

Describing motion II: Vectors

SYMBOL AND UNIT SUMMARY

Symbol	Quantity	Unit
$\underline{v}, \underline{v}_i, \underline{v}_f$	vector	
\underline{v}	velocity	ms^{-1}
\underline{v}_i	initial velocity	ms^{-1}
\underline{v}_f	final velocity	ms^{-1}
$\Delta \underline{v}$	change in velocity	ms^{-1}
\underline{p}	momentum	Ns
\underline{p}_i	initial momentum	Ns
\underline{p}_f	final momentum	Ns
$\Delta \underline{p}$	change in momentum	Ns
\underline{V}_{AB}	velocity of A relative to B	ms^{-1}

Vectors and scalars

A vector is any quantity which has a direction associated with it, whereas a scalar has no direction associated with it. Vector quantities include force, momentum, displacement, velocity, acceleration and impulse. Scalars include mass, time, energy, distance, work, power and speed.

A vector quantity has a magnitude stated in appropriate units and a direction. The direction may be stated in terms of north, south, east and west, or perhaps in terms of the vertical and horizontal, depending on the situation.

In print, symbols for vector quantities are identified in various ways such as heavy type, an arrow under or over the symbol (e.g. \vec{a}) or some other means (e.g. \underline{a}). These conventions are often not used where it is assumed that the quantity is known to be a vector.

A vector quantity can be conveniently represented in a drawing by means of a line with an arrow on it. The arrow gives the direction of the vector and the magnitude is shown by the length of the line according to some scale.

In one dimension the direction of a vector is indicated by the use of + and - signs.

Adding vectors

Vector quantities can be added by representing them by lines with arrows and placing them nose to tail. The lines represent the magnitude of each vector by their length according to some scale. The arrows give directions. The sum of two vectors v_1 and v_2 is shown in the diagram below.

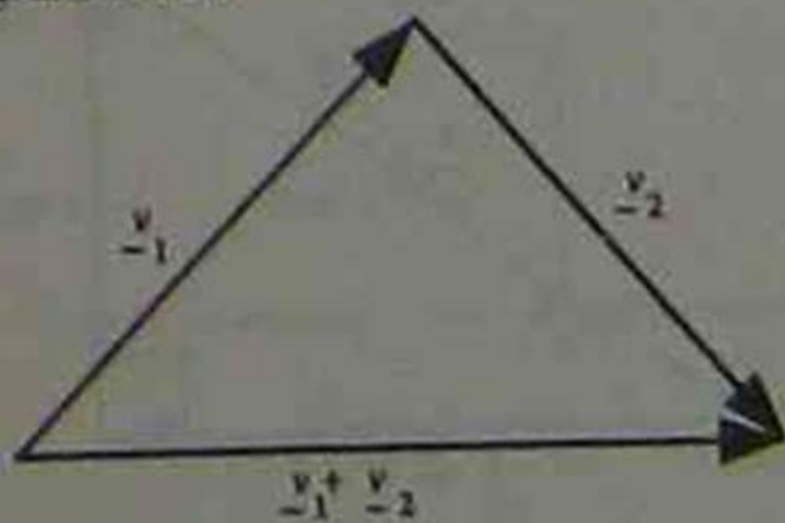


Fig. 8.1

The answer is obtained from an accurate scale drawing, or the use of the cosine rule, or Pythagoras' theorem.

Components of vectors

A vector can be thought of as being the sum of two other vectors at right angles to each other. These two vectors are called the components of the original vector.

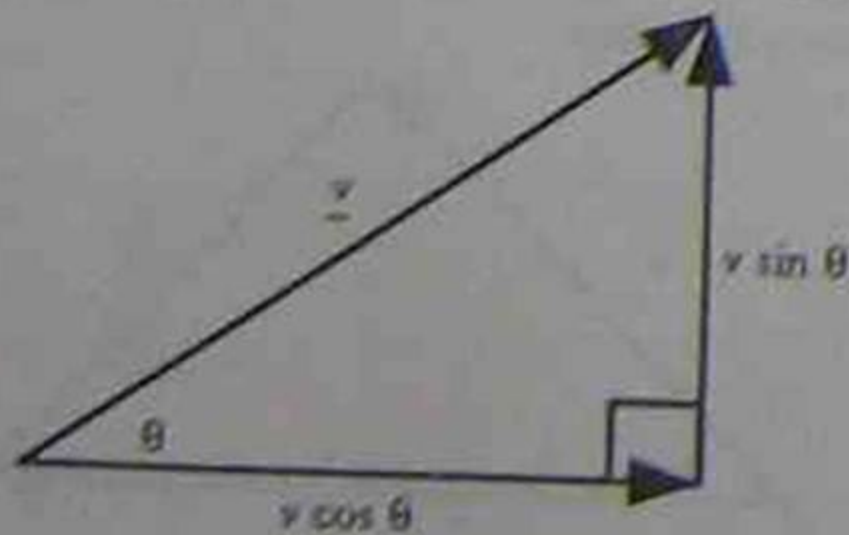


Fig. 8.2

The component of a vector \underline{v} at an angle θ to that vector has magnitude $v \cos \theta$.

When adding vectors we can simply add the components of the vectors as shown below.

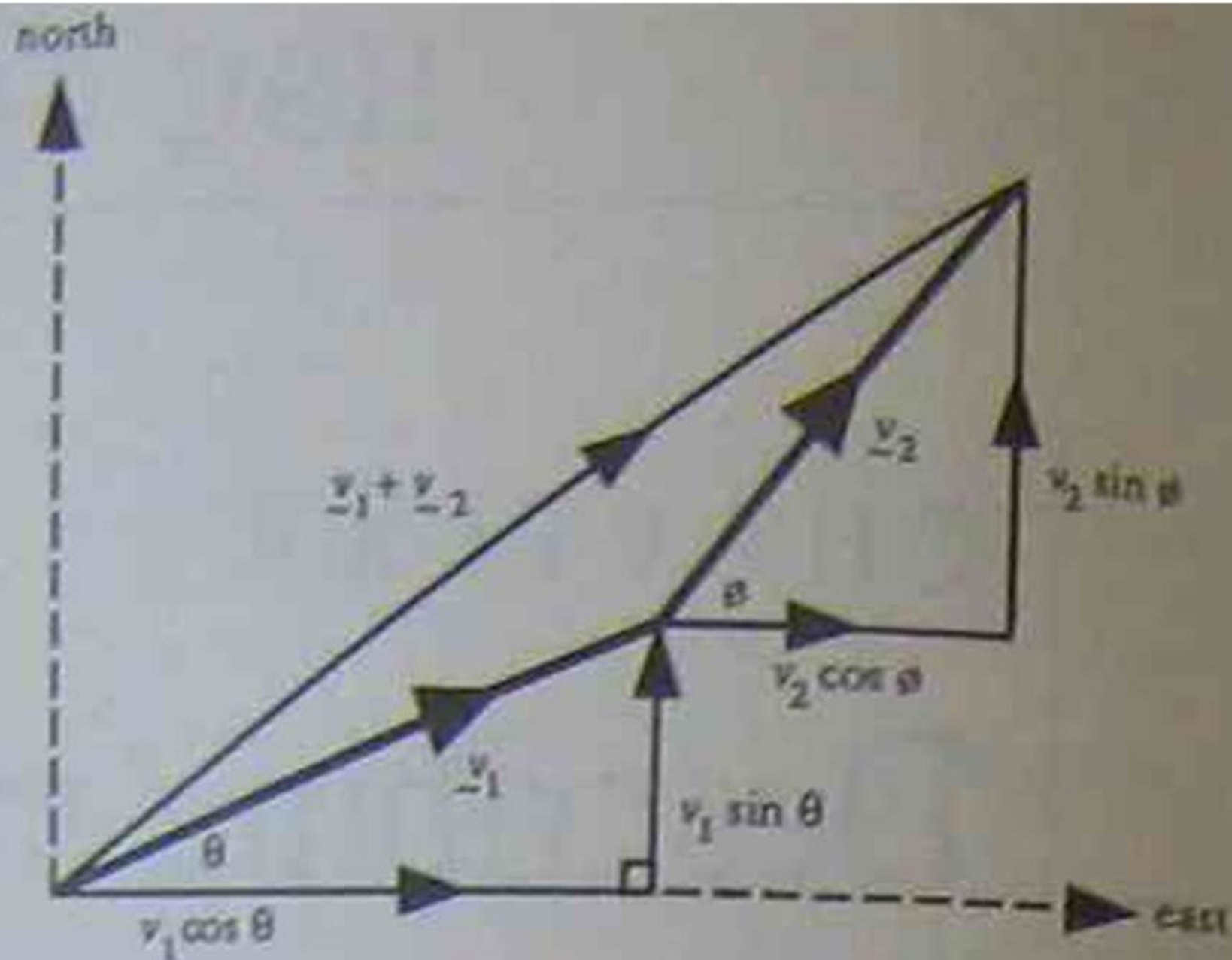


Fig. 8.3

Fig. 6.5

Northerly component of resultant

$$\begin{aligned} &= v_1 \cos (90^\circ - \theta) + v_2 \cos (90^\circ - \phi) \\ &= v_1 \sin \theta + v_2 \sin \phi \end{aligned}$$

Easterly component of resultant

$$= v_1 \cos \theta + v_2 \cos \phi$$

Northerly component \div easterly component gives the tan of the direction.

Subtracting vectors

The meaning of $-\underline{v}$ is a vector equal in magnitude to \underline{v} but having the opposite direction as shown in Figure 8.4(a).

When we subtract \underline{v}_2 from \underline{v}_1 , we ADD to \underline{v}_1 a vector equal to \underline{v}_2 in magnitude, with the opposite direction to \underline{v}_2 as shown in Figure 8.4(b) and (c).

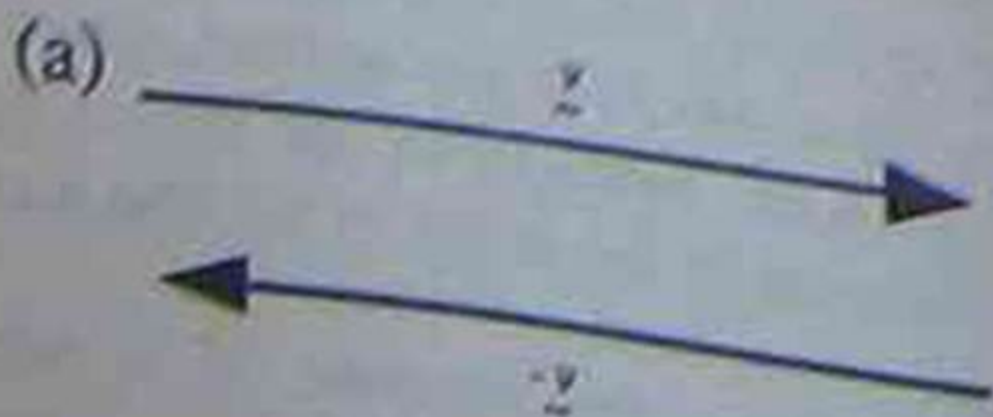


Fig. 8.4(a)

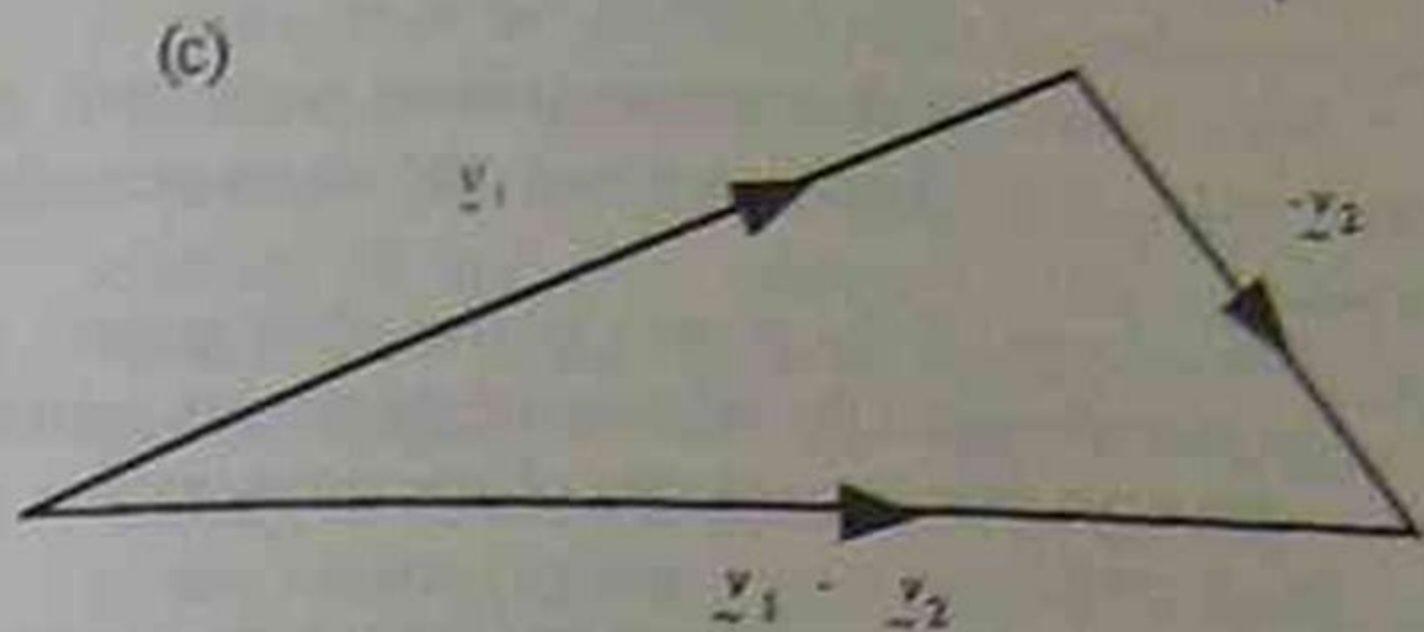
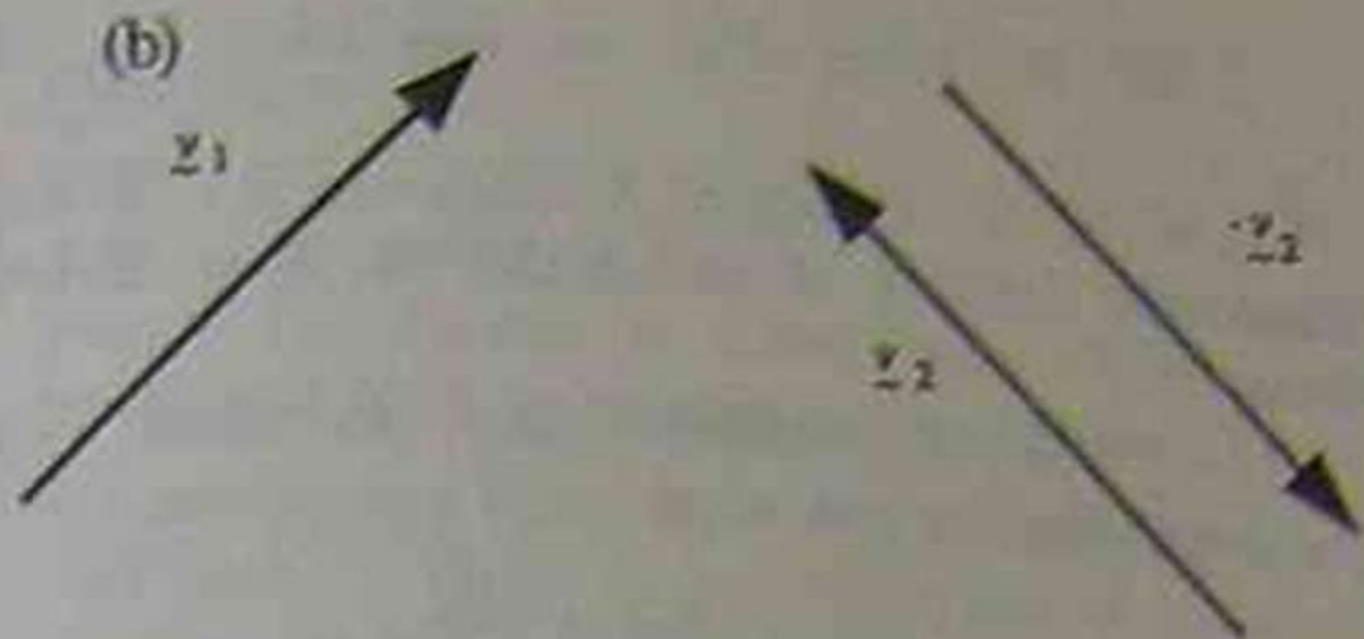


Fig. 8.4

The most common application of the subtraction of vectors is to determine change in velocity or change in momentum.

Change in velocity = final velocity - initial velocity.

$$\Delta \underline{v} = \underline{v}_f - \underline{v}_i$$

Change in momentum = final momentum
- initial momentum.

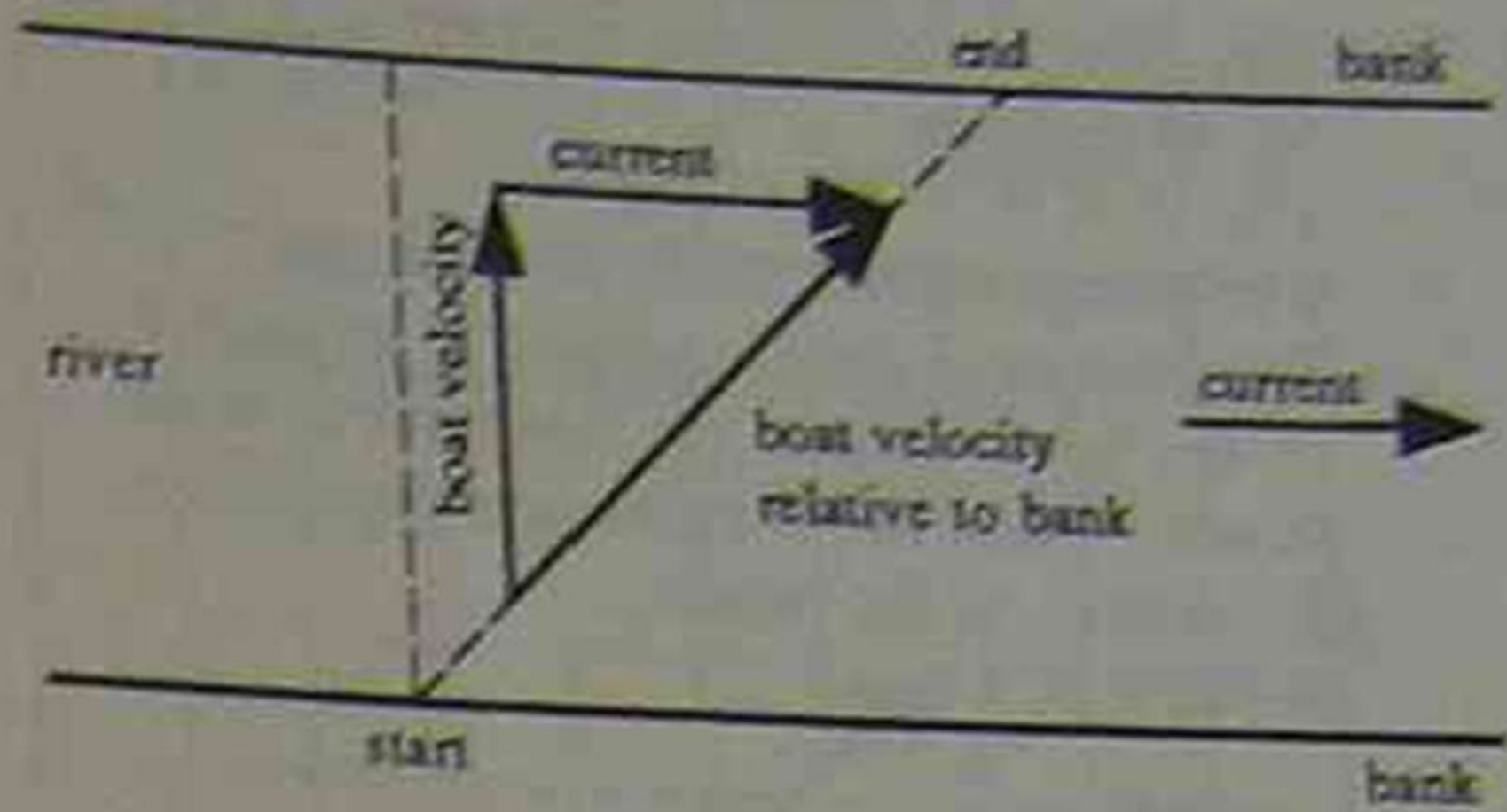
$$\Delta \underline{p} = \underline{p}_f - \underline{p}_i$$

Crossing rivers

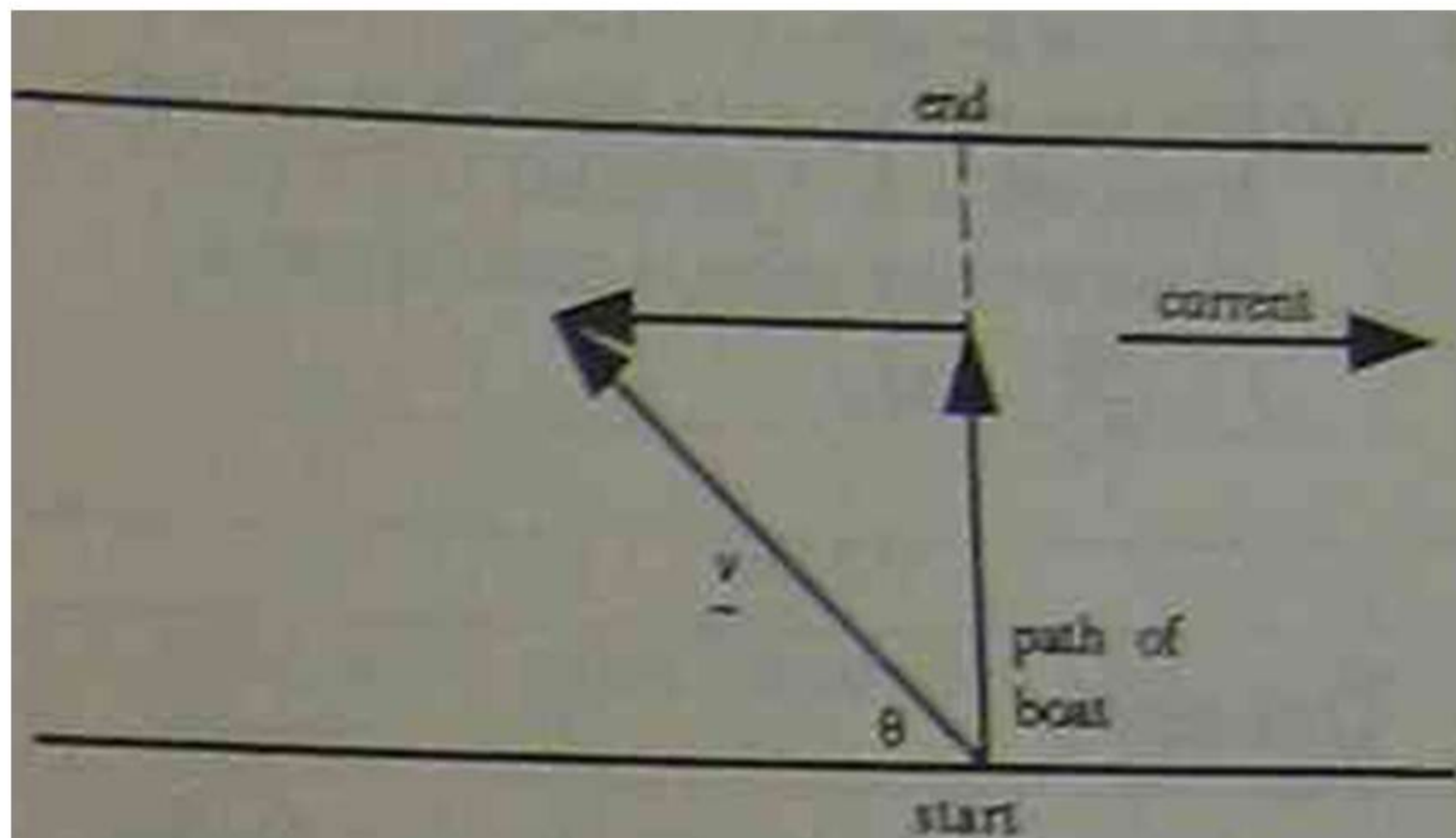
When a boat crosses a river there is usually a current tending to take the boat downstream.

The quickest crossing of the river occurs when the boat always has direction at right angles to the bank (and the current which is parallel to the bank). The time taken to cross the river is simply the width of the river divided by the velocity of the boat. The boat will be carried downstream a distance found by multiplying the velocity of the current by the time taken to cross the river. This is shown in Figure 8.5(a).

If the boat is to reach a point directly opposite the start point, the boat must point upstream at an angle that makes the component of the boat's velocity upstream equal and opposite to the velocity of the current downstream. This is shown in Figure 8.5(b). If the current is faster than the boat, the boat cannot cross directly to the point opposite—it cannot avoid being carried downstream.



(a)



$$v \cos \theta = \text{current velocity}$$

(b)

Fig. 8.5

EXAMPLE

A boat capable of 10 m s^{-1} crosses a river with a current downstream of 4 m s^{-1} . The river is 400 m wide. Calculate:

- (a) the shortest possible time for crossing the river;
- (b) how far downstream the boat is carried by the current if it crosses in the shortest period of time;
- (c) the direction the boat must point so that it crosses directly to the point opposite;
- (d) the time taken to cross the river if the boat crosses directly.

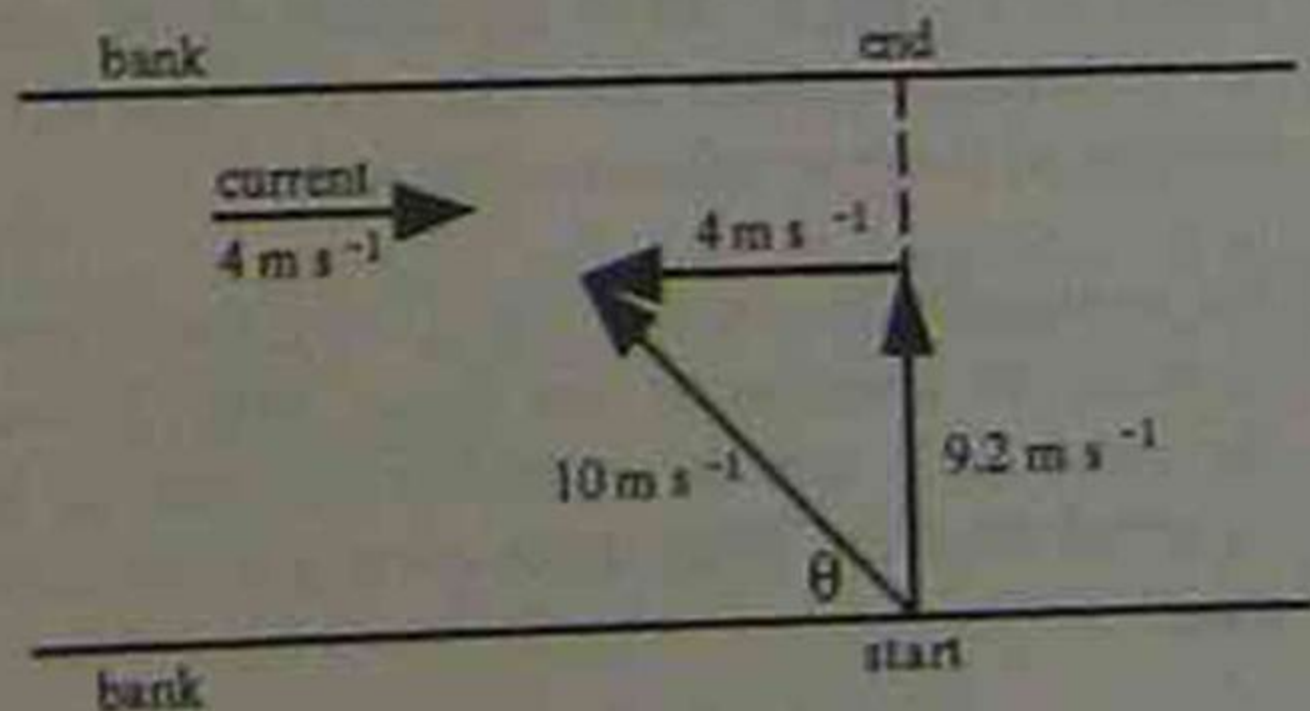


Fig. 8.6

Answer

- (a) The boat makes the quickest crossing if it heads directly across the current.

$$t = \frac{s}{v} = \frac{400}{10} = 40 \text{ s}$$

Key facts and equations

- A vector is a quantity that has magnitude and direction. A scalar has magnitude only.
- The following quantities are vectors: displacement, velocity, acceleration, momentum and force.
- The following quantities are scalars: mass, time, work, energy, power, distance and speed.
- A vector can be represented by a line. The length of the line represents the magnitude of the vector according to some scale, and the direction of the line shows the direction of the vector.

- Vectors can be added by drawing them to scale so that they are nose to tail.
- The component of a vector \underline{y} in a direction θ to that vector is $y \cos \theta$.
- The meaning of $-\underline{y}$ is a vector equal in magnitude to \underline{y} having the opposite direction to \underline{y} .
- In determining relative velocities we can use the equations:

$$\underline{V}_{AB} = -\underline{V}_{BA}$$

$$\underline{V}_{AC} = \underline{V}_{AB} + \underline{V}_{BC}$$