CHAPTER 4

Mechanical interactions I: Work, energy and power

Background knowledge: You need to have worked through Chapters 1, 2 and 3.

SYMBOL AND UNIT SUMMARY

Symbol	Quantity	Unit
W	work	J or N m or kg m^2 s^{-2}
E	energy	
KE	kinetic energy	
PE	potential energy	
P	power	W or J s^{-1} or N m s^{-1} or kg m^2 s^{-3}

Work

Work in a scientific sense is defined as the product of force and displacement:

$$W = Fs$$

where F and s are in the same direction.

Fig. 4.1

Work done is 1 newton metre or 1 joule when a force of 1 newton moves its point of application 1 metre in the direction of the force.

NB When the displacement is not in the direction of the force, e.g. pulling a trolley where the applied force is at angle θ to the direction of displacement, work is calculated by multiplying the component of the force in the direction of the displacement, $F \cos \theta$, by the displacement s.

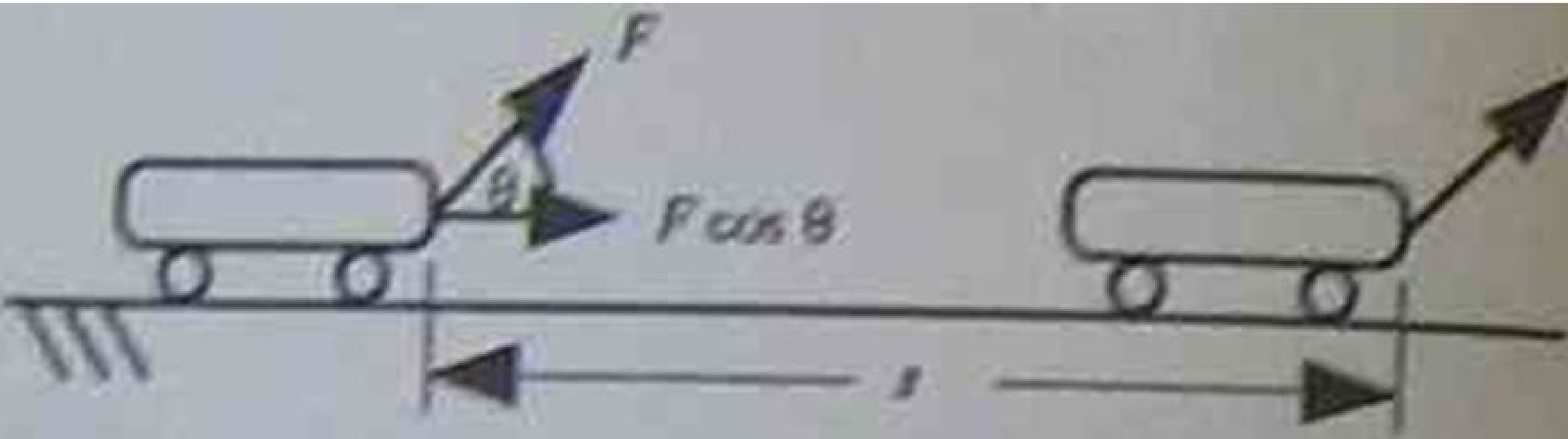


Fig. 4.2

$$W = Fs \cos \theta$$

Work is a scalar quantity (it needs size only to describe it) with SI units of N m or J.

Quantities that require size and direction to describe them are called **vectors** (see Chapter 8).

EXAMPLE

Calculate the work done:

(a) when a force of 50 N moves its point of application 3.0 m in the direction of the force:

(b) when a force of 50 N moves its point of application 3.0 m in a direction of 60 degrees to the line of action of the force.

Answer

(a) $W = Fs = 50 \times 3.0\ \text{N m} = 150\ \text{N m}$

(b) $W = Fs \cos\theta$

$= 50 \times 3.0 \times \cos 60^\circ\ \text{N m}$

$= 75\ \text{N m}$

Work and energy

Energy is a measurement of the capacity of a body or system to do work. It is a scalar quantity with SI units of N m or J.

Mechanical energy is a form of energy possessed by a body or system because of its motion or position.

Work and force versus displacement graphs

The area under a graph of force F (N) on the vertical axis against displacement s (m) of the point of application of the force on the horizontal axis is a measure of work done in joules. This F versus s graph is also called a work diagram. The work diagram when the force F and s are in the same direction and F is (a) constant and (b) varies uniformly, is given in Figure 4.4.

4.4.

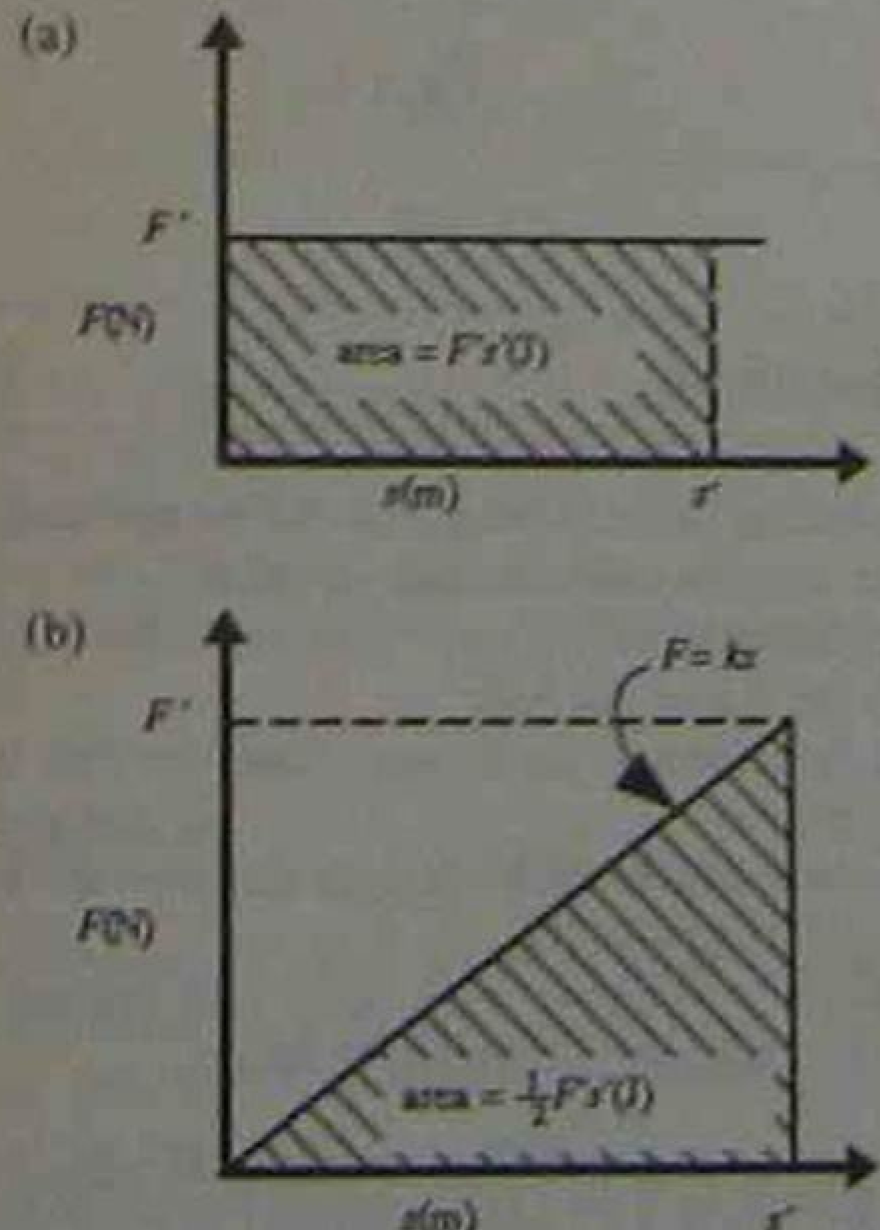


Fig. 4.4

The equation of graph (b) is $F = ks$ where k (N m^{-1}) is the slope of the graph. As Robert Hooke discovered in the 1600s, the force required to stretch a coiled spring is proportional, but opposite in direction, to its displacement, that is, $F = -ks$. This is the mathematical form of Hooke's Law for springs. k is called the force constant of the spring. See also the section on mass on a spring in Chapter 11, 'Simple harmonic motion'.

EXAMPLE

The engine of a car supplies a constant force of 2 kN and moves a cargo 10 m. If the force is suddenly changed to 4 kN and the cargo moved an extra 30 m:

(a) sketch the F versus s graph for this event;

(b) calculate the total work done by the engine.

Answer

(a)

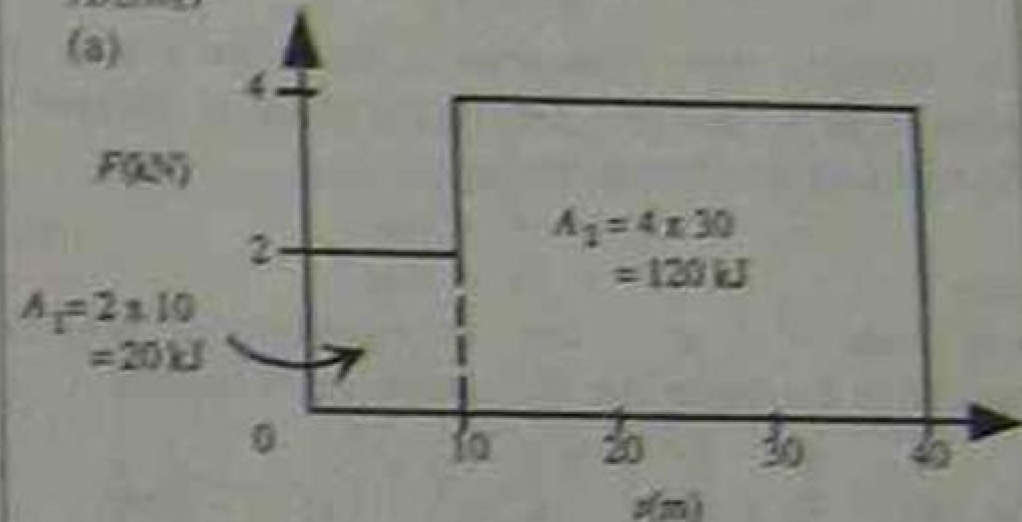


Fig. 4.5

(b) Work done

$$W = A_1 + A_2$$
$$= (20 + 120)\ \text{kJ}$$
$$= 140\ \text{kJ}$$

graph.

Kinetic energy and work

Kinetic energy (KE) is the energy an object has by virtue of its motion.

Assuming an object does not absorb energy, the amount of work a moving object can do is equal to the work done in giving it its velocity. The mechanical work done W by a uniform force F to move a body through a displacement s is calculated as we saw in the last section by

$$W = Fs$$

$$W = Fs$$

The constant force F required to accelerate a body of mass m as we saw in Chapter 2 is given by Newton's Second Law of Motion, namely:

$$F = ma$$

thus $$W = Fs = mas$$

and since $v^2 = u^2 + 2as$, an equation of uniform motion in a straight line at constant acceleration,

$$as = (v^2 - u^2)/2$$

i.e. $$W = Fs = m(v^2 - u^2)/2 = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

Work done equals the change in the $\frac{1}{2}$ mass $\times$ (velocity)^2 terms:

- $\frac{1}{2}mv^2$ is called the final KE of translation of the body;
- $\frac{1}{2}mu^2$ is the initial KE of the body.

That is, work done = change in KE

or $W = Fs =$ KE, where $u = 0$

The KE of a body of mass m moving with velocity v is given by

$$\mathrm{KE} = \frac{1}{2}mv^2 \qquad (1)$$

When the KE of a body is known, the velocity of the body can be calculated from

$$v = \sqrt{2\mathrm{KE}/m}$$

Note that the KE of a body of fixed mass is proportional to its velocity squared, and KE of a body moving at constant velocity is proportional to its mass.

EXAMPLE

What is the KE of a model car of mass 2 kg when its velocity in $m\,s^{-1}$ is:

(a) 0 (b) 1 (c) 2 (d) 3

(e) State the ratio of KE of the car when its velocity in $m\,s^{-1}$ is in turn 0, 1, 2, 3.

(f) Sketch the graphs of KE of the car against:

(i) its velocity v as v changes from $v = 0$ to $v = 3\ m\,s^{-1}$;

(ii) its $(\text{velocity})^2$, v^2, as v changes from $v = 0$ to $v = 3\ m\,s^{-1}$.

Answer

(a) $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times 0^2\ J = 0\ J$

(b) $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times 1^2 = 1\ J$

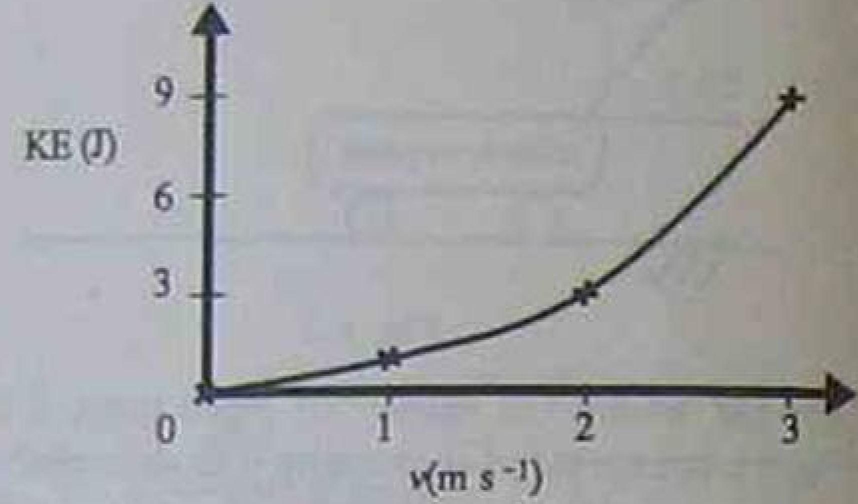
(c) $KE = \frac{1}{2} \times 2 \times 2^2 = 4\ J$

(d) $KE = \frac{1}{2} \times 2 \times 3^2 = 9\ J$

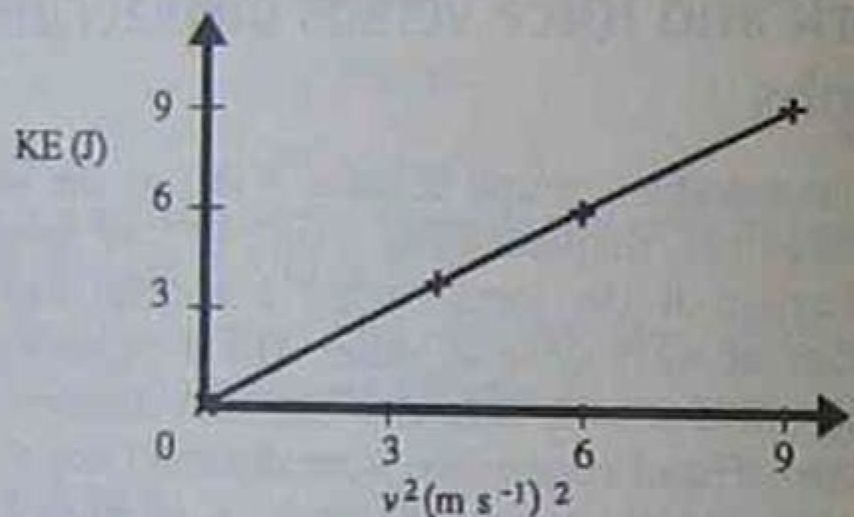
(e) Ratio of KE is 0:1:4:9

(f)

KE (J)	0	1	4	9
$v\ (ms^{-1})$	0	1	2	3
$v^2\ (ms^{-1})^2$	0	1	4	9



(*f*) (i)



(*f*) (ii)

Fig. 4.6

Potential energy

As well as KE, another form of mechanical energy is stored energy or potential energy, PE. PE of a body may be due to:

1. ITS SHAPE

As we saw in the section on work and force versus displacement graphs, a stretched coil spring has PE equal to the work done to stretch the spring.

Elastic PE in a coil spring $= \frac{1}{2}Fs^2$

and since $F = -ks$ (Hooke's Law),

where k is the force constant of the spring,

$$\text{PE} = \tfrac{1}{2}ks^2$$

$$PE = \frac{1}{2}ks^2$$

Similarly a compressed spring contains elastic PE which can do useful work as in a car's suspension. This stored energy can do work, e.g. drive a clock.

2. ITS POSITION IN A FIELD

For example, a raised object in a gravitational field has gravitational PE (GPE) of position equal to the work done on the object against the gravitational attraction of the Earth. The GPE of an object as it is raised at constant speed through vertical height h = force × displacement. The force F (up) equals the size of the weight W (down) = mg, and when the PE of the body at the lower position is arbitrarily set at zero then:

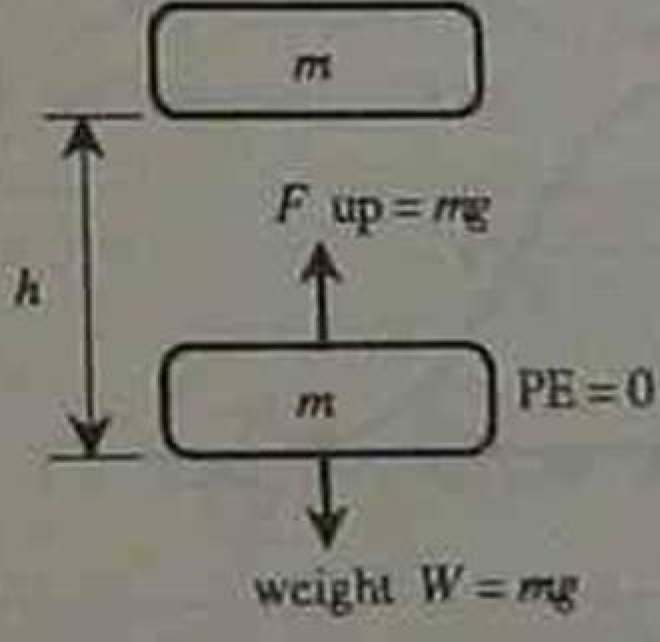


Fig. 4.7

$$\text{GPE} = m \times g \times h$$

where g = gravitational field strength taken as 9.8 N kg^{-1} or 10 N kg^{-1} near the Earth's surface.

Electric potential energy is measured in terms of the work needed to move charges in an electric field; see potential difference in Chapter 5, 'Electrostatics'.

EXAMPLE

An object of mass 30 kg is lifted through a vertical height of 10 m.

(a) Calculate how much work is done against the Earth's gravitational field.

(b) What is the gain in GPE of the object?

Take $g = 9.8\ \mathrm{m\,s^{-2}}$.

Answer

(a) $W = Fs = mgh = 30 \times 9.8 \times 10\ \mathrm{J}$

$= 2940\ \mathrm{J}$

(b) 2940 J since work done on the mass is stored as GPE of the mass.

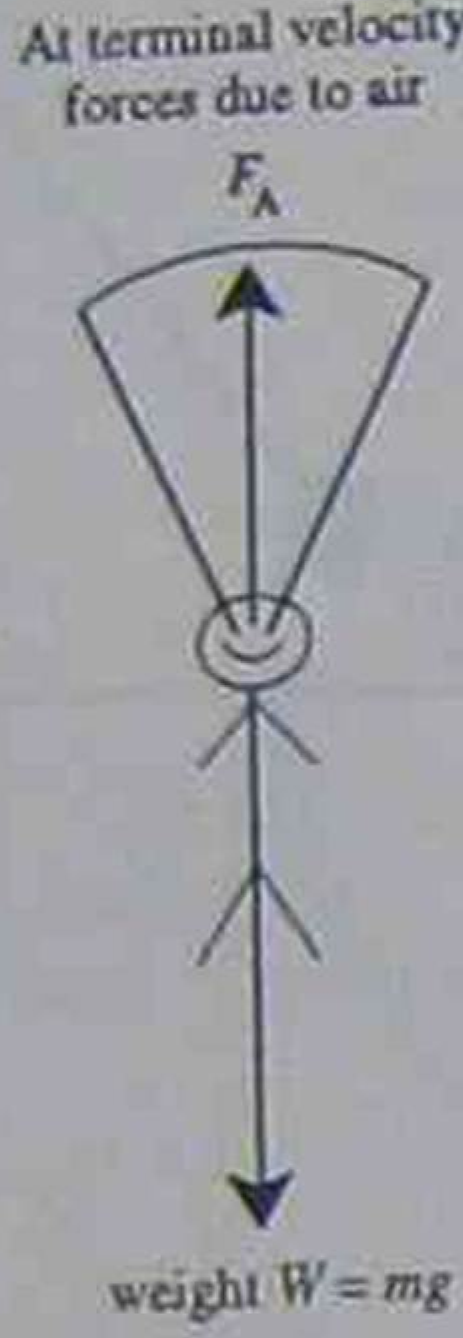
Energy transformations

There are many forms of energy: KE, PE, heat, light, sound, electrical, magnetic, chemical and nuclear. In nature and in man-made devices, energy is changed or transformed from one form to another. An electrical stove changes electrical energy into heat energy. In any transformation, the Law of Conservation of Energy applies; in other words energy is neither created nor destroyed, only changed from one to one (or more) other forms.

So the total energy put into a device equals the total energy that leaves the device. A machine is a device that converts energy into another form.

At time $t = 0$ a skydiver jumps from a plane, loses potential energy and gains kinetic energy and after time T_t reaches a constant velocity, called terminal velocity V_t. The vertical velocity of the diver changes with time as shown in Figure 4.10(b).

(a)



(b)

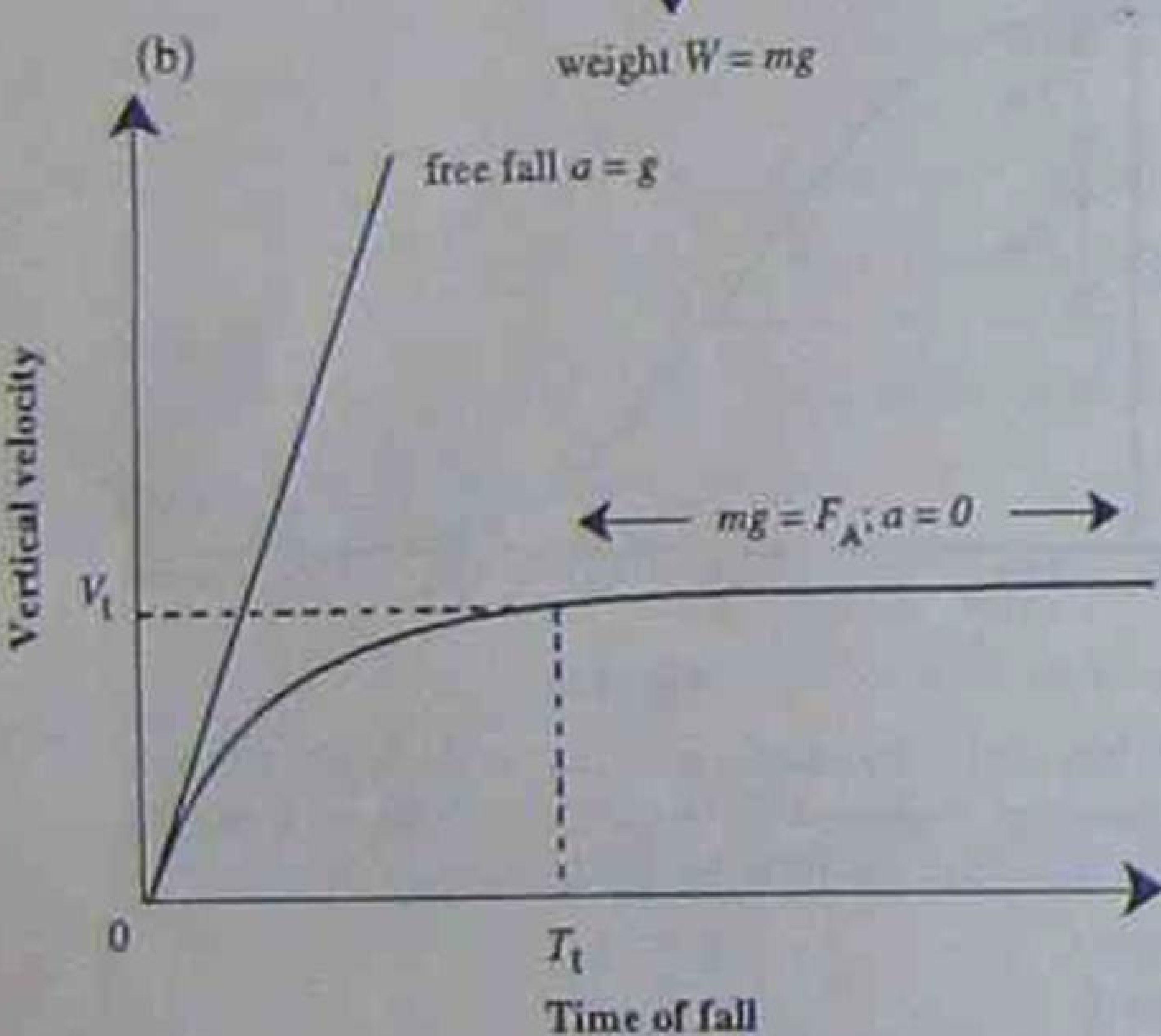

(b)
weight $W = mg$
free fall $a = g$
$mg = F_A; a = 0$
Vertical velocity
V_t
0
T_t
Time of fall

Fig. 4.10

While falling at a constant terminal velocity, the downward weight of the skydiver and equipment (mg) is balanced by upward force (F_A) due to air resistance. The KE ($1/2mv^2$) is constant and loss in GPE ($mgh = mgV_t t$) equals work done against air resistance, where h is the vertical distance fallen in time t.

Questions

11. From library and industry research determine what energy transformations occur in the following types of electric power stations:

(a) a coal fired station?
(b) a hydroelectric station?
(c) a nuclear station?

12. Name a device that converts:
  (a) light energy to electrical energy;
  (b) electrical energy to sound;
  (c) electrical energy to KE;
  (d) light energy to PE.

13. A 1000 kg jet of water travelling at 2.0 $m\,s^{-1}$ strikes the blade of a turbine on an electrical generator. Calculate:
  (a) the initial KE of the water;
  (b) the energy output of the generator if it is 100% efficient.

KE and PE transformations of falling objects

A freely falling object will lose a certain amount of GPE (mgh) which is transformed into KE so that at any part in its fall mechanical energy is conserved, so

$$KE + PE = \text{constant}$$

At A KE = 0 J GPE = mgh

At C KE = $\frac{1}{2}mv^2_C$ GPE = 0

At B, loss in PE = gain in KE,

therefore $mgh' = \frac{1}{2}mv^2_B$

At C $mgh'' = \frac{1}{2}mv^2_C$

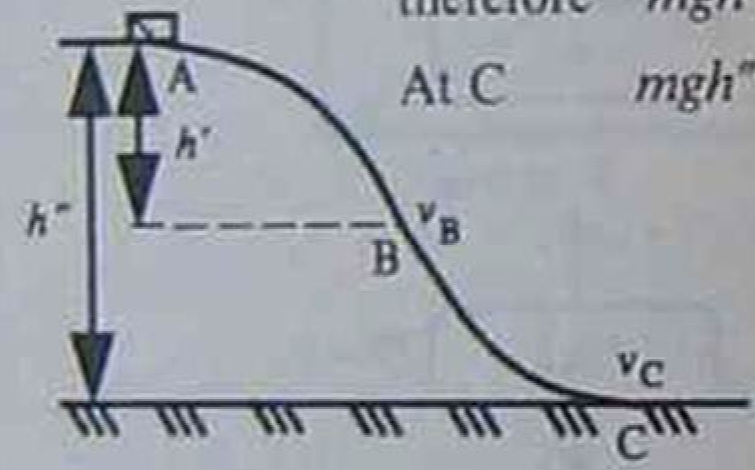


Fig. 4.11

EXAMPLE

A car of mass 1000 kg moving at $10\ \mathrm{m\,s^{-1}}$ along a level road comes to a dip in the road. The bottom of the dip is 10 m below the level road as shown in Figure 4.12.

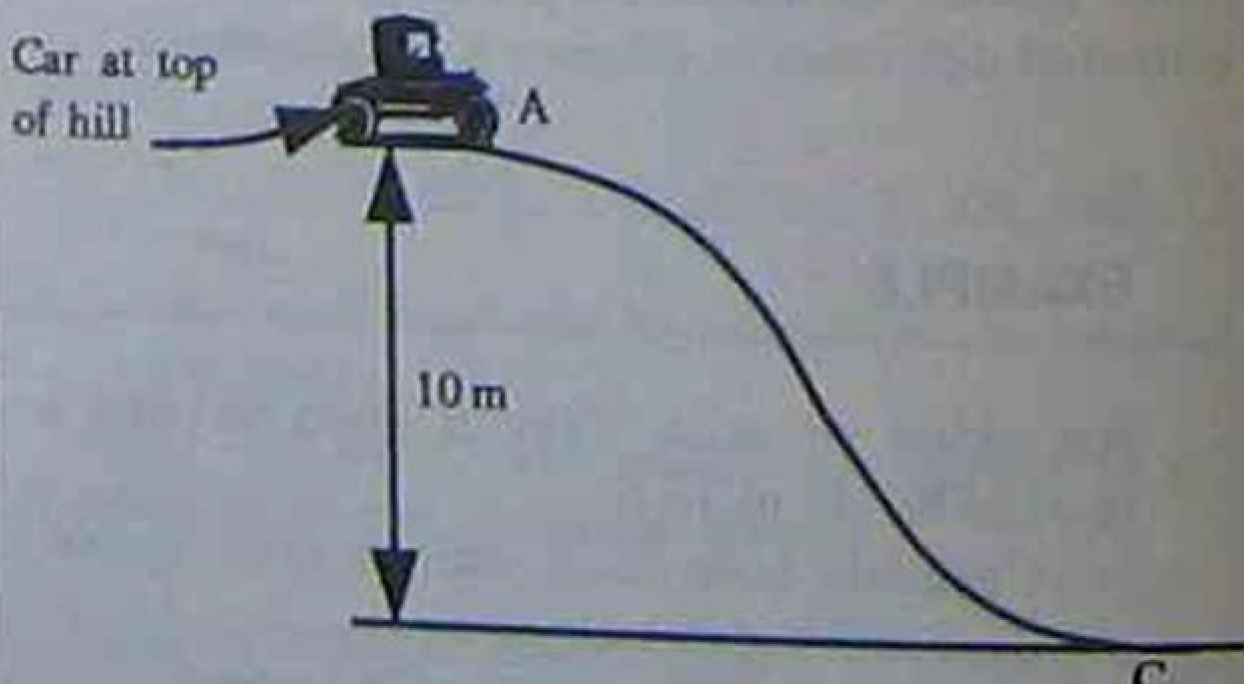


Fig. 4.12

Calculate:

(a) PE of car at A;

(b) KE of car at A;

(c) PE of car at C;

(d) total mechanical energy of A;

(e) KE of car at C;

(f) velocity of car at C;

(g) velocity at C if the car moves from rest at A.

Take $g = 9.8\ m\,s^{-2}$.

Answer

(a) PE at A $= mgh$

$$= 1000 \times 9.8 \times 10 \text{ J}$$

$$= 98 \text{ kJ}$$

(b) KE at A $= \frac{1}{2}mv_A^2$

$$= \frac{1}{2} \times 1000 \times (10)^2 \text{ J}$$

$$= 50 \text{ kJ}$$

(c) PE at C is given a value of zero.

(d) total mechanical energy at A

$$= \text{PE} + \text{KE}$$

$$= (98 + 50) \text{ kJ}$$

$$= 148 \text{ kJ}$$

(e) KE at C = KE at A + gain in KE from loss in PE as car falls from A to C

$$= 50 \text{ kJ} + 98 \text{ kJ}$$

$$= 148 \text{ kJ}$$

(f)

$$v = \sqrt{2 \times \text{KE}/m}$$

$$= \sqrt{2 \times 148 \times 1000/1000} \text{ m s}^{-1}$$

$$= 17.2 \text{ m s}^{-1}$$

(g)

$$v = \sqrt{2 \times 98 \times 1000/1000} \text{ m s}^{-1}$$

$$= 14 \text{ m s}^{-1}$$