

(b) standing waves in a spring

EXAMPLES

I. In Figure 16.22(a), S_1 and S_2 are signal generators producing waves of frequency f in phase. A microphone is moved along a line joining the centres of the two speakers, each emitting a sound of 50 kHz. The microphone detects positions of maximum intensity every 3.5 cm.

- (a) Explain these observations.
- (b) The microphone is moved from a maximum 14 cm along the line XY. How many minima are detected?
- (c) Calculate the speed of sound.

Answer

(a) The longitudinal progressive sound waves from each speaker produce a longitudinal standing wave along the line AB between the speakers. This wave has positions of maximum intensity $\lambda/2$

apart. Therefore $\lambda/2 = 3.5 \text{ cm}$, so $\lambda = 7.0 \text{ cm}$.

(b) Minima are half-way between maxima, therefore 4 minima are detected in 14 cm.

$$\begin{aligned}(c) v &= f\lambda = 5.0 \times 10^3 \times 7.0 \times 10^{-2} \text{ ms}^{-1} \\ &= 350 \text{ ms}^{-1}\end{aligned}$$

2. A transmitter T produces waves of amplitude a which fall normally onto a sheet of aluminium as shown in Figure 16.23.

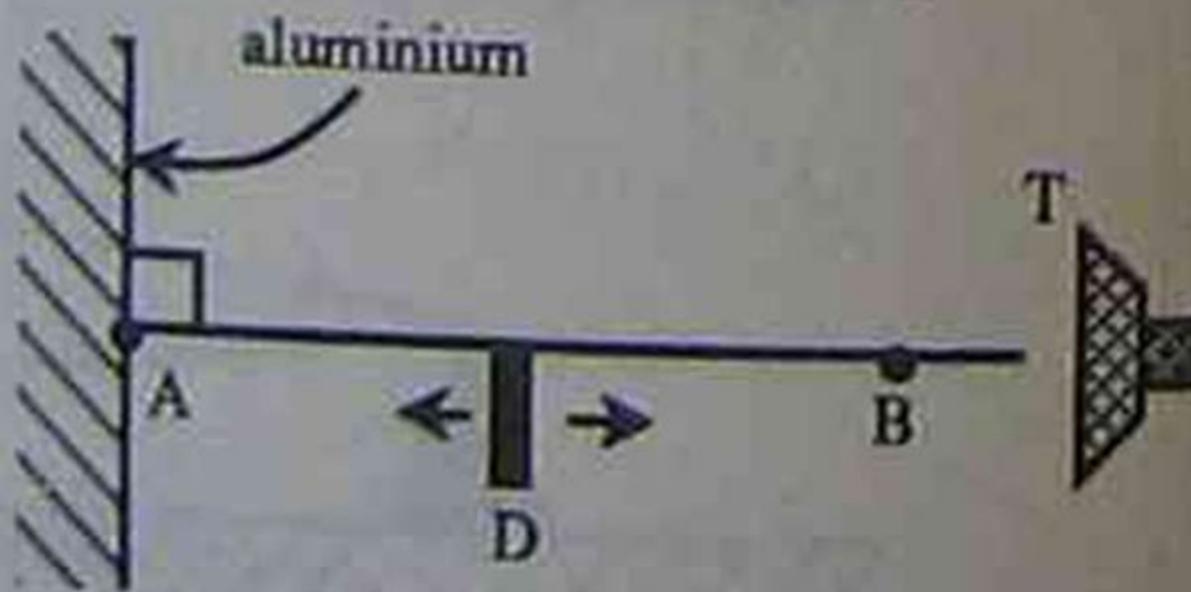


Fig. 16.23

The detector D moves along the line AB and identifies 9 nodal positions in 10 cm.

- (a) Sketch a profile of the standing wave formed by reflection. Label your sketch to show the position of nodes N, antinodes A, and the amplitude of the standing wave in terms of the amplitude a of the waves from the transmitter.
- (b) What is the wavelength of the waves?
- (c) Calculate the frequency of the waves. Take the velocity of the waves to be $3.0 \times 10^8 \text{ ms}^{-1}$.

Answer

(a)

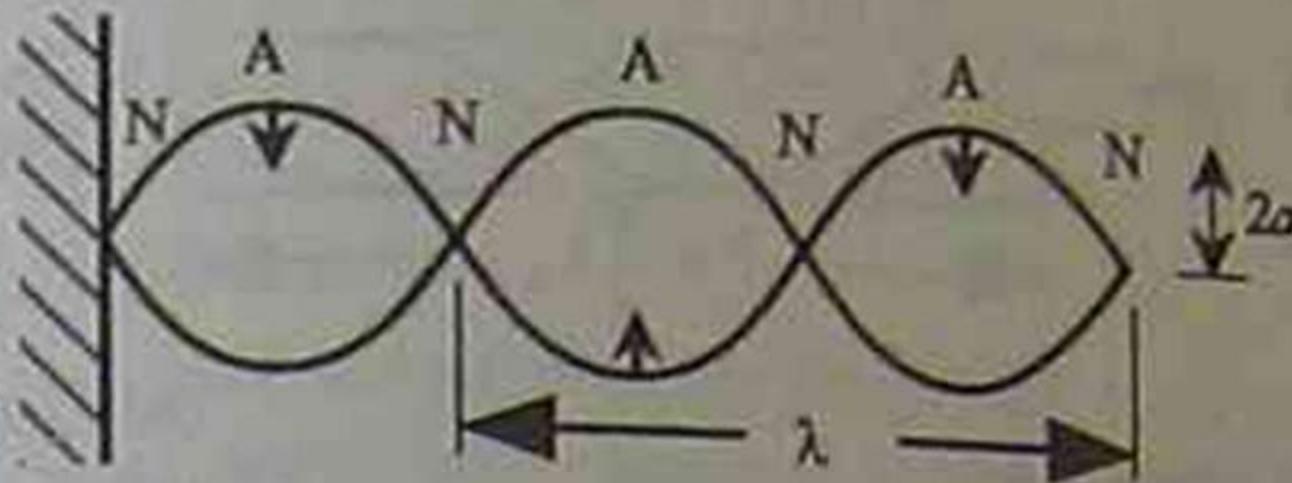


Fig. 16.24

(b) 9 nodes in 10 cm; this means $8 \times \lambda/2$ in 10 cm or 4 waves in 10 cm, therefore $\lambda = 10/4 \text{ cm} = 2.5 \text{ cm}$.

(c) $v = f\lambda$, therefore

$$f = v/\lambda = 3.0 \times 10^5 / 2.5 \times 10^{-2} \text{ Hz}$$
$$= 1.2 \times 10^{10} \text{ Hz}$$

Standing waves in strings and wires

A plucked string clamped at both ends results in a transverse progressive wave incident on a clamp being

WAVES II 175

reflected. The incident and reflected wave have the same frequency, wavelength and amplitude and superimpose to form a transverse standing wave. A number of possible standing-wave patterns or modes of vibration are produced simultaneously. Each mode has a node at the clamps.

The first mode

The first mode, i.e. the simplest or fundamental mode contains a node at either end and an antinode in the middle and vibrates at a frequency called the fundamental frequency. The fundamental mode is also called the first tone and is always the first harmonic.

A harmonic is a wave of frequency an integer multiple of the fundamental frequency.

The second mode or first overtone of vibration contains a node in the middle and has half the wavelength of the fundamental and therefore twice the frequency, and is therefore the second harmonic.

Possible modes of vibration of a stretched string are:

mode (M)	frequency f	wavelength λ	length L
1st fundamental: (1st harmonic)	f_1	$\lambda_1 = 2L$	L
2nd 1st overtone: (2nd harmonic)	$2f_1$	$\lambda_2 = \frac{2L}{2}$	$\frac{\lambda_1}{2}$
3rd 2nd overtone: (3rd harmonic)	$3f_1$	$\lambda_3 = \frac{2L}{3}$	$\frac{\lambda_2}{2}$
Mth	Mf_1	$\lambda_M = \frac{\lambda_1}{M}$	standing waves in a string or wire fixed at both ends

Fig. 16.25

A musical note is an enriched fundamental tone and is a complex wave formed by the simultaneous superposition of harmonics. The process of identifying the harmonics in a complex wave is called Fourier analysis.

EXAMPLE

- (a) What is (i) the wavelength, and (ii) the frequency of the fundamental, first, second and third overtones of a transverse standing wave in a string stretched between two clamps 50 cm apart. Take the velocity of waves in a string to be 130 ms^{-1} .
- (b) Are all possible harmonics producible in the stretched string described in (a)?

Answer

(a) Fundamental

$$\begin{aligned}(i) \lambda_1 &= 2 \times L \\ &= 2 \times 0.5 \text{ m} \\ &= 1.0 \text{ m}\end{aligned}$$

$$\begin{aligned}(ii) f_1 &= v/\lambda_1 \\ &= \frac{130}{1.0} \text{ Hz} \\ &= 130 \text{ Hz}\end{aligned}$$

1st overtone

$$\begin{aligned}(i) \lambda_2 &= L \\ &= 0.5 \text{ m}\end{aligned}$$

$$\begin{aligned}(ii) f_2 &= \frac{130}{0.5} \text{ Hz} \\ &= 260 \text{ Hz}\end{aligned}$$

2nd overtone

2nd overtone

$$\begin{aligned}(i) \lambda_3 &= \frac{2}{3} \times L \\&= \frac{2}{3} \times 0.50 \text{ m} \\&= 0.333 \text{ m}\end{aligned}$$

$$\begin{aligned}(ii) f_3 &= \frac{130}{0.333} \text{ Hz} \\&= 390 \text{ Hz}\end{aligned}$$

3rd overtone

$$\begin{aligned}(i) \lambda_4 &= \frac{1}{2} \times L \\&= 0.50/2 \text{ m} \\&= 0.25 \text{ m}\end{aligned}$$

$$\begin{aligned}(ii) f_4 &= \frac{130}{0.25} \text{ Hz} \\&= 520 \text{ Hz}\end{aligned}$$

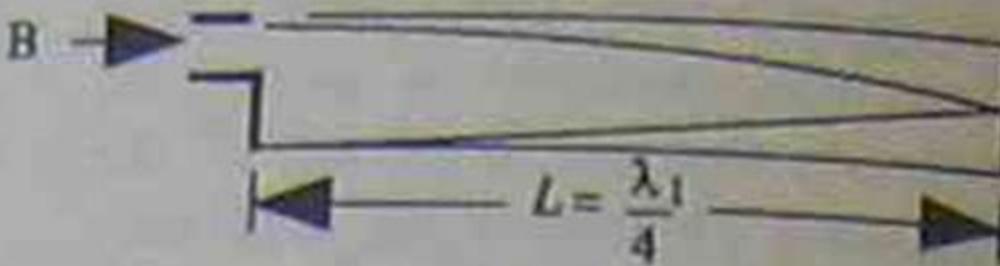
(b) Yes

Standing waves in pipes

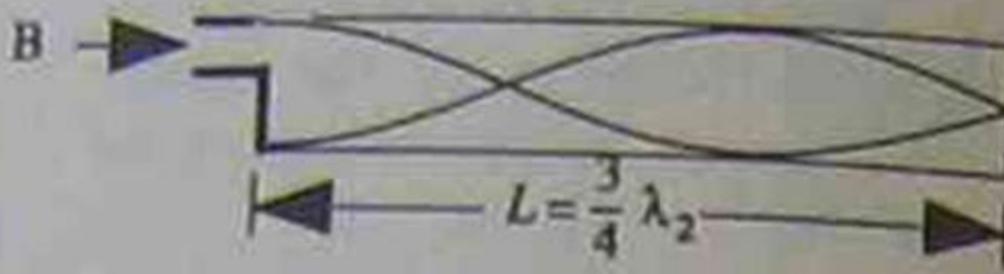
Longitudinal standing waves can be produced in air pipes contained in organs, musical instruments and our vocal tract by longitudinal progressive waves being reflected from the ends of the pipes. The frequency of the standing wave is the same as the natural frequency of the air in the pipe. The pipe may be a 'closed' pipe, open at both ends.

Modes of vibration in:

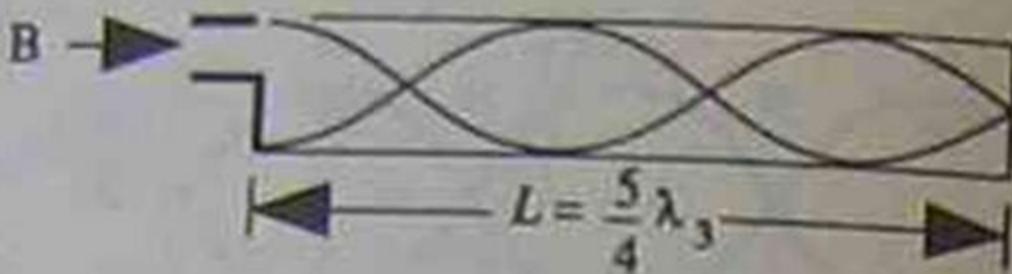
A. closed pipe



fundamental;
(1st harmonic)
 $\therefore f_1 = \frac{v}{4L}$



1st overtone;
(3rd harmonic)
 $\therefore f_2 = 3 \frac{v}{4L} = 3f_1$



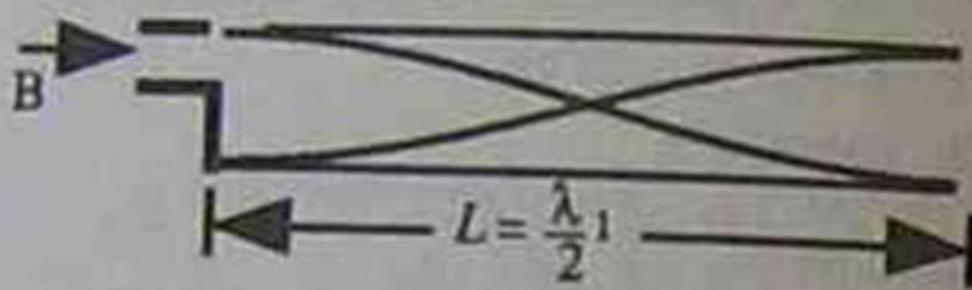
2nd overtone;
(5th harmonic)
 $\therefore f_3 = 5 \frac{v}{4L} = 5f_1$

B = blast of air

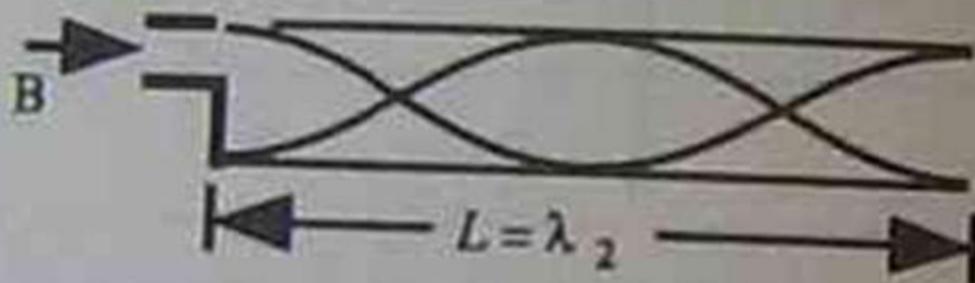
only odd harmonics
possible

Fig. 16.26

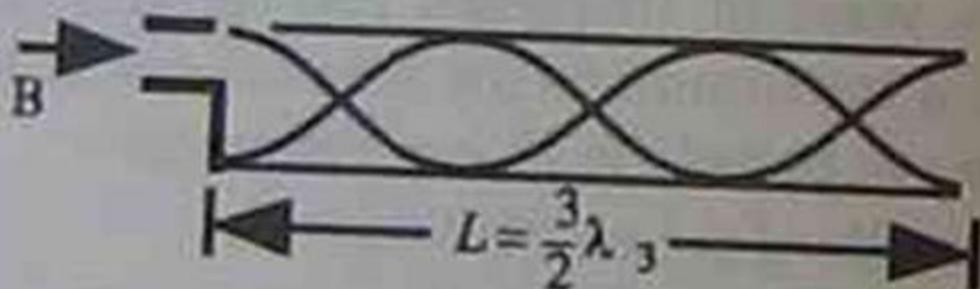
B. open pipe



fundamental:
(1st harmonic)
 $\therefore f_1 = \frac{v}{2L}$



1st overtone:
(2nd harmonic)
 $\therefore f_2 = \frac{v}{L} = 2f_1$



2nd overtone:
(3rd harmonic)
 $\therefore f_3 = \frac{3}{2} \frac{v}{2L} = 3f_1$

B = blast of air

all harmonics
possible

EXAMPLES

1. For a closed pipe of length 1.36 m determine the wavelength and frequency of the following modes of vibration:
- (a) fundamental;
 - (b) 1st overtone;
 - (c) 2nd overtone.
 - (d) Which harmonics are produced in a closed pipe?

Take the velocity of sound to be 340 ms^{-1} .

Answer

Answer

(a) For the fundamental, which is the 1st harmonic:

$$L = \lambda_1/4,$$

therefore

$$\lambda_1 = 4 \times L = 4 \times 1.36 \text{ m} = 5.44 \text{ m.}$$

$$f_1 = v/\lambda_1 = 340/5.44 = 62.5 \text{ Hz.}$$

(b) The 1st overtone has the same velocity but a wavelength $\frac{1}{3}$ that of the fundamental, hence:

$$\lambda_2 = 5.44/3 = 1.81 \text{ m,}$$

and therefore a frequency 3 times greater, hence the 1st overtone is the 3rd harmonic:

$$\begin{aligned}f_2 &= 3 \times f_1 = 3 \times 62.5 \text{ Hz} \\&= 187.5 \text{ Hz}\end{aligned}$$

(c) The 2nd overtone is the 5th harmonic, so:

$$\lambda_3 = 5.44/5 \text{ m} = 1.09 \text{ m,}$$

$$\begin{aligned}\text{and } f_3 &= 5 \times f_1 = 5 \times 62.5 \text{ Hz} \\&= 312.5 \text{ Hz}\end{aligned}$$

(d) Odd harmonics.

2. The second lowest possible frequency for a standing wave in a pipe of length 0.5 m open at both ends is 650 Hz. What is the velocity of the standing wave?

Answer

The second lowest frequency standing wave

in an open pipe has a wavelength equal to the length of the pipe, therefore:

$$\begin{aligned}v &= f \times \lambda = 650 \times 0.5 \text{ ms}^{-1} \\&= 325 \text{ ms}^{-1}\end{aligned}$$

ADDITIONAL WORKED EXAMPLES

1. (a) What is resonance?
(b) Describe an experiment to determine the velocity of sound in air.
(c) If the first and second resonant lengths of an air pipe open at one end are L_1 , (22 cm) and L_2 , (67 cm), and the velocity of sound in air is 346 ms^{-1} , what is:
(i) the third resonant length, L_3 ? (ii) the frequency of the sound?

Answer

- (a) See text.
- (b) See Figure 16.20(b); if known then $v = f/2(L_2 - L_1)$
- (c) (i) $\lambda/2 = (L_2 - L_1) = (67 - 22) \text{ cm} = 45 \text{ cm}$.
therefore $\lambda = 90 \text{ cm}$.
The third resonant length $L_3 = 1.25 \times \lambda = 1.25 \times 90 \text{ cm} = 112.5 \text{ cm}$.
- (ii) $v = f/\lambda$,
therefore $f = v/\lambda = 346/0.90 = 384 \text{ Hz}$.

2. (a) Sketch the fundamental and first two overtones for a note of frequency 500 Hz produced by air in:
(i) an open pipe, (ii) a closed pipe.
Label your sketches to show nodes, N; antinodes, A; harmonics and frequencies.
(b) If the velocity of sound in air present in the pipes in (a) is 350 m s^{-1} , what is the length of (i) the open pipe, and (ii) the closed pipe.

Answer

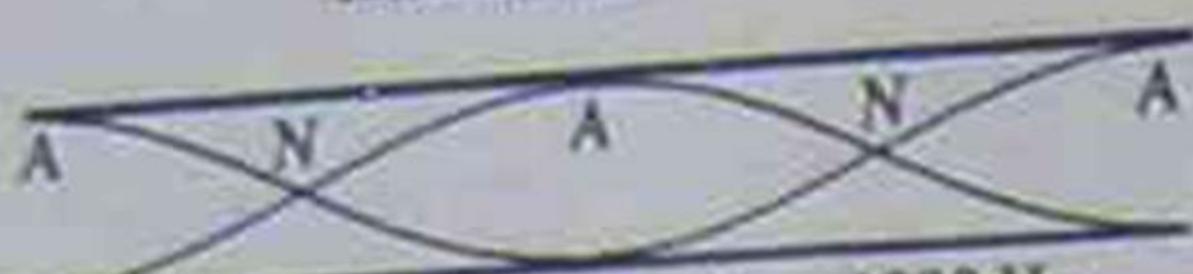
(a)

① open pipe



$$L = \frac{1}{2} \lambda$$

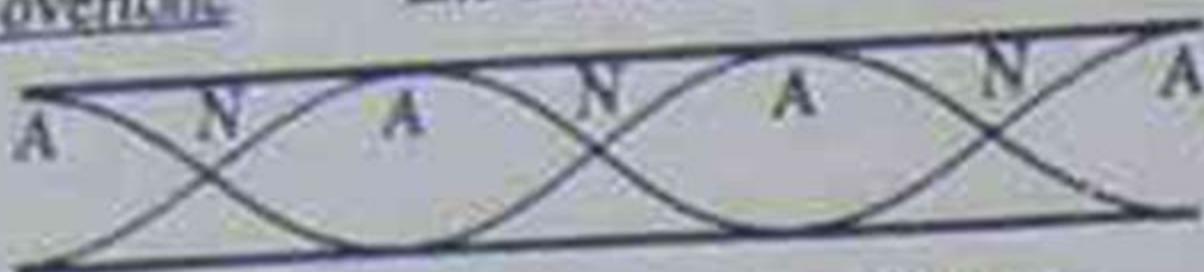
fundamental mode 500 Hz



$$L = \frac{2}{3} \lambda$$

first overtone

2nd harmonic 1000 Hz



$$L = \frac{3}{4} \lambda$$

second overtone

3rd harmonic 1500 Hz

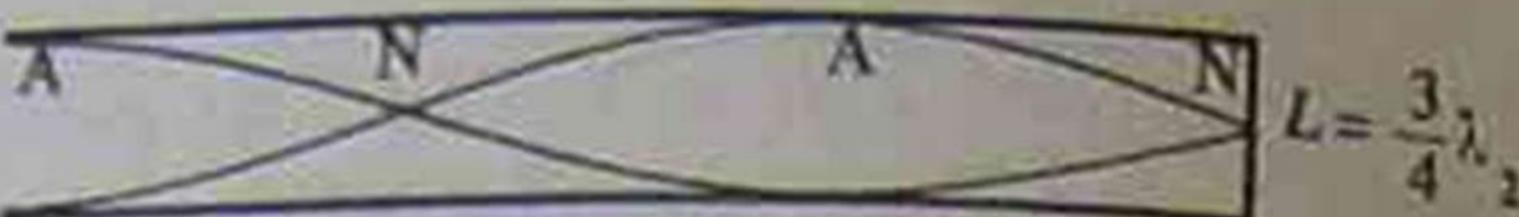
A = antinode N = node

Fig.

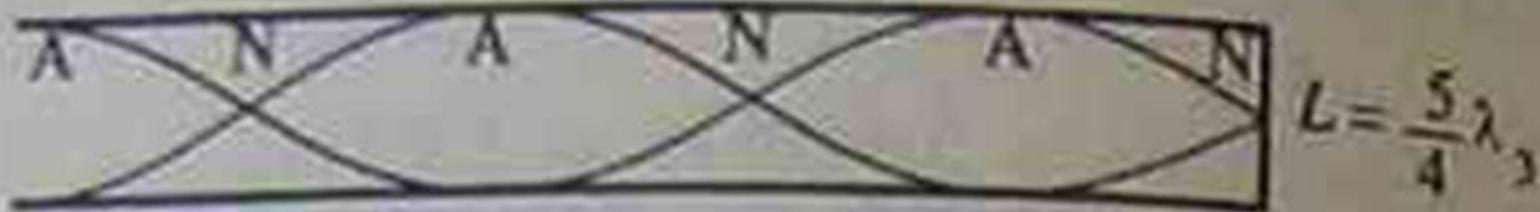
(ii) closed pipe



fundamental mode 500 Hz



3rd harmonic 1500 Hz



5th harmonic 2500 Hz

A = antinode N = node

Fig. 16.29

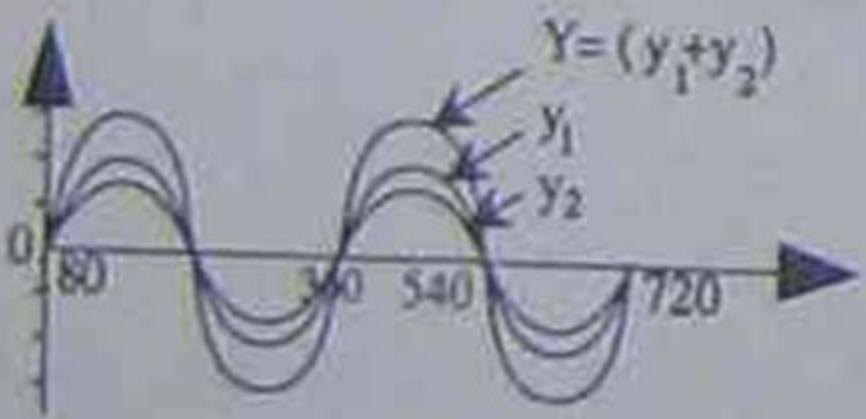
- (b) (i) Length of open pipe $L = \lambda_1/2$,
 $\lambda_1 = v/f_1 = 350/500 = 0.70 \text{ m} = 70 \text{ cm}$,
therefore $L = 35 \text{ cm}$.
- (ii) Length of closed pipe $L = \lambda_1/4$
 $= 70/4 = 17.5 \text{ cm}$.

Key facts and equations

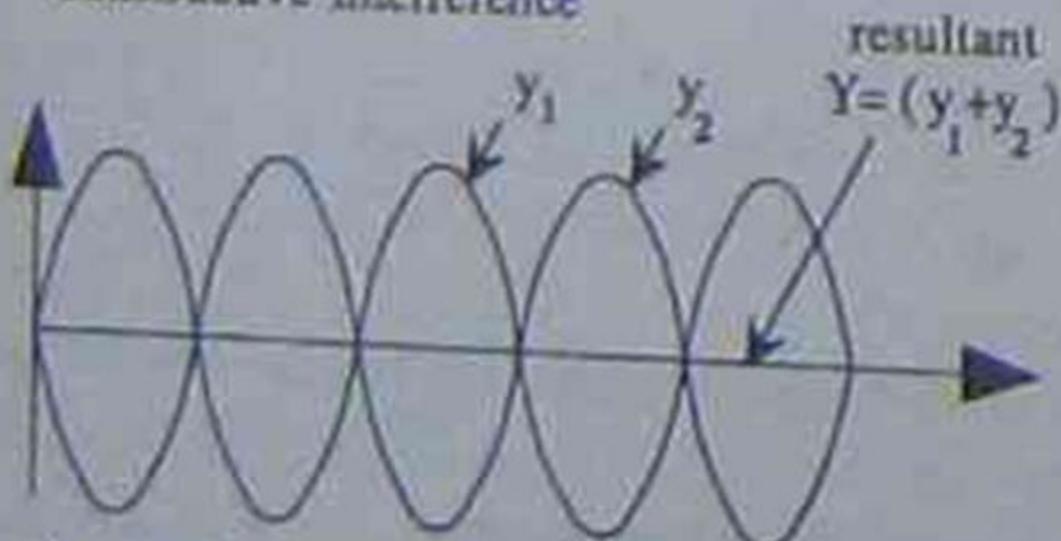
See also Chapter 6, 'Waves I', Key facts and equations, and revise:

- I. Wave terms
- II. Characteristics of waves
- III. Reflection and transmission of pulses and waves
- IV. Interference of pulses and waves

Pulses and waves interfere in accordance with the Principle of Superposition, that is, at any instant the displacement y of a particle in a medium is the sum of the displacements, $y_1 + y_2$, where y_1 is the particle's displacement due to wave 1 alone and y_2 is the particle's displacement due to wave 2 alone. When y_1 and y_2 are in phase, constructive interference occurs.



constructive interference



destructive interference

Fig. 16.30

When y_1 and y_2 are out of phase, destructive interference takes place.

(a) From two sources.

See Figure 16.17.

(b) Resonance or sympathetic vibration is the production of a large-amplitude vibration when a body or system receives an energy pulse with a frequency equal to the natural frequency of the body or system.

- (c) A beat is a regular rise and fall in amplitude of a wave formed by interference of progressive waves travelling with the same speed at about the same amplitude, and close frequencies, f_1 and f_2 . The observed beat frequency:

$$f_B = (f_1 - f_2); f_1 > f_2.$$

See Figure 16.18.

The envelope of a beat has a frequency of $(f_1 - f_2)/2$.

V. Waves (rays) and wave fronts in:

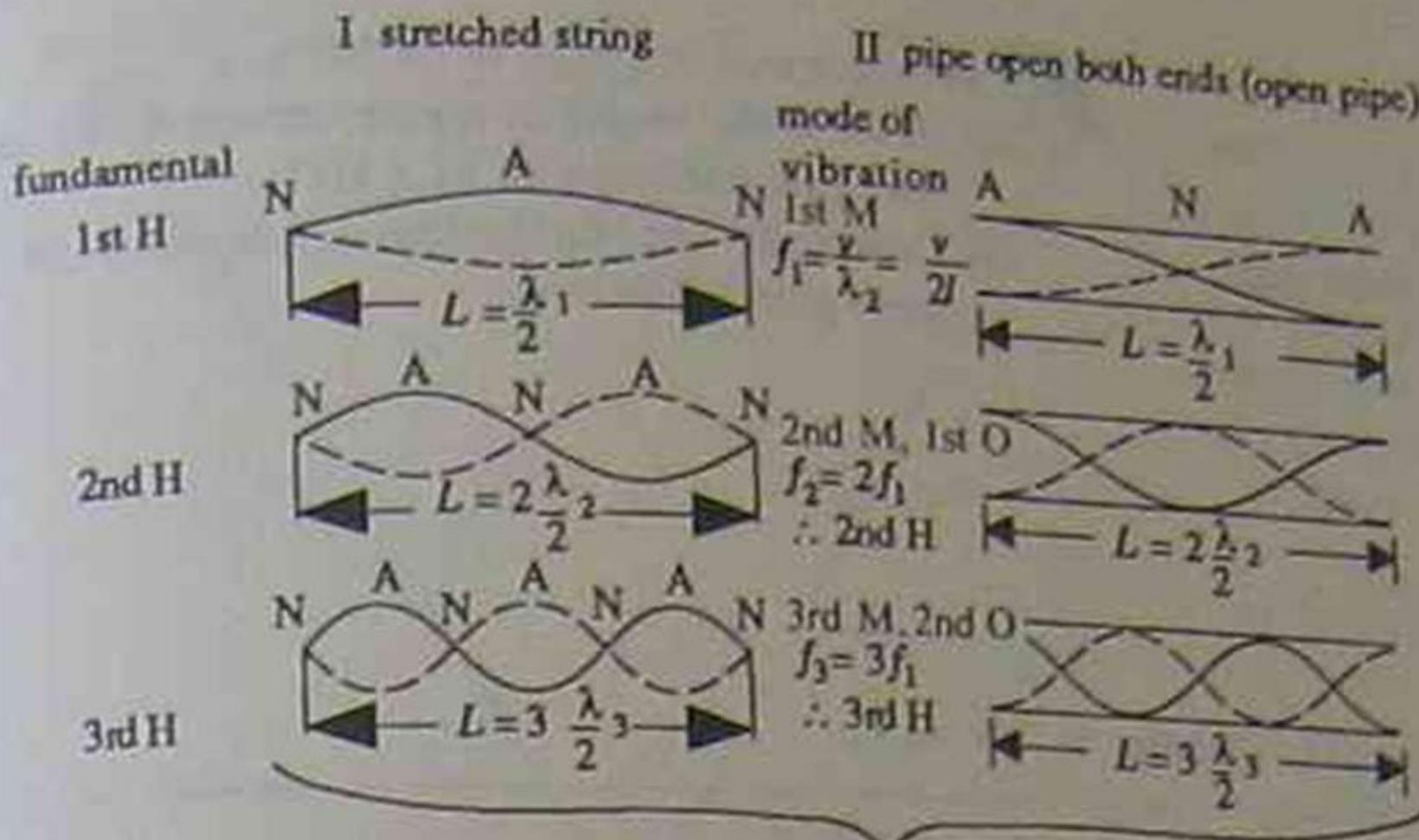
(a) Reflection

The line of a wave front is at right angles to the direction of wave (ray) travel. The rays obey the laws of reflection. These are:

1. The angle of incidence always equals the angle of reflection.
2. The incident ray, the normal and the reflected ray are all in the same plane, which is perpendicular to the reflecting surface. See Figure 16.1.

(b) Refraction

Waves entering a new material may change direction, velocity and wavelength; that is, they may be refracted. The waves obey the laws of refraction.



H = harmonic

N = node

A = antinode

O = overtone

f = frequency

λ = wavelength

M = mode

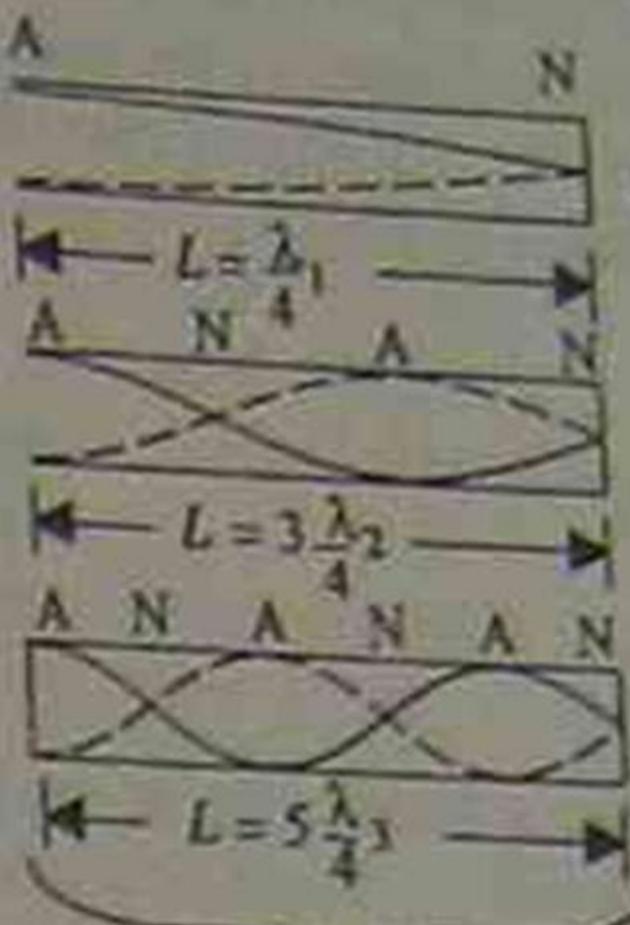
In general

$f_M = Mf_1$ — all harmonics formed.

M = mode number — an integer.

Fig. 16.31

III pipe open one end (closed pipe)



In general

$$f_M = (2M+1)f_1 \quad \text{— odd harmonics present}$$

- (c) Diffraction is the bending of rays around edges or objects in their path, e.g. sound travels 'round' corners owing to diffraction. The shape of the diffraction pattern depends on the ratio of λ/d where d is the size of the object or slit width.

Diffraction patterns of various size slits and objects are shown in Figures 16.9, 16.10 and 16.11.

(d) Dispersion is the separation of waves of different frequencies, when they enter a medium where their velocity depends upon frequency, so each frequency has its own velocity and refractive index. Glass is a dispersive medium for light waves, paraffin wax is a dispersive medium for microwaves and calcite disperses infra-red waves.

VII. Standing (stationary waves)

Standing (stationary) waves are produced when progressive waves travelling in opposite directions with equal frequency, speed and about the same amplitude interfere.

Longitudinal standing waves are produced by the reflection of longitudinal progressive waves in wind instruments and our vocal cords.

Transverse standing waves are produced by the reflection of progressive transverse waves in stringed instruments.

Features of standing waves are:

- (a) Modes of vibration—possible stationary wave patterns.
- (b) Nodes (N)—position of particles with no displacement; successive nodes are $\lambda/2$ apart.
- (c) Antinodes (A)—position of particles with maximum particle displacement; successive antinodes are $\lambda/2$ apart.
- (d) Fundamental or 1st mode or 1st tone—is the lowest frequency (1st harmonic) and the simplest mode of vibration.

- (e) Overtone—is any tone with a frequency greater than that of the fundamental.
- (f) Harmonic—is a wave whose frequency is a whole number multiple of the fundamental frequency.

Figure 16.31 shows the modes of vibration for standing waves in:

- I. A string clamped at both ends.
- II. An air column in a pipe open at both ends, i.e. an 'open pipe'.
- III. An air column in a pipe open at one end, i.e. a 'closed pipe'.

NB A standing wave is established in a body or system at resonance.