CHAPTER 3

Mechanical interactions I: Momentum in one dimension

Background knowledge: Before attempting this unit you should be familiar with Chapters 1 and 2.

SYMBOL AND UNIT SUMMARY

Symbol	*Quantity*	*Unit*
p	momentum	kg m s^{-1} or N s
Δp	change of momentum or impulse	kg m s^{-1} or N s
F	force	kg m s^{-2} or N
W	weight	kg m s^{-2} or N

Linear momentum

Momentum is a useful idea to help describe the effect of forces in collisions or during the change in motion of a body.

Linear momentum is the product of mass and velocity of a body and has a direction the same as the velocity.

$$p = mv \text{ kg m s}^{-1} \text{ or N s}$$

The momentum in one direction is regarded as a positive quantity and the momentum in the opposite direction takes a negative sign.

EXAMPLE

A car of mass 600 kg is initially travelling at 20 m s^{-1} north and three seconds later has a velocity of 30 m s^{-1} south. Calculate the car's:

(a) initial momentum, p_i

(b) final momentum, p_f

Answer

(a) Let momentum to the north be +ve

$$\begin{aligned} p_i &= mu \\ &= 600 \times (+20) \text{ N s} \\ &= +1.2 \times 10^4 \text{ N s or} \\ &\quad 1.2 \times 10^4 \text{ N s north} \end{aligned}$$

(b)

$$\begin{aligned} p_f &= mv \\ &= 600 \times (-30) \text{ N s} \\ &= -1.8 \times 10^4 \text{ N s or} \\ &\quad 1.8 \times 10^4 \text{ N s south} \end{aligned}$$

Changes in momentum or impulse

Change in momentum Δp, also called **impulse**, is a vector quantity (see Chapter 8). Impulse is calculated by taking the initial momentum p_i from the final momentum p_f.

$$\Delta p = (p_f - p_i) = (mv - mu)$$

EXAMPLE

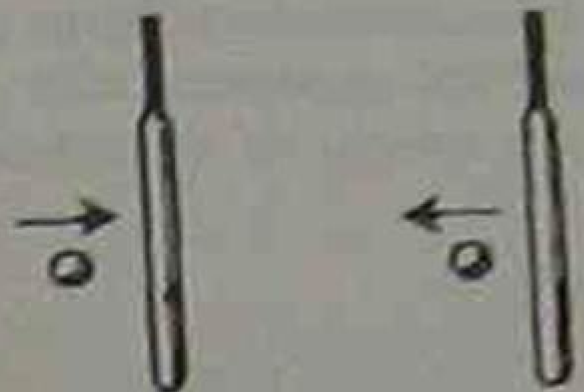


Fig. 3.1

A ball of mass 0.50 kg moves horizontally to the right at $10\ \mathrm{m\,s^{-1}}$ and strikes a stationary vertical bat. Calculate the impulse if:

(a) the ball stops when it hits the bat;

(b) the ball rebounds at $10\ \mathrm{m\,s^{-1}}$ along its original path.

Answer

Answer

Consider vectors to the right to be positive and those to the left negative.

(a) $p_i = mu$
$= 0.50 \times (+10)$ N s
$= +5.0$ N s

$p_f = mv$
$= 0.50 \times 0$ N s
$= 0.00$ N s

therefore $\Delta p = (p_f - p_i)$
$= [0.00 - (+5.0)]$ N s
$= -5.0$ N s or
5.0 N s to the left.

(b) In this case
$p_f = 0.50 \times (-10)$ N s
$= -5.0$ N s

Hence $\Delta p = [-5.0 - (+5.0)]$ N s
$= -10$ N s, i.e. 10 N s to the left.

The answer can be shown on a scaled line with 1 cm = 4.0 N s.

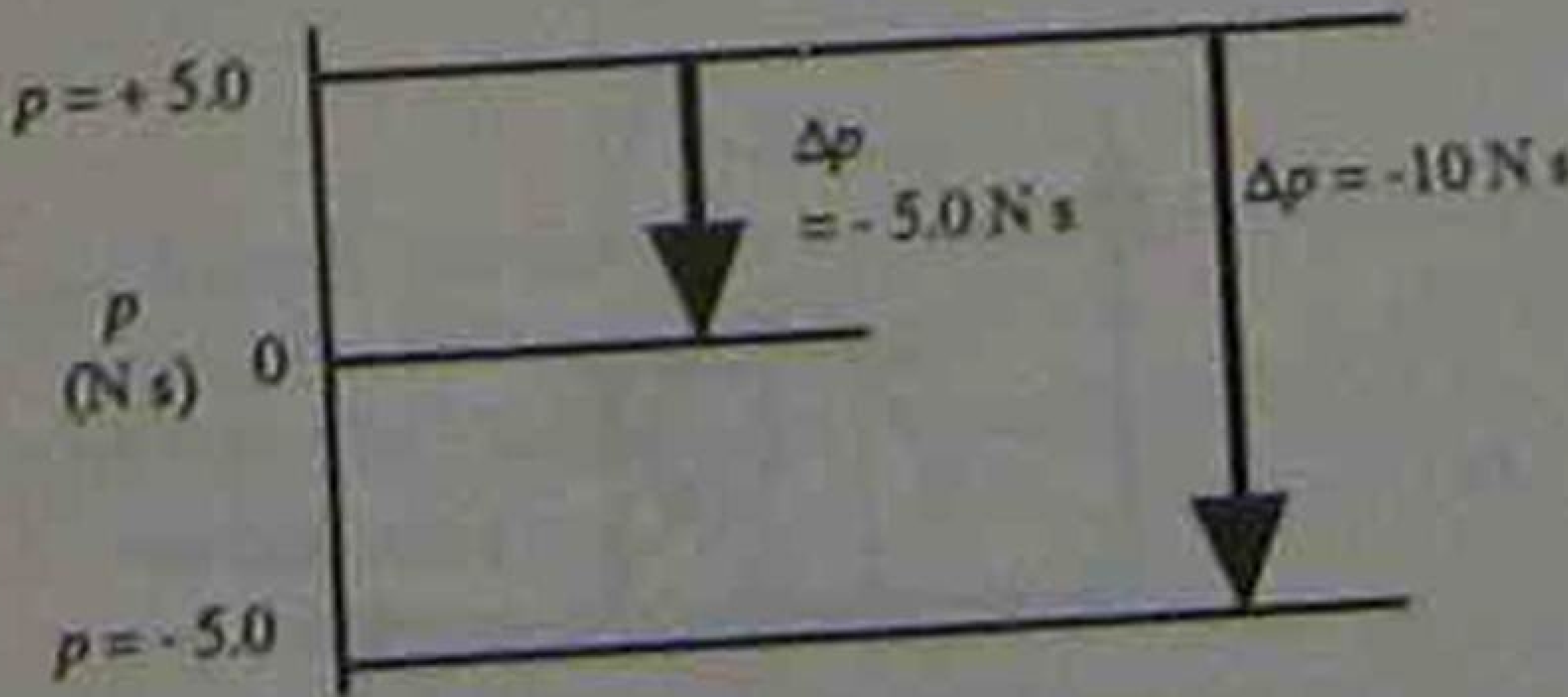


Fig. 3.2

Conservation of linear momentum

Law of conservation of linear momentum

Provided there are no external forces acting on a system of colliding or exploding bodies, then along any direction:

$$\left.\begin{array}{c}\text{Total linear}\\ \text{momentum before the}\\ \text{collision/explosion}\end{array}\right\} = \left\{\begin{array}{c}\text{Total linear}\\ \text{momentum after the}\\ \text{collision/explosion}\end{array}\right.$$

A system separated from external forces is called an isolated system.

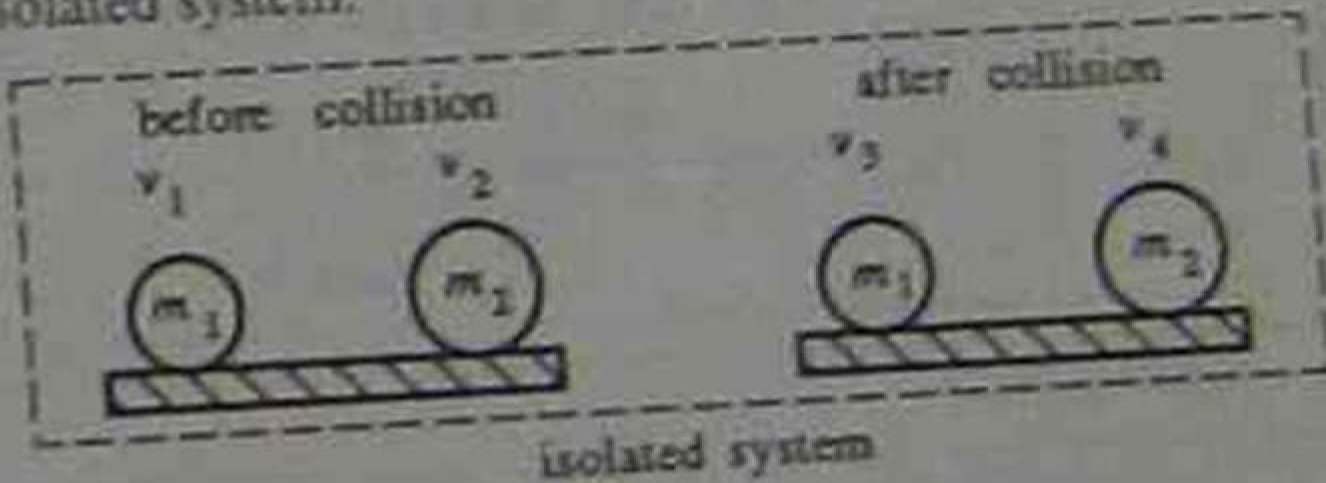


Fig. 3.3

Fig. 3.3

Mathematically,

let p = momenta before impact or explosion,

and p' = momenta after impact or explosion.

For conservation of momentum:

$$p_1 + p_2 = p_1' + p_2' \qquad (1)$$

$$m_1v_1 + m_2v_2 = m_1v_3 + m_2v_4$$

If m_2 is at rest before collision and m_1 and m_2 stick together (coalesce) on impact and their combined mass has velocity V, then Equation 1 simplifies to:

$$m_1v_1 = (m_1 + m_2)V$$

NB In 'Energy and collisions' (Chapter 4) we will see that in an elastic collision/explosion both momentum and kinetic energy (energy due to motion) are conserved.

EXAMPLE

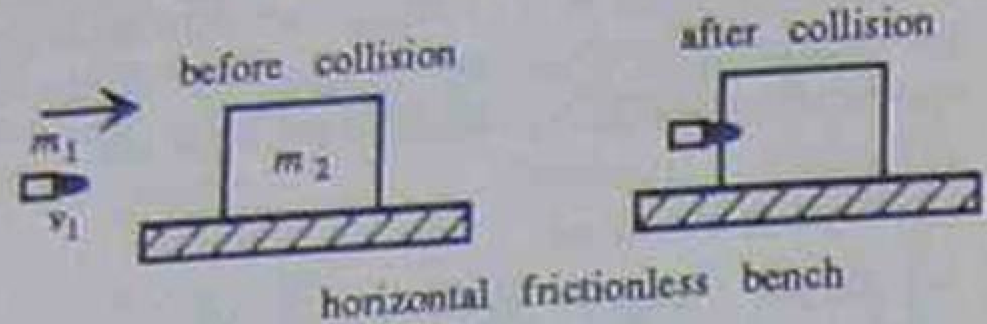


Fig. 3.4

A bullet of mass 3.00 g travelling at 700 m s^{-1} collides and coalesces with a stationary block of wood of mass 1997 g.

(a) Calculate the total initial momentum of the system.

(b) What is the total momentum of the system after the collision?

(c) For the composite body after the collision calculate its:

(i) mass;

(ii) velocity.

Answer

(a) $p_1 = m_1v_1$

$= 0.00300 \times 700$ N s

$= 2.10$ N s in the direction of v_1

(b) From the Law of Conservation of Momentum, the same answer as in (a).

(c) (i) $m' = (m_1 + m_2)$

$= (0.003 + 1.997)$ kg

$= 2.000$ kg

(ii) From conservation of momentum,

$m_1v_1 = (m_1 + m_2)V$

$2.10 = 2.000\,V$

i.e. $V = 2.10/2.000$ m s^{-1}

$= 1.05$ m s^{-1} along the line of impact

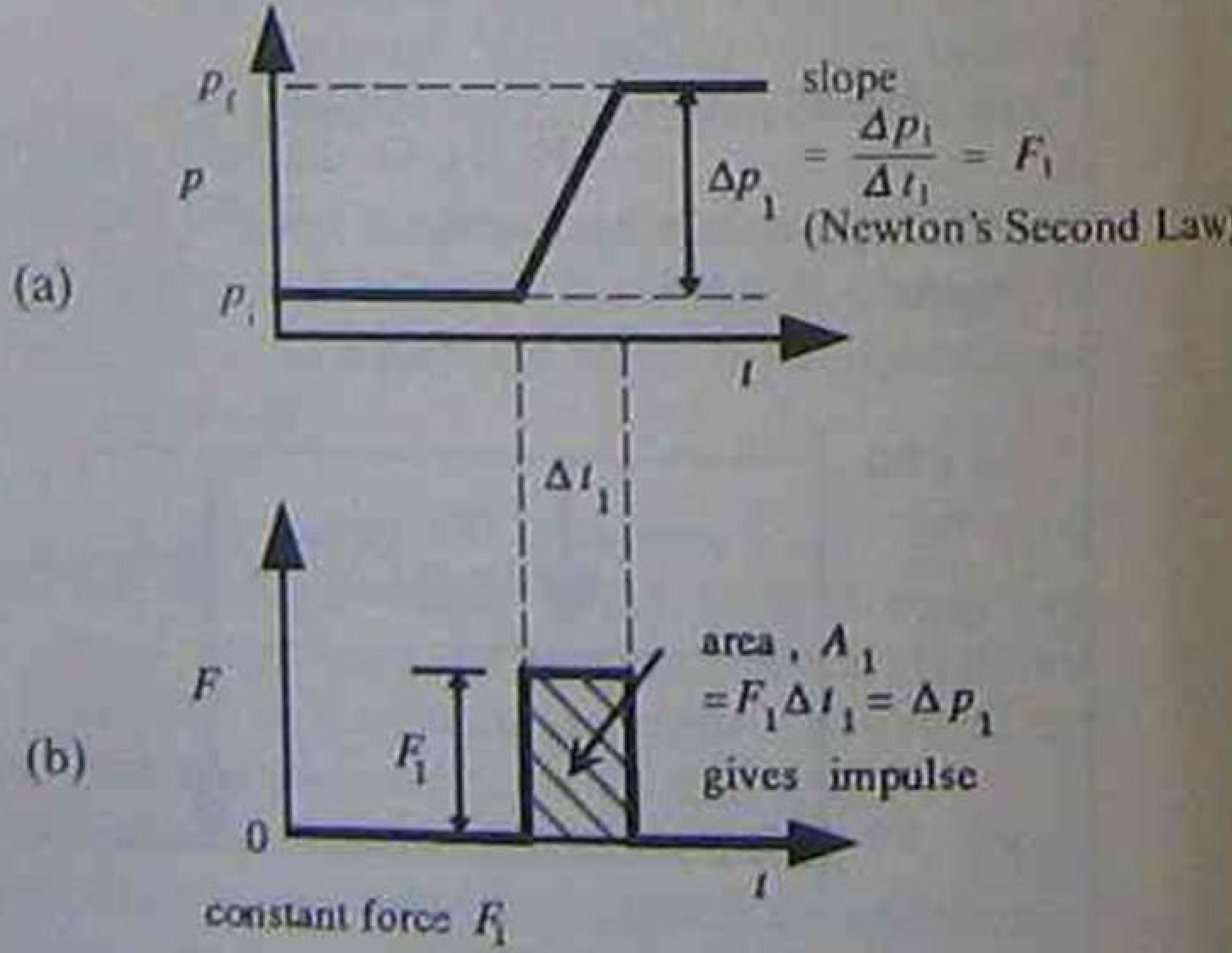
Graphing momentum

When momenta change, for example in a collision or explosion, a momentum versus time (p/t) graph and a corresponding force versus time (F/t) graph can be used to describe momenta before, during, and after a collision or explosion. The area under the force-against-time graph gives a measure of a force–time product or impulse. Two special cases need to be considered:

Case A

When the force changing momentum is constant.

Consider an event in which a body with constant momentum p_i experiences first a constant force F_1 for time Δt_1 in the direction of p_i and the momentum is changed to p_f. This event can be described graphically using a p/t graph (see Figure 3.6(a)) and an F/t graph as shown in Figure 3.6(b).

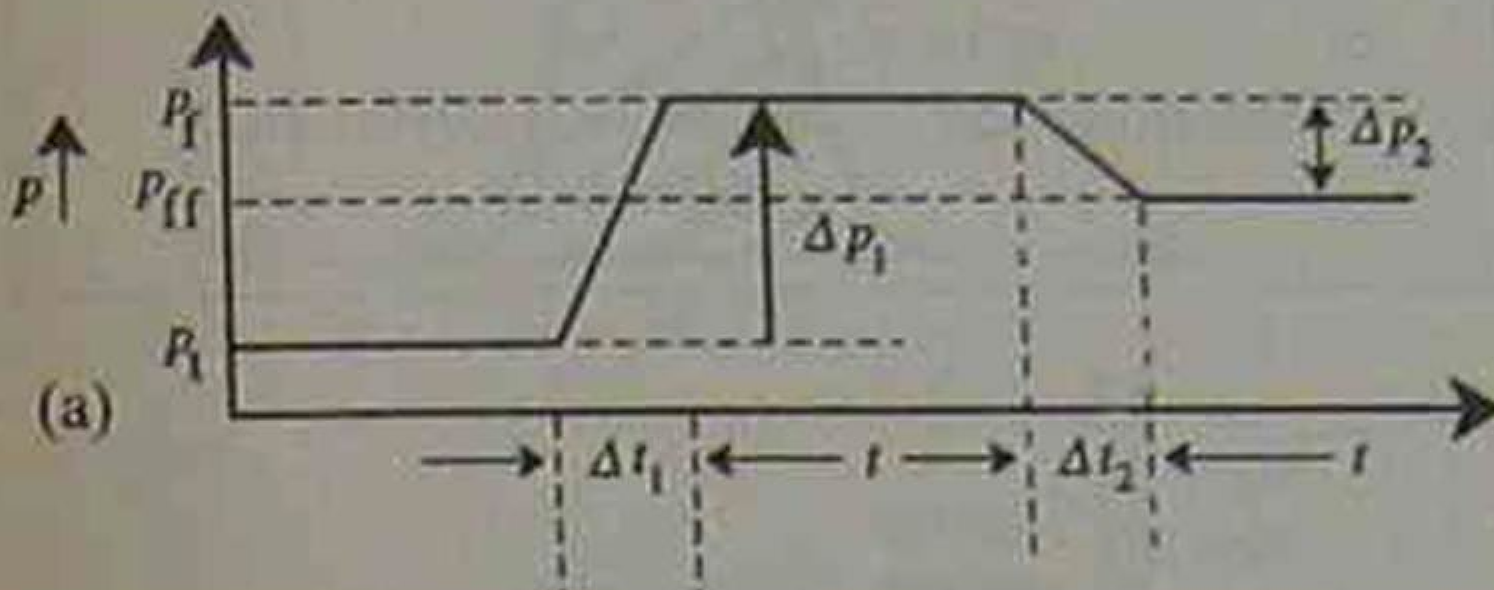


p_f
p
p_i
(a)
slope
$= \frac{\Delta p_1}{\Delta t_1} = F_1$
(Newton's Second Law)
Δp_1
t
Δt_1
F
(b)
F_1
0
area, A_1
$= F_1 \Delta t_1 = \Delta p_1$
gives impulse
t
constant force F_1

Fig. 3.6

At time t after the first force ceased to act, a second constant smaller force F_2 is applied for time Δt_2 *opposite to the direction* of p_i and the new momentum is now p_f.

The slope of the p/t graph represents the size and sign of the force, F, in the corresponding F/t graphs for the time that F_1 and F_2 act as shown in Figure 3.7.



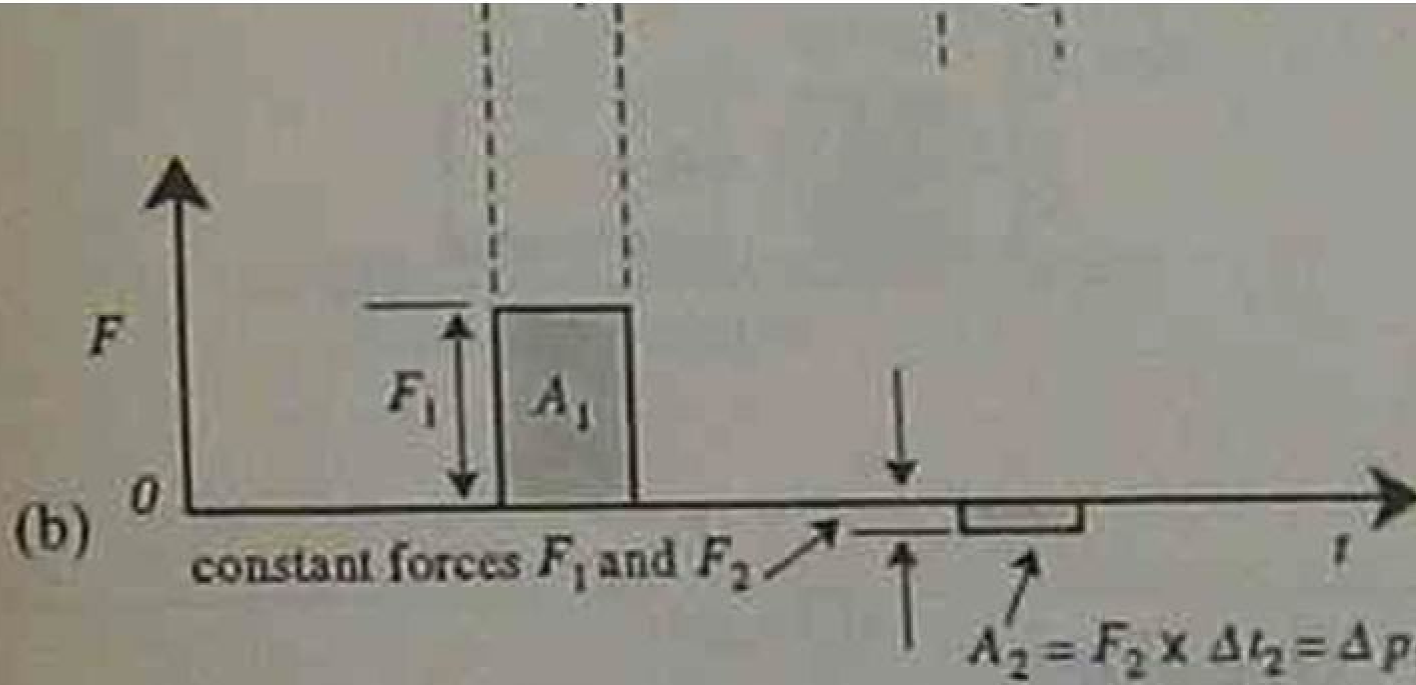


Fig. 3.7

The area between the graph and the axis above the time axis represents change in momentum Δp_1 in a particular direction, and the area A_2 under the F/t graph and below the time axis represents a change in momentum Δp_2 in the opposite direction. If during a particular event areas A_1 and A_2 are equal, net change of momentum is zero.

EXAMPLE

The force versus time graph for a ball of mass 0.2 kg and velocity 100 m s^{-1} on striking a bat travelling in the same direction as the velocity is shown in Figure 3.8.

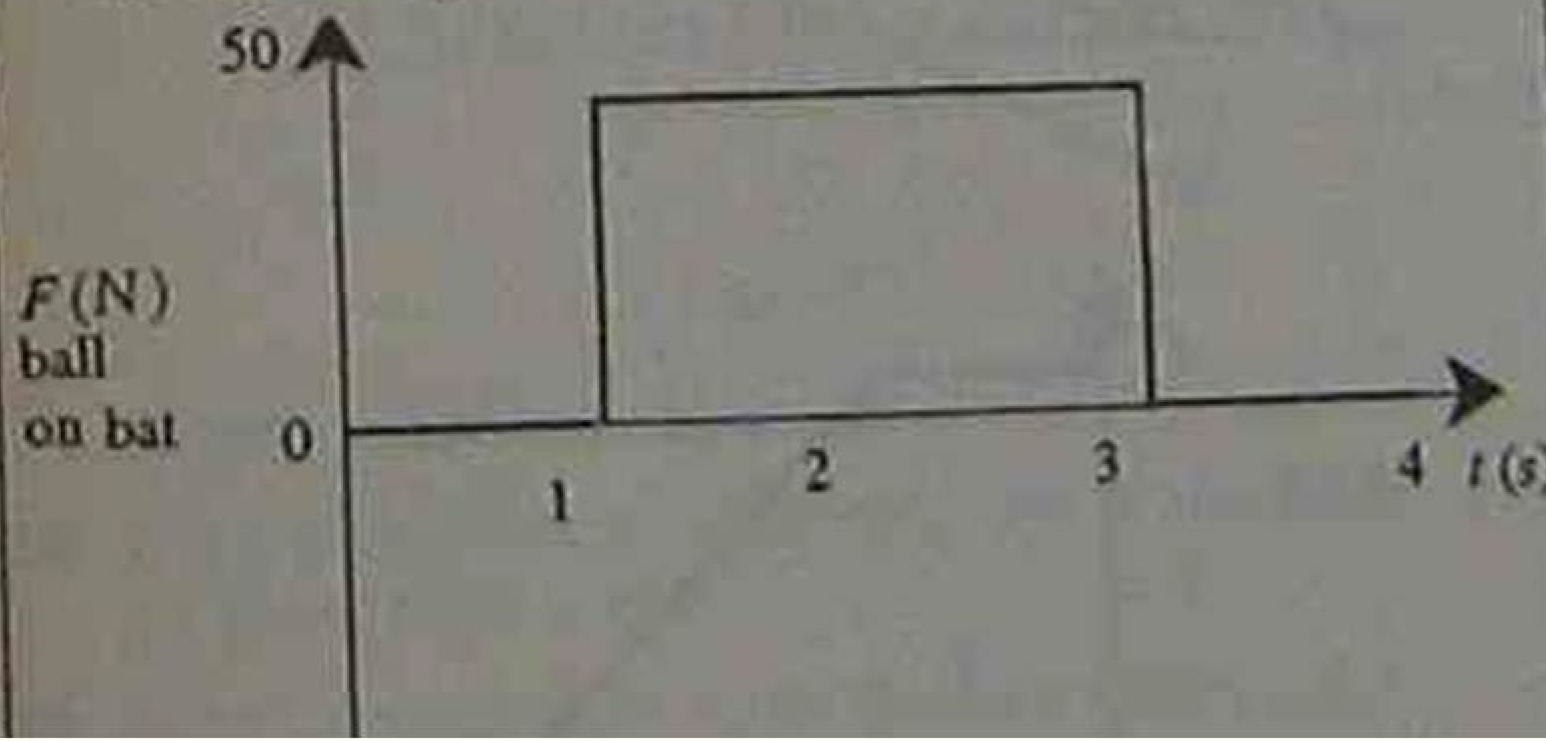


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Fig. 3.8

(a) Sketch the force of bat on ball versus time from $t = 0$ s to $t = 4$ s.

(b) Calculate the change in momentum of the ball during:

(i) the 2nd second;

(ii) the 3rd second.

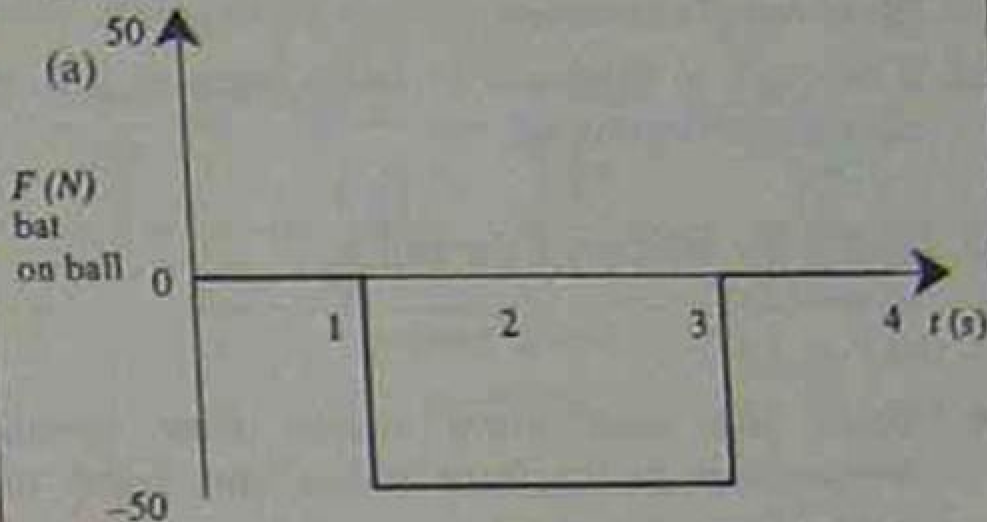
(c) What is the average force between the bat and the ball during the strike?

(d) Sketch the corresponding momentum versus time graph.

(e) Determine the final velocity of the ball.

Answer

(a)



(b) Δp = area under F/t graph = $F \times \Delta t$

(i) $\Delta p = 50\text{ N} \times 1\text{ s}$

$= 50$ Ns in the direction of the original momentum.

(ii) $\Delta p = 50\text{ N} \times 1\text{ s}$

$= 50$ Ns in the direction of the original momentum.

(c) $F = \Delta p / \Delta t$

$= 100/2\text{ N}\cancel{s}/\cancel{s}$

$= 50$ N in the direction of the original momentum

momentum.

(d)

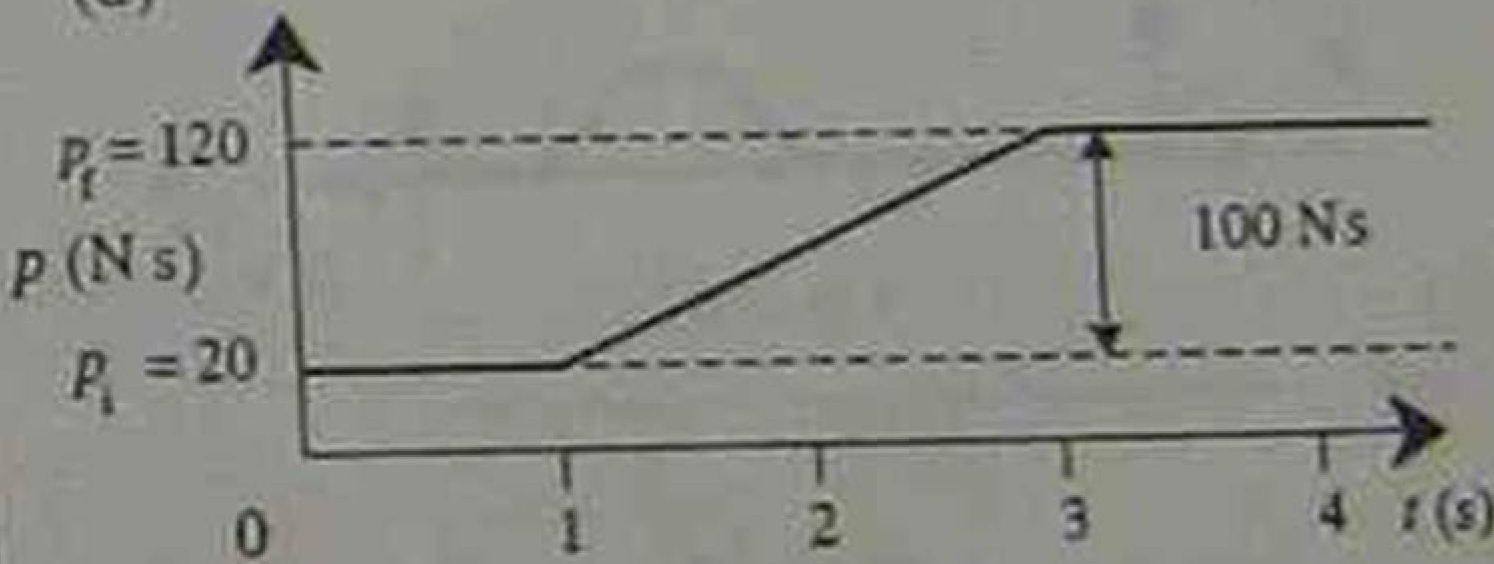


Fig. 3.10

(e) $v = u + \Delta v$ but $\Delta v = \Delta p / m$

$$= 100/0.2 \text{ ms}^{-1}$$
$$= 500 \text{ ms}^{-1}$$

$= (100 + 500) \text{ ms}^{-1}$

$= 600 \text{ ms}^{-1}$ in the ball's original direction.

EXAMPLE

The force against time for a golf club striking a stationary ball of mass 100 g is shown in Figure 3.13.

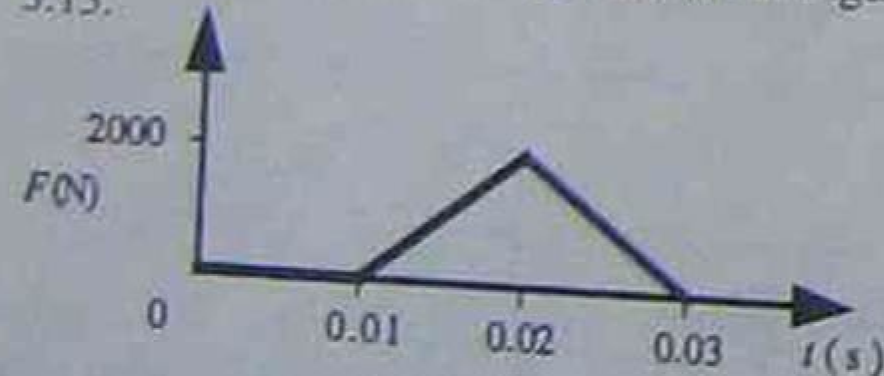


Fig. 3.13

What is:

(a) the impulse imparted to the ball?
(b) the average force on the ball?
(c) the speed of the ball after the strike?

Answer

(a) Area under F/t graph = impulse, therefore the impulse imparted to the ball is represented by the area of a triangle
$= +\frac{1}{2}(0.03 - 0.01) \times 2000$ N s
$= +20$ N s or 20 N s in the direction of $+F$.

(b) $F(\text{average}) = \Delta p / \Delta t = +20/0.02$ N

$= +1000$ N

Alternative answer:

F changes at a uniform rate, therefore

$F(\text{average}) = (\text{minimum force} + \text{maximum force})/2$

$= (0 + 2000)/2$ N

$= +1000$ N

(c) $m\Delta v = \Delta p$

therefore $\Delta v = \Delta p/m = +20/0.1$ m s^{-1}

$= +200$ m s^{-1}

Collisions, and momentum versus time graph problems

During a collision in which momentum is conserved, momentum is transferred from one object to another and there is no change in the total momentum of the system before, during or after the collision. Momentum changes with time can be comprehensively described in a momentum versus time graph. Consider a collision between ball X moving to the right and a stationary ball Y. As a result of the collision, ball X rebounds and ball Y moves to the right, as shown in Figure 3.18.

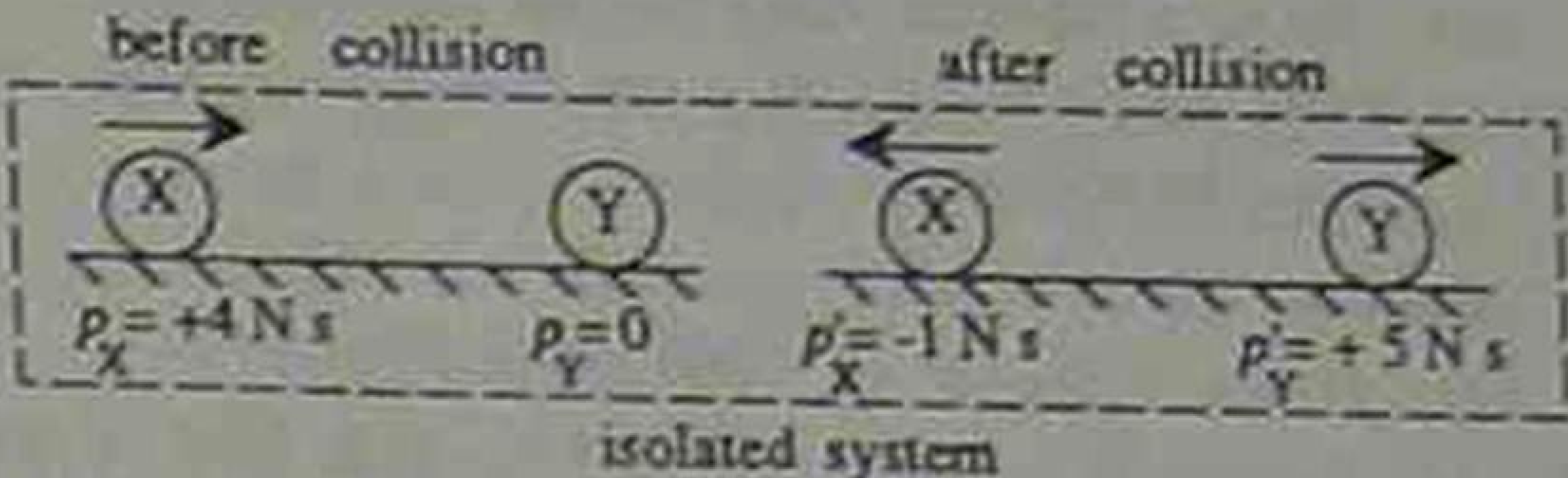


Fig. 3.18

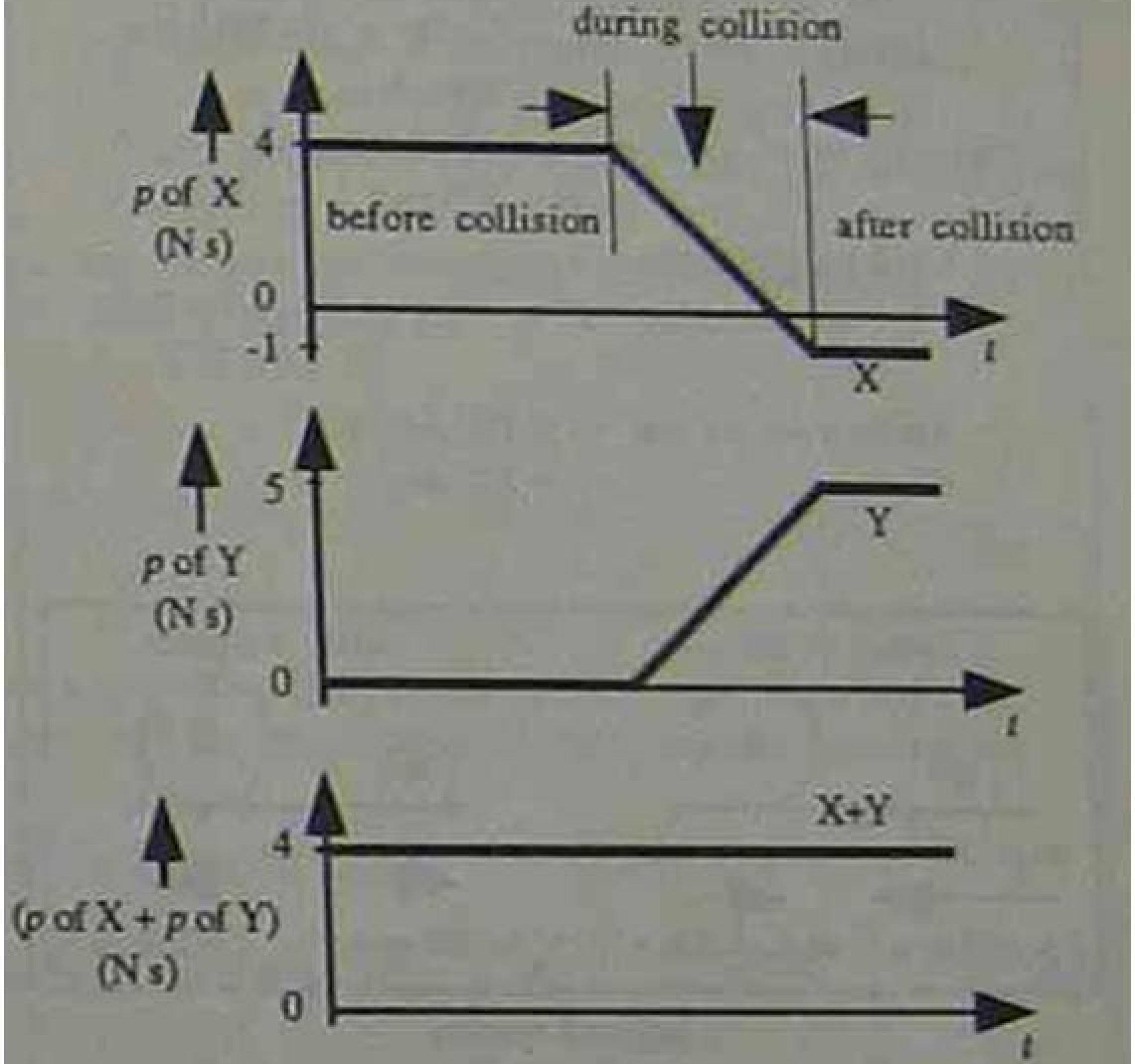


Fig. 3.19

The momentum versus time graph for ball X, ball Y and the system before, during and after the collision are shown in Figure 3.19.

EXAMPLE

Two balls, A of mass 2.0 kg, and B of mass 5.0 kg, collide at time T. The momentum versus time graph for both balls before the collision and the momentum of ball A after the collision are shown in Figure 3.20.

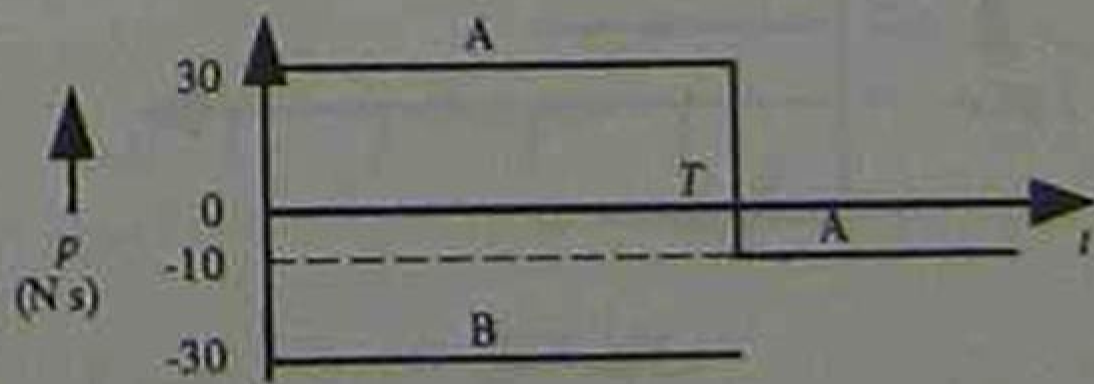


Fig. 3.20

(a) Calculate:

(i) the pre-collision velocity u_A of ball A;

(ii) the pre-collision velocity u_B of ball B;

(iii) total momentum of the system.

(b) Calculate:

(i) the post-collision velocity v_A of ball A;

(ii) the post-collision momentum p_B' of ball B;

(iii) the velocity v_B of ball B.

(c) Draw a labelled diagram to represent the mass and velocity of each ball before and after the collision.

Answer

(a) (i) $u_A = p_A/m = +30/2.0\ \text{m s}^{-1}$
$= +15\ \text{m s}^{-1}$

(ii) $u_B = p_B/m = -30/5.0\ \text{m s}^{-1}$
$= -6.0\ \text{m s}^{-1}$

(iii) $p = p_A + p_B = [+30 + (-30)]\ \text{N s}$
$=$ zero N s

(b) (i) $v_A = p_A'/m = -10/2.0\ \text{m s}^{-1}$
$= -5.0\ \text{m s}^{-1}$

(ii) to conserve momentum $p_B' = -p_A' = -(-10\ \text{N s})$
$= +10\ \text{N s}$

(iii) $v_B = p_B'/m = +10/5.0\ \text{m s}^{-1}$
$= +2.0\ \text{m s}^{-1}$

(c)

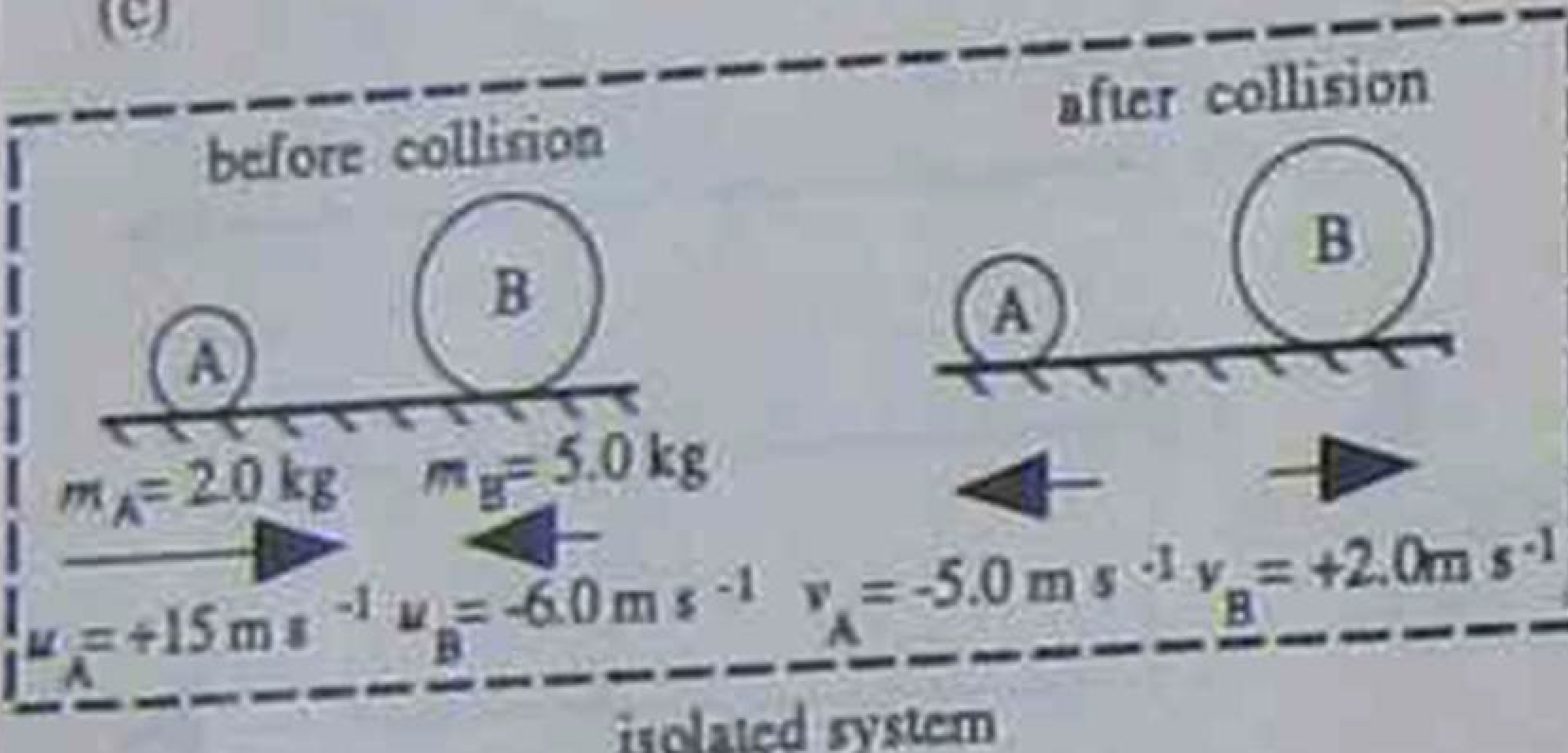

before collision
after collision
A
B
A
B
$m_A = 2.0$ kg
$m_B = 5.0$ kg
$u_A = +15$ m s^{-1}
$u_B = -6.0$ m s^{-1}
$v_A = -5.0$ m s^{-1}
$v_B = +2.0$ m s^{-1}
isolated system

Fig. 3.21

Impulse and force problems

As you saw in Chapter 2 on force, one form of Newton's Second Law is:

$$F = ma \qquad (1)$$

and in Chapter 1 on motion in one dimension,

$$\text{acceleration } a = \Delta v/\Delta t$$
$$= (v - u)/\Delta t \qquad (2)$$

When we combine Equations 1 and 2

$$F = (mv - mu)/\Delta t = m\Delta v/\Delta t$$

$$\text{i.e. } F = \Delta p/\Delta t$$

$$\text{hence } \Delta p = F \times \Delta t \quad \text{or} \quad m\Delta v = F \times \Delta t$$

Thus the change of momentum = applied force × time. When Δt is small, the applied force is called the impulsive force. For a given impulse, F is large when Δt is small, e.g. when a hammer hits a nail, and F is small when Δt is large, e.g. when a ball is caught by hands moving backwards with the ball.

EXAMPLE

A pile driver of mass 50 kg moving down at $3.0\ \mathrm{m\,s^{-1}}$ strikes a pile and rebounds up at $1.0\ \mathrm{m\,s^{-1}}$. If the pile driver is in contact with the pile for 0.50 s, calculate:

(a) the impulse imparted to
  (i) the pile driver;
  (ii) the pile;

(b) the average force exerted on
  (i) the pile driver;
  (ii) the pile.

Answer

Consider impulse down as +ve

(a) (i) Δp (pile driver) $= p_f - p_i$

$= mv - mu$

$= [50 \times (-1.0) - 50 \times (+3.0)]$ N s

$= -200$ N s or 200 N s up.

(ii) Δp (pile) which equals, and is in opposite direction to, Δp imparted to pile driver

$= +200$ N s or 200 N s down.

(b) (i) F (on pile driver)

$= \Delta p / \Delta t$

$= -200/0.50$ N

$= -400$ N or 400 N up

F (on pile) is equal in magnitude but opposite in direction to the force on the pile driver

$= +400$ N or 400 N down.

Take $g = 9.8\ \mathrm{m\,s^{-2}}$.

ADDITIONAL WORKED EXAMPLES

1. A shell of mass $m = 20$ kg is fired horizontally from a gun on a level frictionless surface. Before firing, the mass of gun plus shell is 2020 kg. After firing, the shell moves down with uniform acceleration the 2.0 m long gun barrel in 0.02 s. Calculate:
   (a) the average and final velocity of the shell as it travels down the barrel;
   (b) the impulse imparted to the shell;
   (c) the average impulsive force on the shell as it travels down the barrel;
   (d) the recoil velocity V of the gun.

Answer

(a) $v_{av} = (u + v)/2 = s/t$

$(0 + v)/2 = +2.0/0.02\ \mathrm{m\,s^{-1}} = +100\ \mathrm{m\,s^{-1}}$

$v = +200\ \mathrm{m\,s^{-1}}$ or $200\ \mathrm{m\,s^{-1}}$ down the barrel.

(b) $\Delta p = m\Delta v = 20 \times (+200)\ \mathrm{N\,s} = +4000\ \mathrm{N\,s}$

(c) $F = \Delta p/\Delta t = +4000/0.02\ \mathrm{N} = +200\ \mathrm{kN}$

(d) For conservation of linear momentum:

total momentum before explosion = total momentum after explosion

$0 = p'(\text{gun}) + p'(\text{shell})$.

therefore $0 = (M - m)V + mv$

hence $V = -mv/(M - m) = -20 \times 200/2000\ \mathrm{m\,s^{-1}}$

$= -2.0\ \mathrm{m\,s^{-1}}$

2. Figure 3.23 shows the force versus time graph for a body of mass 20 kg initially moving to the right at 6.0 m s^{-1}.

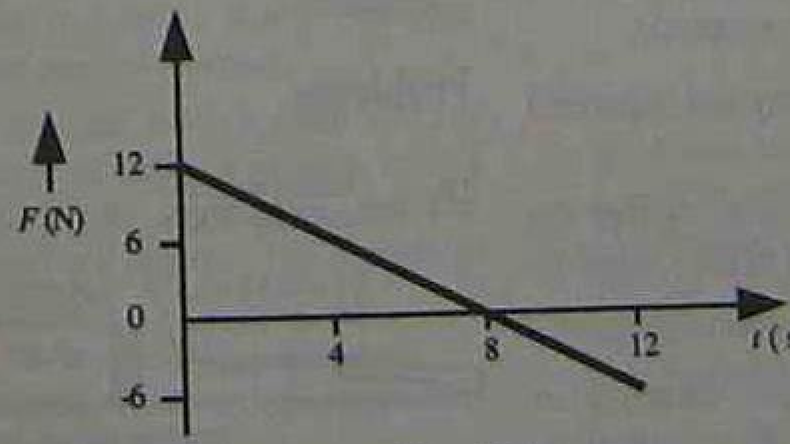


Fig. 3.23

Calculate:

(a) the initial momentum of the body;

(b) the impulse from $t = 0$ s to (i) $t = 4$ s (ii) $t = 8$ s (iii) $t = 12$ s;

(c) the average force on the body;

(d) the velocity of the body after (i) 8 s (ii) 12 s.

Answer

(a) $p = mu = 20 \times (+6.0)$ N s $= +120$ N s or 120 N s to the right.

(b) Using the idea that the area under the F/t graph = impulse:
if the area is above the t axis the impulse takes a particular sign (say positive) and areas below the t axis and the graph are assigned the opposite sign (negative).

(i) Therefore, impulse from $t = 0$ s to $t = 4$ s

$= [(+6 \times 4) + (\frac{1}{2} \times 6 \times 4)]$ N s

$= +36$ N s or 36 N s to the right,

(ii) similarly, impulse from $t = 0$ s to $t = 8$ s

$= +\frac{1}{2} \times 12 \times 8$ N s

$= +48$ N s,

$= +48$ N s,

(iii) likewise, impulse from $t = 0$ s to $t = 12$ s

$= +(48 - 12)$ N s

$= +36$ N s.

(c) From Newton's Second Law:

$F(\text{average}) = \Delta p / \Delta t = +36/12$ N $= +3$ N

Alternative answer for (c):

The force changes at a constant rate, therefore the average force

$F(\text{ave}) =$ (initial force + final force)/2

$= [(+12 + (-6)]/2$ N $= +6/2$ N $= +3$ N

(d) (i) $v = u + \Delta v$. After 8 s, $\Delta v = \Delta p / m = +48/20$ m s^{-1} $= +2.4$ m s^{-1}

$= [(6 + (+2.4)]$ m s^{-1}

$= +8.4$ m s^{-1}

(ii) Similarly after 12 s, $\Delta v = +36/20$ m s^{-1} $= +1.8$ m s^{-1}

hence $v = [(6 + (+1.8)]$ m s^{-1}

$= +7.8$ m s^{-1}

Key facts and equations

- Linear momentum is the product of mass and velocity. Momentum p is regarded as +ve in one direction and −ve in the opposite direction: $p = mv$
- Change in momentum or impulse:
  $$\Delta p = (p_f - p_i) = [(p_f + (-p_i)]$$
  Impulse can be determined graphically, mathematically and from the area under an F/t graph.
- From Newton's Second Law, unbalanced force F is the rate of change of momentum $F = \Delta p / \Delta t$.

- An isolated system is a system of any size separated from external forces.

- For collisions/explosions in an isolated system the Law of Conservation of Momentum applies, namely:

$$\left.\begin{array}{c}\text{total linear}\\ \text{momentum before}\end{array}\right\} = \text{total linear momentum after}$$

$p_1 + p_2 = p_1' + p_2'$ where $'$ means after collision/explosion,

and $m_1v_1 + m_2v_2 = m_1v_3 + m_2v_4$.

- An elastic collision/explosion is one in which both momentum and kinetic energy are conserved. See also Chapter 4, 'Energy and collisions'.

- Collisions/explosions can be described and analysed using a momentum versus time graph and corresponding force versus time graph.

- The area under the force versus time graph yields the impulse.

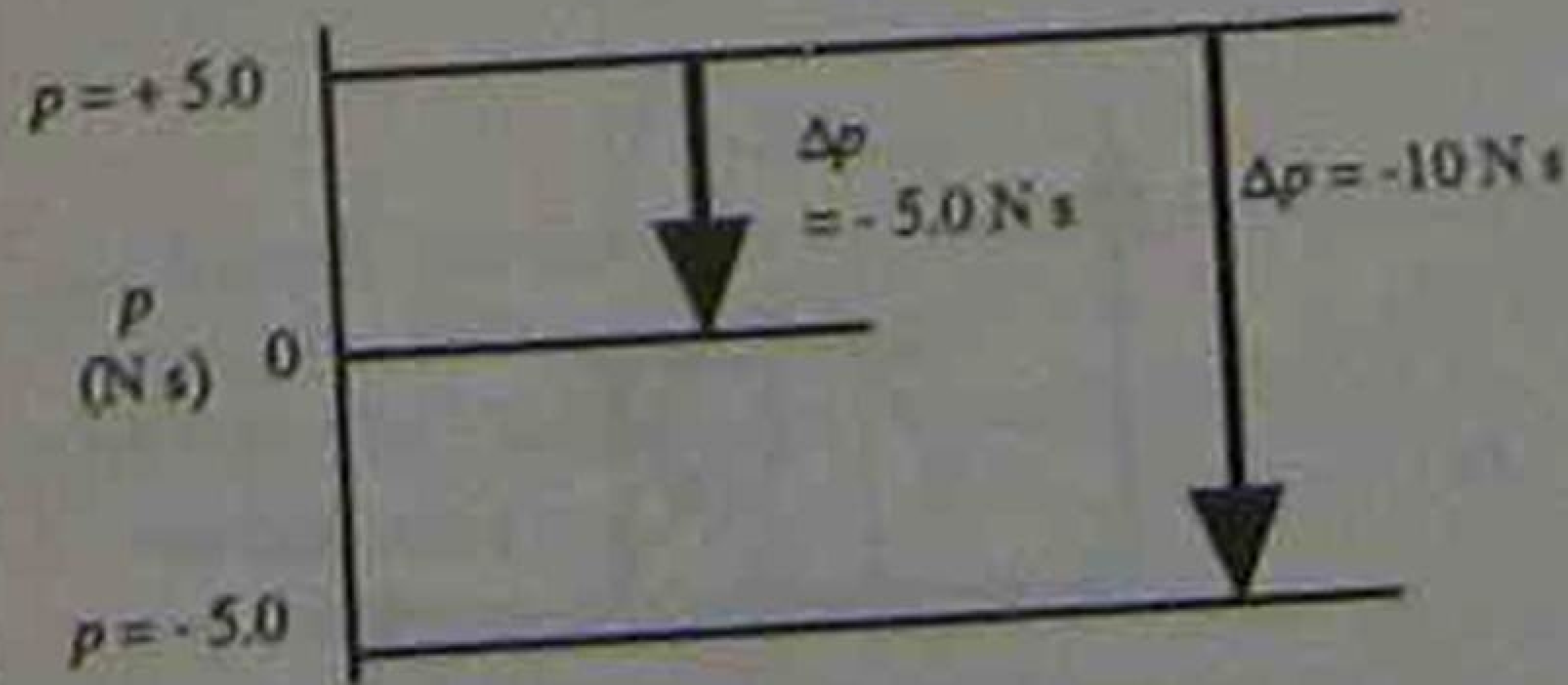


Fig. 3.2