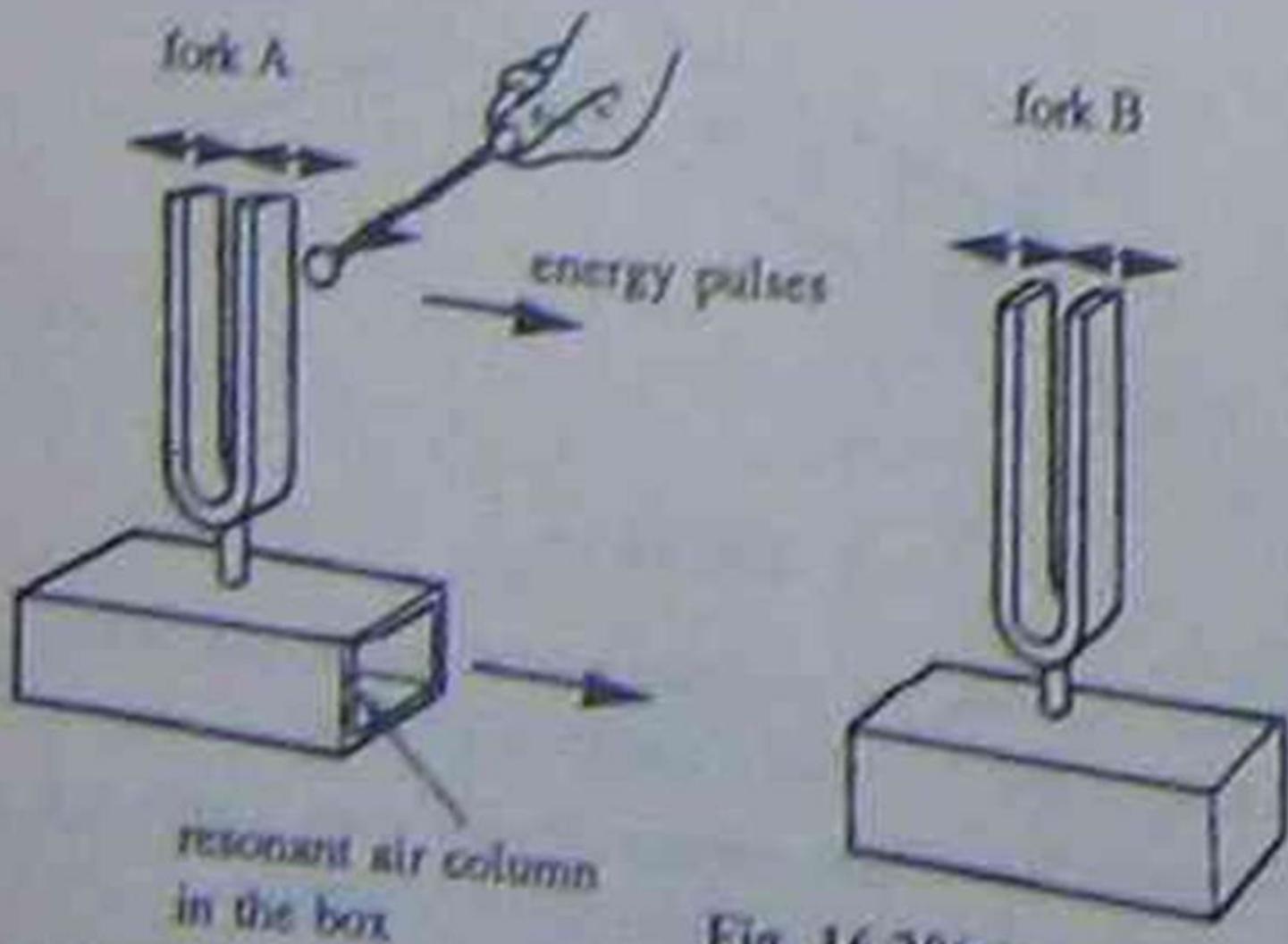


Fig. 16.20(a)

Energy pulses from tuning fork A give rise to sympathetic vibrations in fork B.

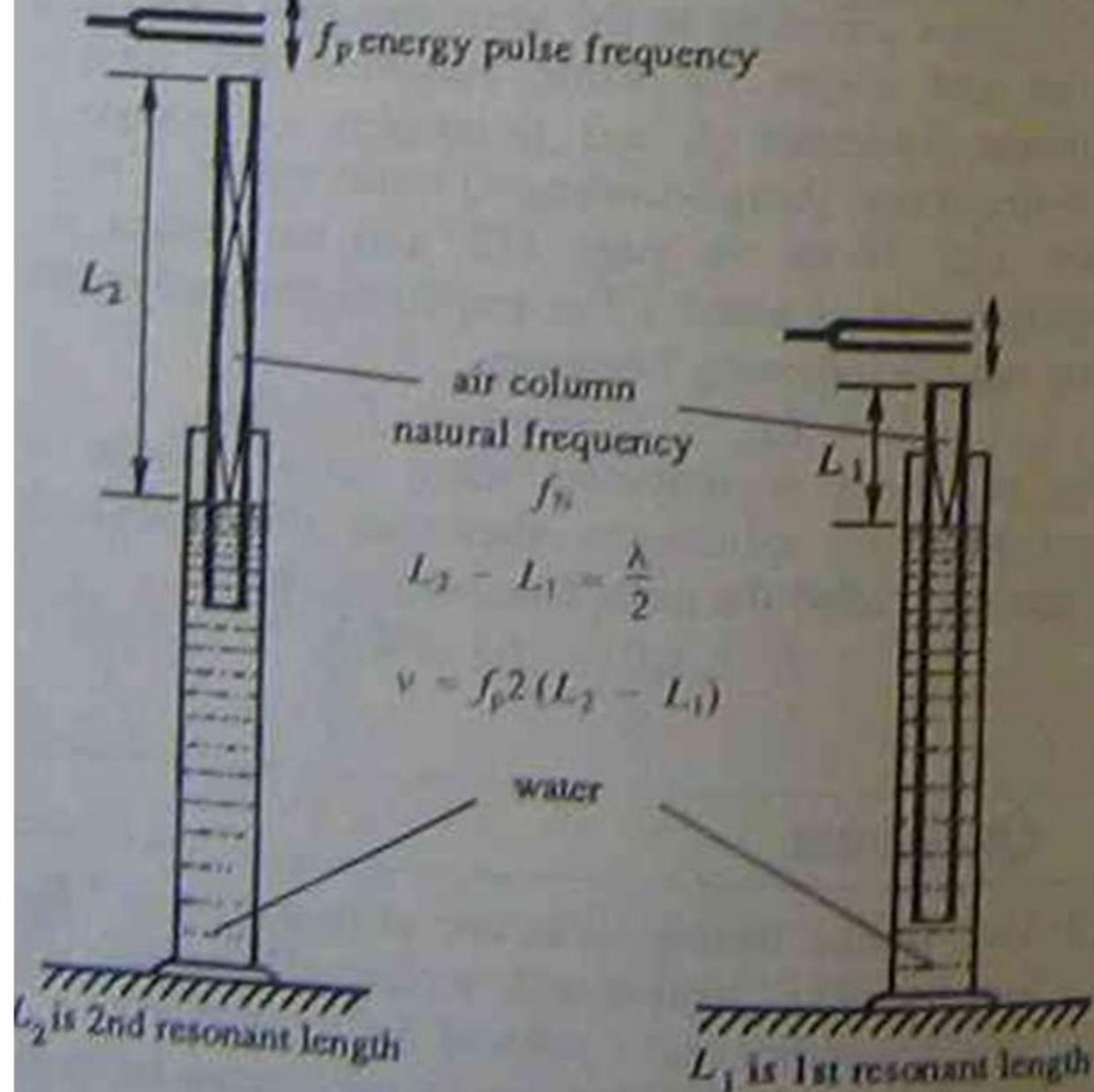
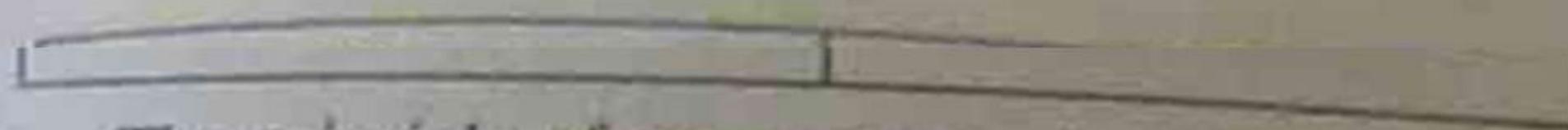


Fig. 16.20(b) Resonant air column

When the natural frequency f_n of the air column is the same as the frequency f_o of the energy pulse applied to the column, resonance and large-amplitude (i.e. louder) sound waves are produced.



The principle of resonance in air columns can be used to determine experimentally the speed of sound in air, as shown in Figure 16.20(b).

<i>Source of impulses</i>	<i>Object in resonance</i>
<p>Vibrating — air near a tuning fork or a person's lips</p>	<p>air columns in pipes, (e.g. in wind instruments, ears, vocal cords)</p>
<p>tuning fork or plectrum</p>	<p>stretched wires or strings (e.g. stringed instruments)</p>
<p>car engine</p>	<p>car body</p>
<p>electromagnetic waves</p>	<p>tuning circuit in radio, TV</p>
<p>feet marching in step or wind on a bridge</p>	<p>bridges — have been destroyed by resonance</p>

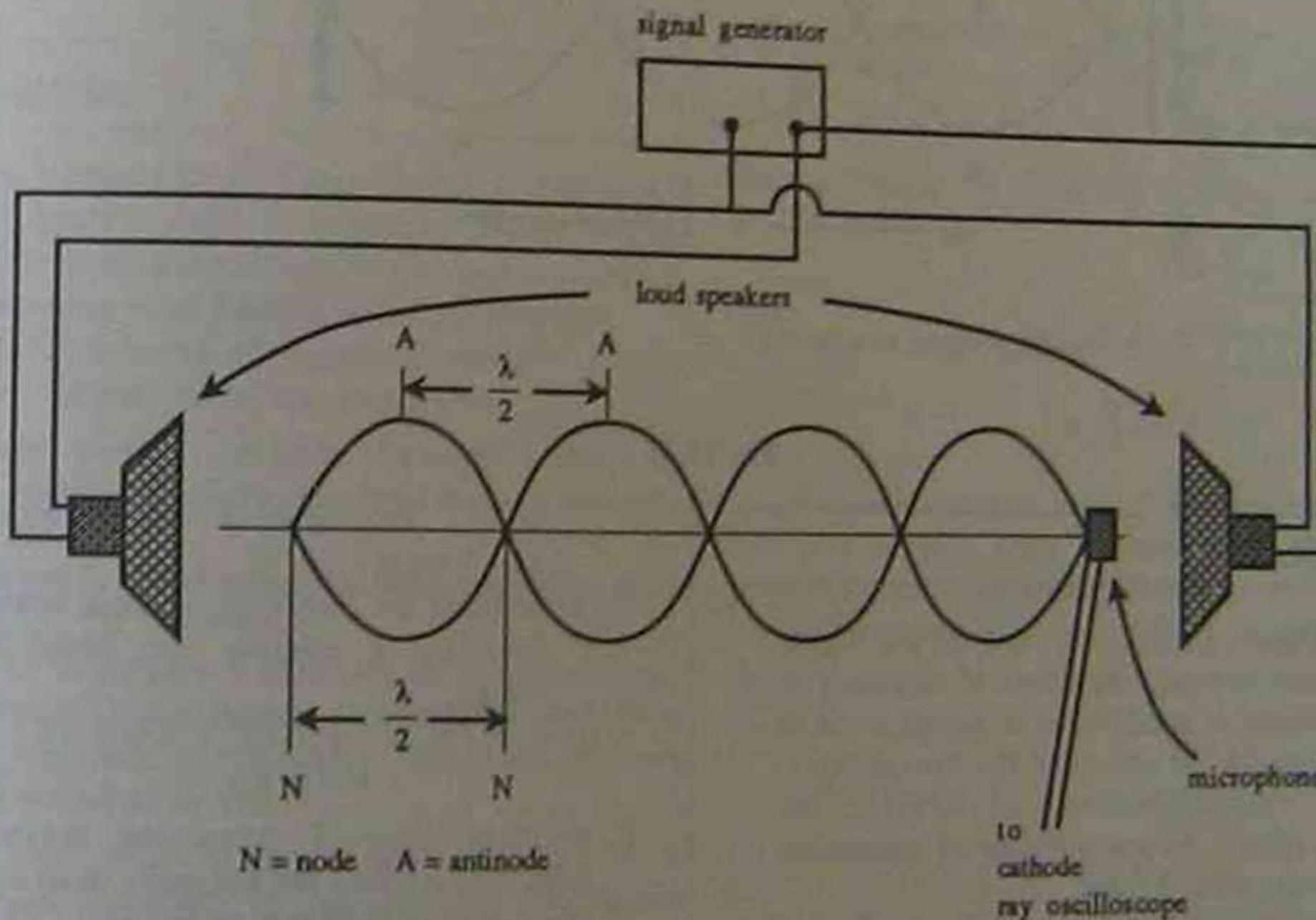
Standing (stationary) waves in wires, string and pipes

Standing waves are formed by the interference of two progressive waves travelling in opposite directions with equal amplitude, frequency and wavelength. The two waves may be from separate sources, e.g. two loudspeakers, or an incident wave and a reflected wave, as in string and wind musical instruments.

Types

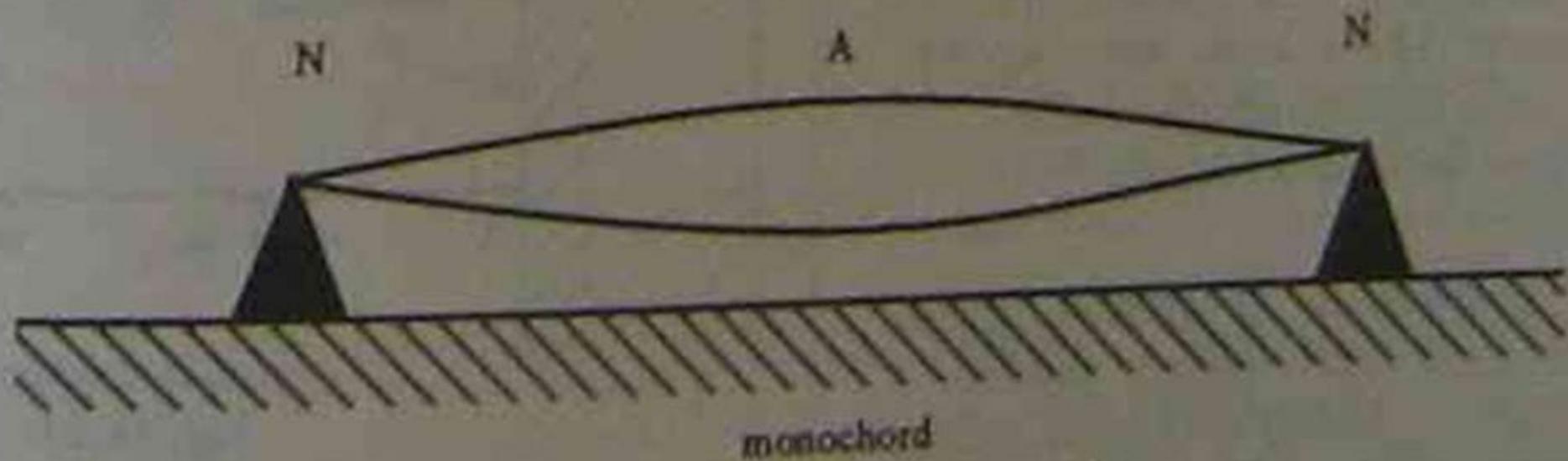
Transverse progressive waves give rise to transverse standing waves, e.g. in a stretched string or wire, and longitudinal progressive waves produce longitudinal standing waves, e.g. in organ pipes and our own vocal cords.

Particles at positions in the standing wave of complete destructive interference have no displacement. These positions are called nodes, N , and are half a wavelength apart. Midway between nodes are positions of maximum constructive interference called antinodes, A . If the amplitude of the interfering progressive waves is a then the amplitude at an antinode is $2a$.



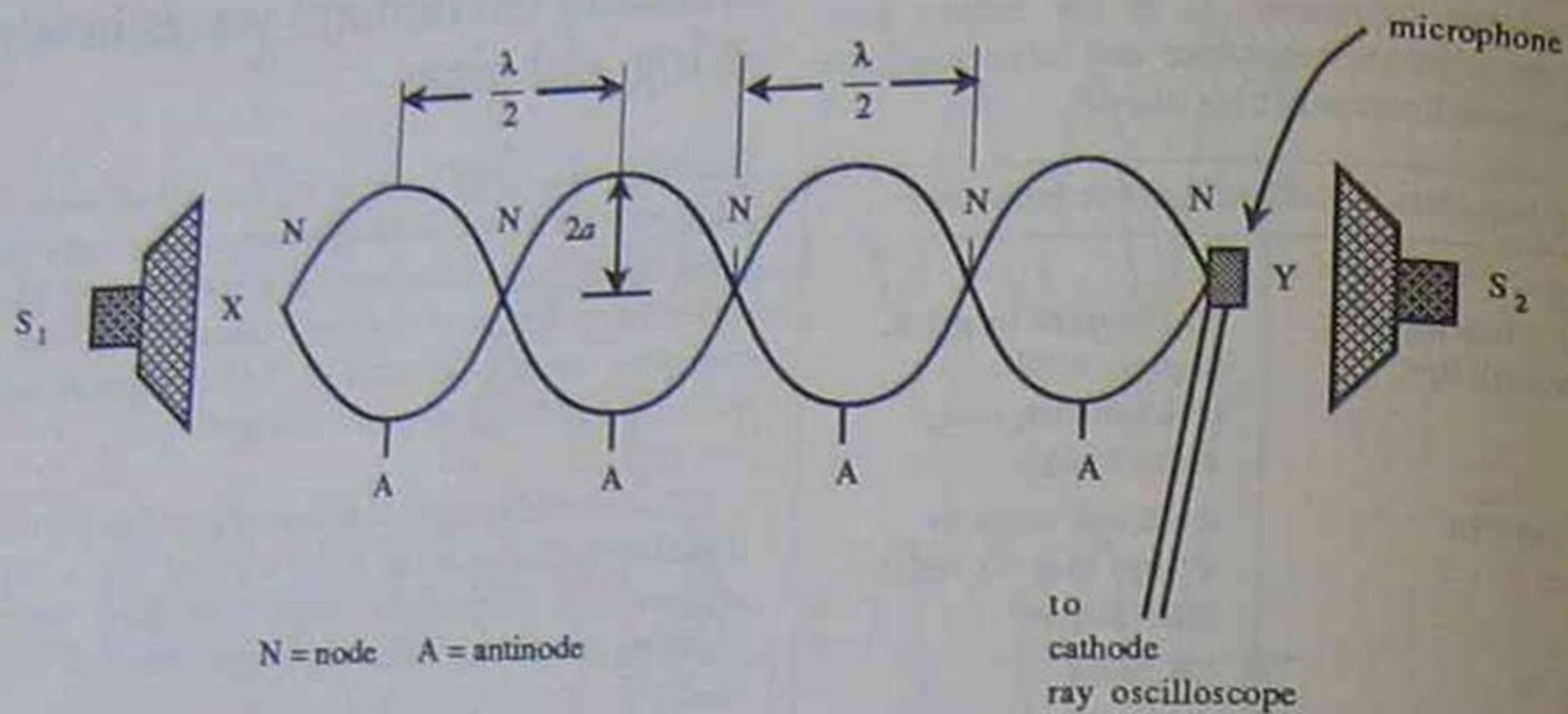
longitudinal standing wave in air

longitudinal standing wave in air



transverse standing wave in a stretched string

Fig. 16.21



(a) standing waves in air