

EXAMPLE

31

- (a) Select one linear accelerator and explain how the particles are accelerated in the machine.
- (b) The accelerating voltage on a Cockcroft and Walton accelerator is 700 kV. The rest mass of an accelerated proton is 1.6724×10^{-27} kg and the charge on the proton is 1.6×10^{-19} C. Using non-relativistic physics, calculate, for the accelerated proton, its:
- KE in joules and electron-volts, given
 $1\text{ eV} = 1.6 \times 10^{-19}\text{ J}$;
 - velocity.
- (c) Accelerators are expensive to build and operate. List some useful information that can be gathered from experiments with accelerators.

accelerators.

Answer

(a) Select one from the text.

(b) (i) Gain in KE

= electrical work done on the proton

$$KE = qV$$

$$= 1.6 \times 10^{-19} \times 700 \times 10^3 \text{ J}$$

$$= 1.12 \times 10^{-13} \text{ J}$$

$$= \frac{1.12 \times 10^{-13}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 700 \text{ keV}$$

(ii) $\frac{1}{2}mv^2 = KE$

$$\therefore v = \sqrt{\frac{2 \times 1.12 \times 10^{-13}}{1.6724 \times 10^{-27}}} \text{ ms}^{-1}$$

$$= 1.16 \times 10^7 \text{ ms}^{-1}$$

(c) Information can be gathered on nuclear force, nuclear structure and fundamental particles.

Mass defect (deficit) and binding energy

The spectrographically determined mass of an atom is less than the sum of the masses of neutrons, protons and electrons that make up that atom. The missing mass is called the **mass defect**, Δm , of the atom. This Δm is present as energy which binds the atomic particles together in the atom. This energy, called **binding energy**, BE, is therefore the energy released when an atom is formed from its constituent parts. BE is also equal to the work that must be done, or the energy required, to separate the atom into its constituent parts.

BE is calculated from the Einstein relation:

$$BE = \Delta m c^2$$

$$BE = \Delta mc^2$$

where c is the velocity of light = $3 \times 10^8 \text{ m s}^{-1}$. With Δm in kilograms, BE will be in joules. When dealing with small quantities of mass and energy, it is often more convenient to use amu for mass, and MeV for energy. The following data may be required.

$$1 \text{ amu} = 932 \text{ MeV} = 1.66 \times 10^{-27} \text{ kg}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

Rest mass of a neutron, $m_n = 1.00866 \text{ amu}$

Rest mass of a proton, $m_p = 1.00727 \text{ amu}$

Rest mass of an electron, $m_e = 0.00055 \text{ amu}$

In nuclear reactions, mass-energy, nucleons, charge and momentum are conserved. We will now look at fission and fusion, which are two reactions in which the binding energy of the products is greater than that of the reactants, and so energy is released.

EXAMPLE

- (a) Calculate the mass defect, in amu, and its BE, in MeV, of Li_3^6 , whose spectrographic mass is 6.00513 amu.
- (b) Calculate the BE per nucleon for the Li_3^6 nucleus.

Answer

$$\text{(a) } \Delta m = \text{mass of nucleons} + \text{mass of electrons} - \text{mass of atom}$$

$$\begin{aligned}\text{Mass of nucleons} &= \text{mass of } 3\text{n} + \text{mass of } 3\text{p} \\ &= 3 \times 1.00866 + 3 \times 1.00727 \\ &= 3.02598 + 3.02181 \\ &= 6.04779 \text{ amu}\end{aligned}$$

$$\begin{aligned}\text{Mass of electrons} &= 3 \times 0.00055 \\ &= 0.00165 \text{ amu}\end{aligned}$$

$$\text{Mass of atom} = 6.00513 \text{ amu}$$

$$\therefore \text{Mass defect, } \Delta m = 6.04779 + 0.00165 - 6.00513$$

$$= 0.0443 \text{ amu}$$

$$\begin{aligned}\text{BE in MeV} &= 0.0443 \times 932 \text{ MeV} \\ &= 41.2876 \text{ MeV}\end{aligned}$$

(b) There are six nucleons in Li_3^6 , so:

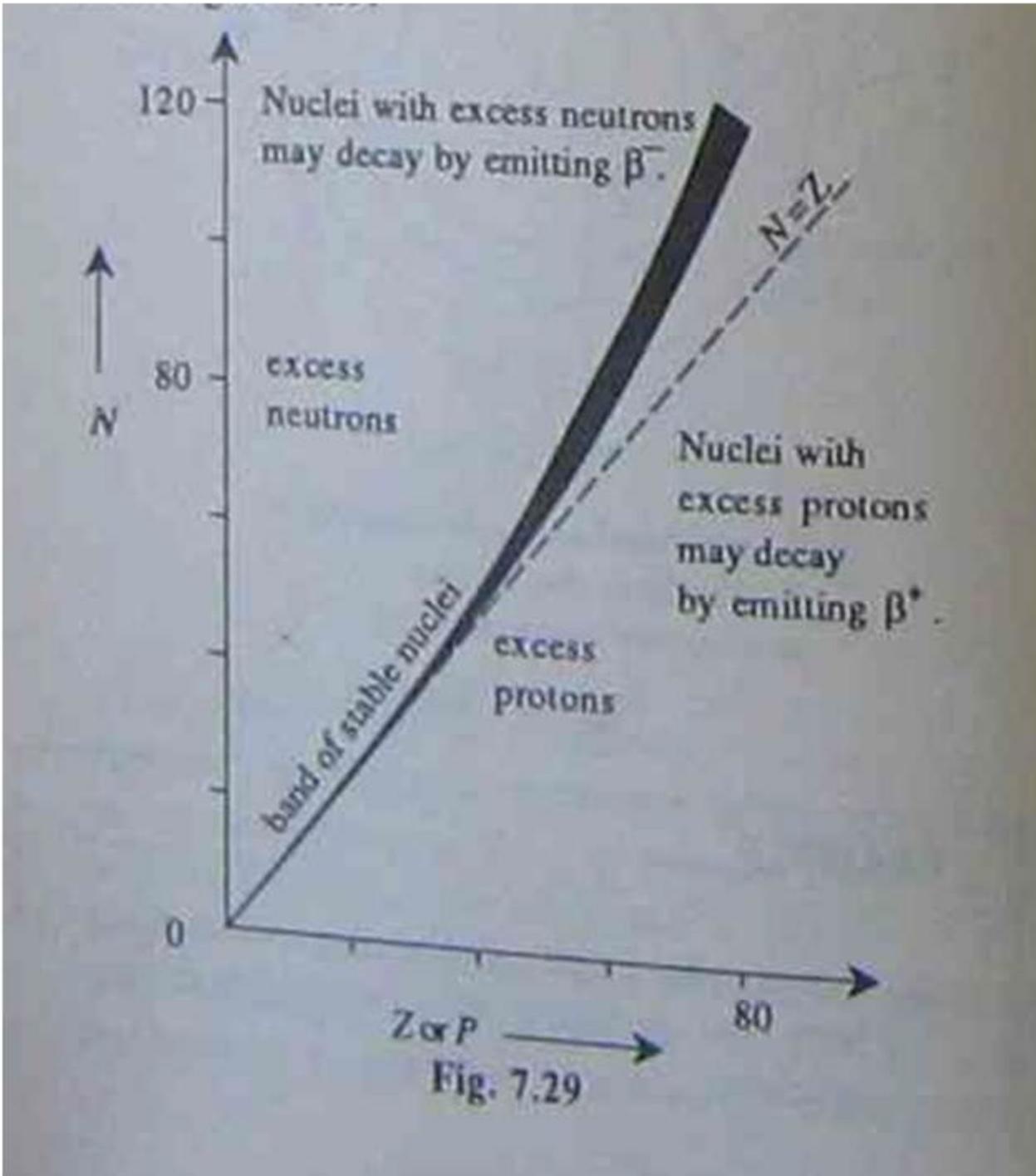
$$\text{BE per nucleon} = 41.2876/6$$

$$= 6.8813 \text{ MeV/nucleon}$$

Nuclear stability

Stable nuclei have large mass defects and large binding energies per nucleon; unstable nuclei are radioactive and may decay by emission of α , β and γ rays.

There are 92 naturally occurring elements, and more than 1000 isotopes of which about 280 are stable. A graph of number of neutrons, N , versus number of protons, Z or P , for stable nuclei, is shown as a narrow band in Figure 7.29.



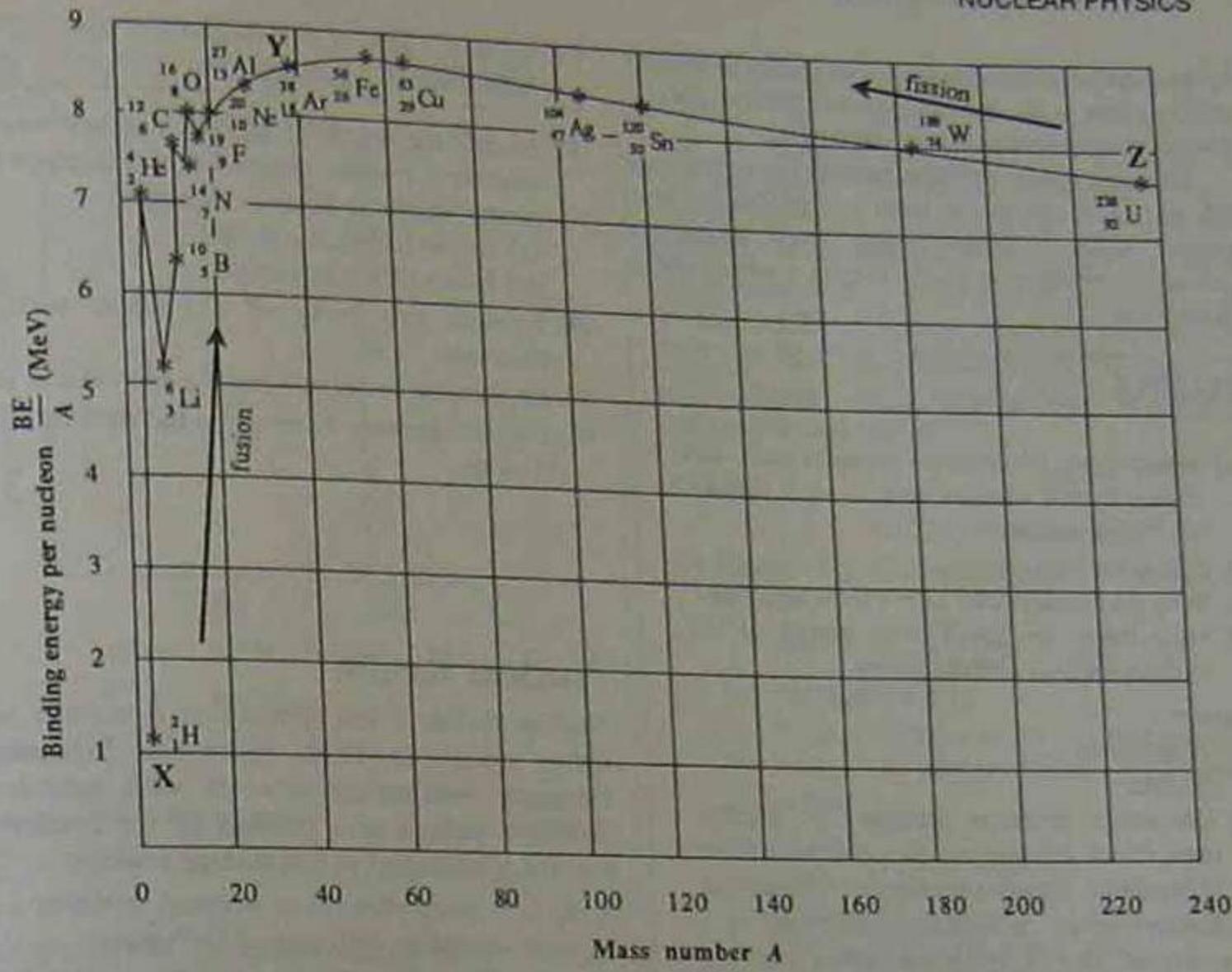


Fig. 7.28

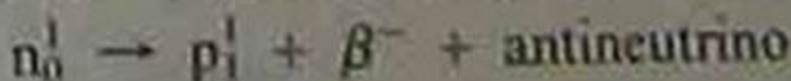
The following information may be deduced from the graph:

(a) For stable nuclei with:

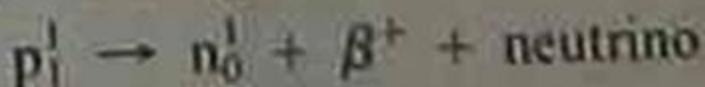
- (i) atomic number less than 20, then $N = P$,
- (ii) atomic number greater than 20, then $N > P$.

The excess neutrons increase the nuclear force and decrease the effect of the repulsive coulombic force between protons (see also 'Nuclear force' section of this chapter).

(b) Unstable nuclei with excess neutrons, i.e. a high ratio of N to P , may become stable by neutron decay and beta (negative electron) emission:



(c) Unstable nuclei with excess protons, i.e. a high ratio of P to N , may become stable by proton decay and positron (positive electron) emission:



Nuclear models

To explain the properties of the nucleus, many models have been proposed. Two important models are:

1. The **liquid drop model**, proposed by Neils Bohr in 1936. He suggested that the nucleus behaves just like a drop of liquid, with the protons and neutrons making up the particles of the liquid. Short-range forces hold the particles together, just like short-range forces hold water molecules together in a drop of water.

The liquid drop model explains:

- (a) 'radioactive decay' as evaporation of particles from the nucleus;
- (b) 'nuclear fission' as two drops momentarily combining to form a large unstable drop which then splits to form smaller drops (see Fig. 7.30).

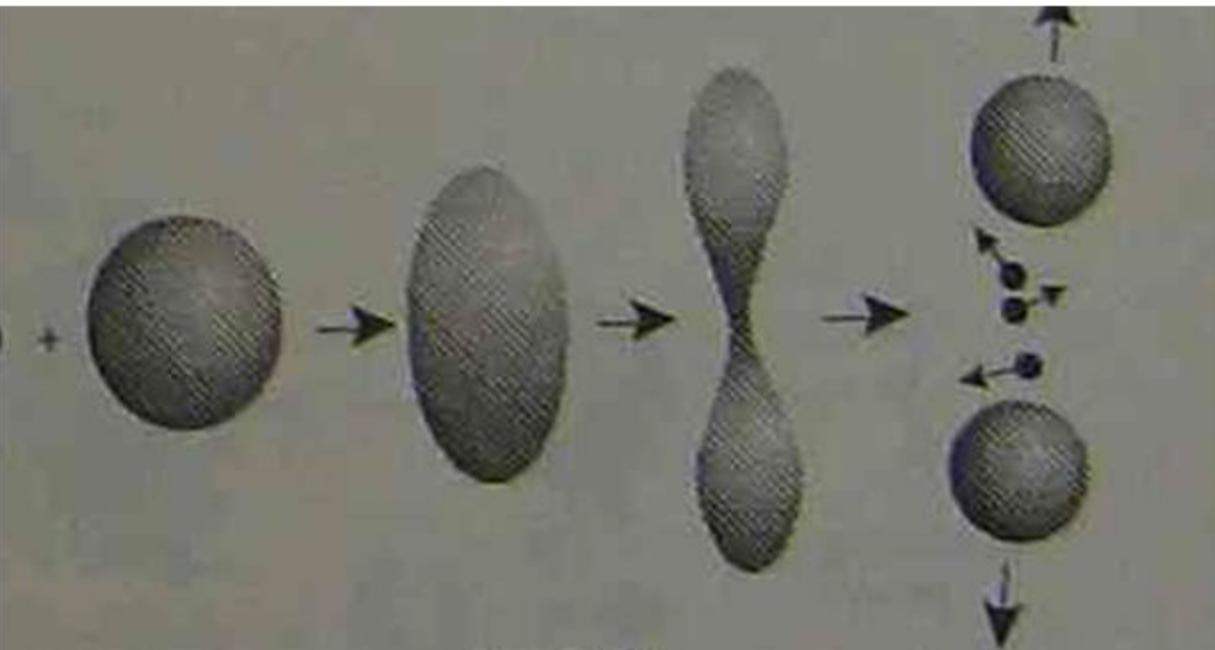


Fig. 7.30

2. The shell model. Figure 7.28 shows the BE per nucleon versus mass number. The peaks on the graph indicate that some nuclei are more stable than others. In 1949, Geoppert-Mayer in America and Jensen in Germany suggested that nuclei with a certain number of nucleons, called a 'magic number', have markedly greater stability. The numbers 2, 8, 20, 50 and 126 are 'magic numbers'.

In the shell model, the nucleons reside in distinct energy levels in the nucleus, just as electrons occupy distinct energy levels outside the nucleus.

The shell model explains gamma ray emission as follows: nucleons move from a high-energy level (excited state) to lower energy levels within the nucleus to produce a lower energy nucleus and a gamma ray.

EXAMPLE

- (a) What type of nuclear decay would you expect from a nucleus with:
 - (i) excess protons?
 - (ii) excess neutrons?
- (b) Why do stable nuclei have excess neutrons?
- (c) How does the liquid drop model of the nucleus explain nuclear fission?

Answer

- (a) (i) positron
(ii) beta
- (b) The extra neutrons increase the nuclear force which counteracts the coulombic force of repulsion between protons in the nucleus.
- (c) Nuclear fission is explained as drops of a neutron and a uranium atom coming together to form a short-lived drop which then splits to form fragments of fission.

Nuclear fission

Nuclear fission is the splitting of a nucleus to form lighter nuclei. In 1939, Hahn and Strassmann in Germany bombarded uranium with neutrons and identified barium as a product of the reaction. This reaction is explained as a two-stage process:

1. A U^{235} atom absorbs a neutron to form a short-lived, vibrating, high-energy U^{236} atom.
2. The U^{236} atom splits to form fragments A and B plus neutron(s) and energy. The process of fission is shown in Figure 7.31.

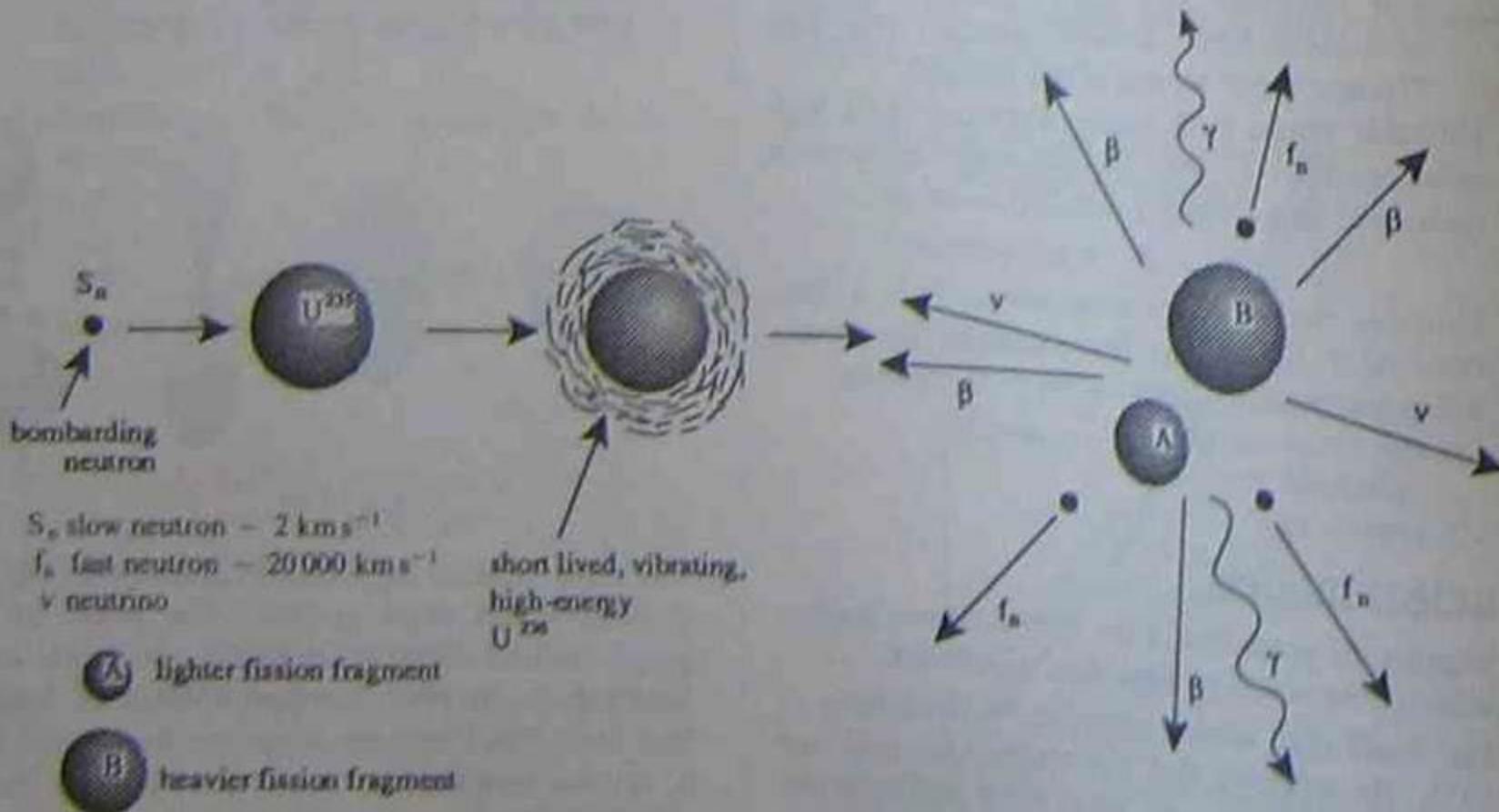


Fig. 7.31

The frequency of emission and the mass number of fragments A and B are displayed graphically in Figure 7.32.

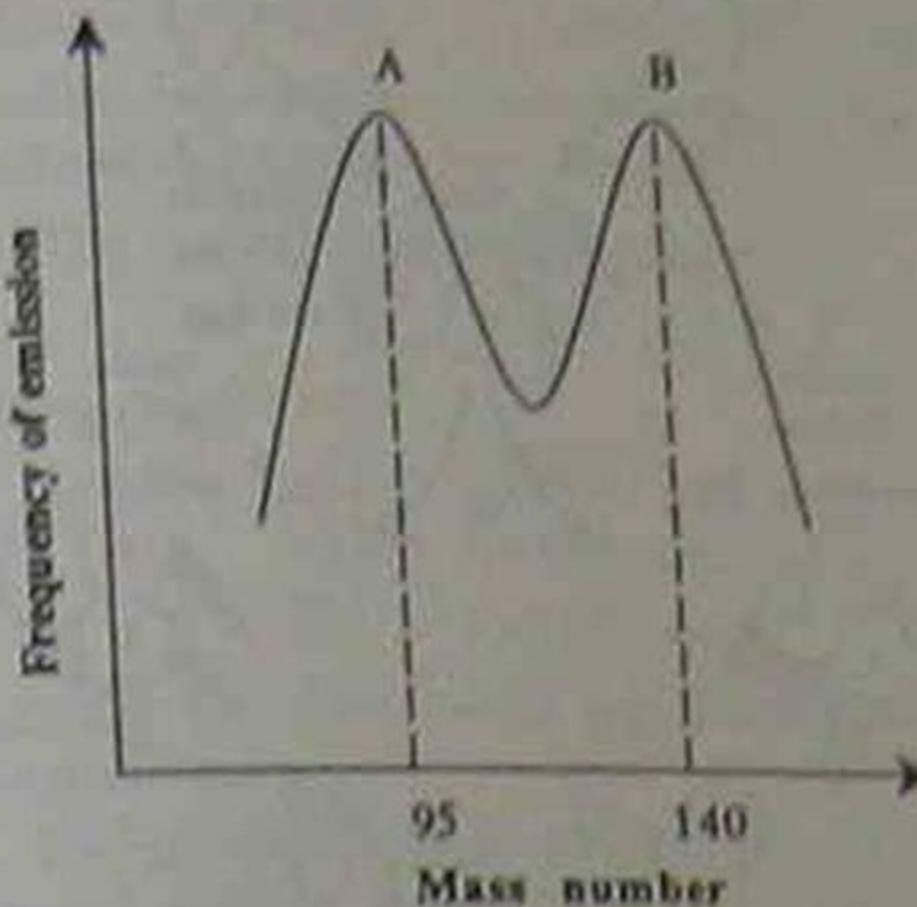
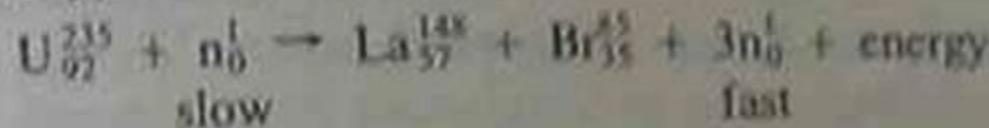


Fig. 7.32

A typical fission reaction is:



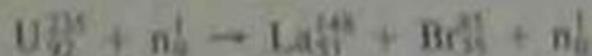
Some features of the fission reaction of U^{235} are as follows:

1. Slow (thermal) neutrons of speed = 2 km s^{-1} are easily captured by a U^{235} nucleus, and therefore are more effective than fast neutrons at causing a fission.
 2. Fast neutrons with velocity = $20\,000 \text{ km s}^{-1}$ and energy $\approx 2 \text{ MeV}$ are produced by the reaction.
 3. A neutron resulting from fission may produce a chain reaction (Fig. 7.33).
 4. The reaction can be controlled by absorbing the neutrons from fission in material such as boron or carbon.

5. The product of the reaction may be radioactive:
e.g. $\text{Ba}^{131} \rightarrow \text{La}^{133} + \beta^- + \text{antineutrino}$
6. Mass is converted into energy, E , of about 200 MeV per fission. This energy is calculated from the Einstein relation $E = \Delta m c^2$, where Δm is the mass defect for the reaction and c is the velocity of light.
7. Approximately 80% of the energy from fission is in the form of KE of fission fragments, approximately 10% is KE of neutrons of fission, and approximately 10% is radiation (beta ray, neutrino and gamma ray) energy.

EXAMPLE

- (a) Balance the following equation of a fission reaction:



- (b) For the reaction in (a):

- What is the mass defect of the reaction?
- What is the energy released, per fission, in MeV?

Data:

$$\text{Mass of U} = 235.044 \text{ amu}$$

$$\text{Mass of La} = 147.915 \text{ amu}$$

$$\text{Mass of Br} = 84.911 \text{ amu}$$

$$\text{Mass of } n = 1.009 \text{ amu}$$

$$1 \text{ amu} = 932 \text{ MeV}$$

- How is the energy evident in the products of the reaction?

Answer

- (a) There should be $3n$ plus energy on the right-hand side of the equation.

- (b) (i) Mass of reactants = mass of U
+ mass of n
= $(235.044 + 1.009) \text{ amu}$
= 236.053 amu

from fission in material such as uranium

2.36/03.2 9

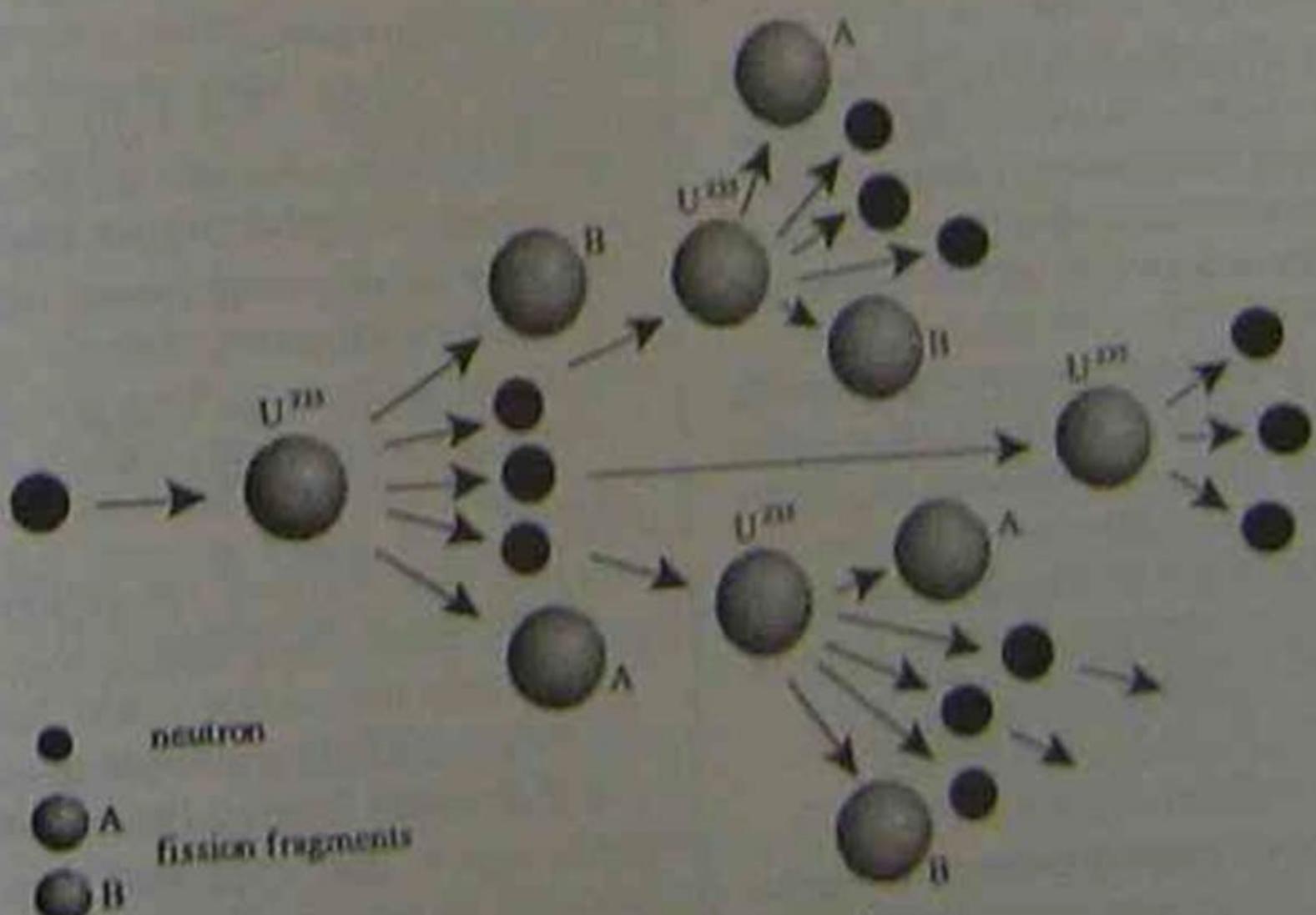


Fig. 7.33 A chain reaction

$$\begin{aligned}\text{Mass of products} &= \text{mass of La} \\&\quad + \text{mass of Br} \\&\quad + \text{mass of } 3\text{n} \\&= (147.915 + 84.911 \\&\quad + 3 \times 1.009) \text{ amu} \\&= 235.853 \text{ amu}\end{aligned}$$

$$\begin{aligned}\text{Mass defect} &= (236.053 - \\&\quad 235.853) \text{ amu} \\&= 0.200 \text{ amu}\end{aligned}$$

$$\begin{aligned}\text{(ii)} E &= 0.200 \times 932 \text{ MeV} \\&= 186.4 \text{ MeV}\end{aligned}$$

(iii) As KE of the particles of the reaction,
and as radiation energy.

Nuclear fusion

Nuclear fusion is a thermonuclear reaction which produces a heavier nucleus from two lighter nuclei. Energy is given out in the reaction. Energy produced by fusion is responsible for the major part of the Sun's energy, stellar energy and energy from the hydrogen bomb.

Heavy isotopes of hydrogen (**deuterium**, H_1^2 or D_1^2 , and **tritium**, H_1^3 or T_1^3 , which are used as fuels in fusion reactors), as well as He_2^3 , are reactants in fusion reactions. Because energy is given out in fusion, the mass defect of the products is greater than that of the reactants, i.e. the mass of reactants is greater than the mass of products. Two typical reactions are:

1. $H_1^2 + H_1^2 \rightarrow H_1^3 + H_1^1 + 4.03 \text{ MeV}$
2. $H_1^2 + H_1^3 \rightarrow He_2^3 + n_0^1 + 3.27 \text{ MeV}$

These reactions are shown diagrammatically in Figure 7.34.

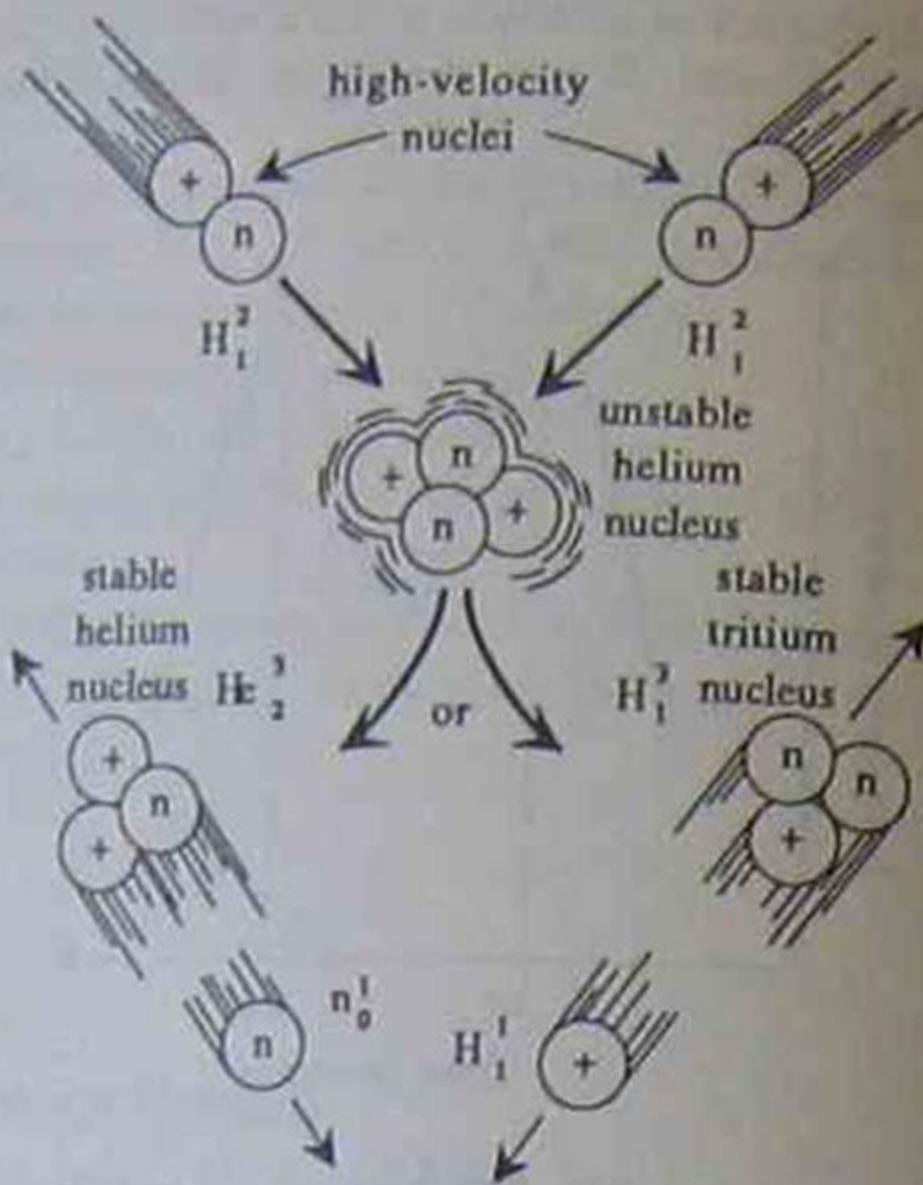


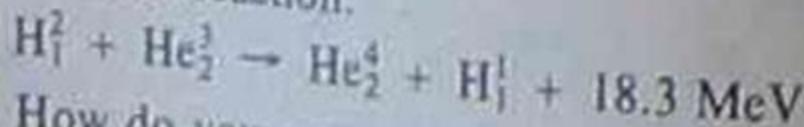
Fig. 7.34 Fusion of deuterium

Before a fusion reaction can occur between positive nuclei 1 and 2 with numbers of protons Z_1 and Z_2 respectively:

1. coulombic force of repulsion, $F \propto Z_1 \times Z_2$, has to be overcome. Nuclei approaching each other need to have high velocities and therefore high KE so that they are more likely to overcome these coulombic forces.
2. temperatures of $\sim 10^9^\circ\text{C}$ are required so that colliding nuclei have sufficient velocity to come close to each other for a long enough time so that nuclear attractive forces can take effect and a fusion reaction can take place.

EXAMPLE

In the fusion reaction:



- (a) How do you account for the 18.3 MeV of energy released during the reaction?
- (b) Why are high temperatures required for the fusion reaction to occur?
- (c) What is the mass change, Δm , in amu, given 1 amu = 932 MeV?

Answer

- (a) The mass defect of the products is greater than that of the reactants.
- (b) To help ensure that nuclei have sufficient velocity (KE) to overcome coulombic forces of repulsion between approaching nuclei.
- (c)
$$\begin{aligned}\Delta m &= 18.3 / 932 \text{ amu} \\ &= 0.196 \text{ amu}\end{aligned}$$

Nuclear energy production

Nuclear energy is produced in fission and fusion reactors. Fission reactors are the only practical reactors today and are used for power generation and research. As we saw in the section on nuclear fission reactions, the energy ΔE ($= \Delta mc^2$) per fission comes from the mass loss Δm during the fission of a U^{235} nucleus.

In thermal reactors, nuclear fission of the fuel atom U^{235} is brought about by 'slow' or thermal neutrons travelling at about 2 km s^{-1} . This fission produces fast neutrons travelling around $20\,000\text{ km s}^{-1}$. These neutrons are slowed down via collisions with atoms of a substance (e.g. graphite, heavy water) called a moderator. The slowed neutrons are more easily captured by the U^{235} nucleus than are fast neutrons,

moderator. The slowed neutrons are more easily captured by the U²³⁵ nucleus than are fast neutrons, which tend to pass straight through the U²³⁵ atom.

The first controlled self-sustaining chain reaction was set up in the USA by Fermi, Bohr, Teller and others in the December of 1942. The world's first commercial thermal reactor was opened in the UK in October 1956.

The smallest quantity of a fissionable material that will sustain a chain reaction in a reactor is called the **critical mass** (or critical size). The reactor operating with this critical mass of fuel is said to be in **critical state**.

Neutrons in a reactor may be lost due to one or more of the following processes.

1. capture by non-fissile material
2. capture by fissile material without fission
3. escape from the fuel

If more than one neutron per fission produces further fissions and energy, the reactor could generate large amounts of energy in a fraction of a second which could melt the fuel and set fire to the reactor. The reactor will shut down if more than one neutron per fission produces another fission.

The essential components of a thermal reactor are as follows:

1. Fuel. The fuel is produced from the uranium ore containing 99.3% U^{238} , 0.7% $^*\text{U}^{235}$, and 0.006% U^{234} . In this ore, there are insufficient $^*\text{U}^{235}$ atoms (which is the isotope fissionable with thermal neutrons) for efficient operation of a thermal reactor. The ore is chemically enriched to 1–3% in $^*\text{U}^{235}$ atoms as follows:

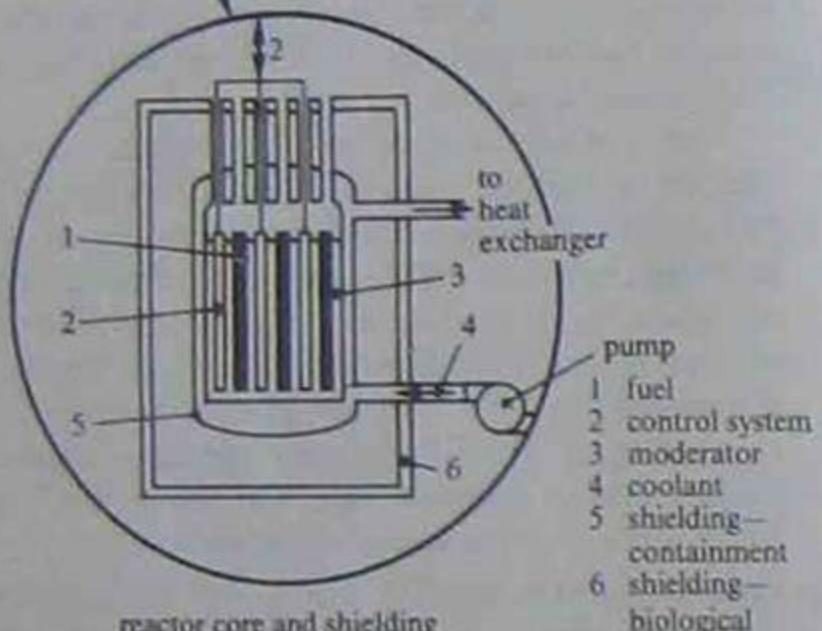
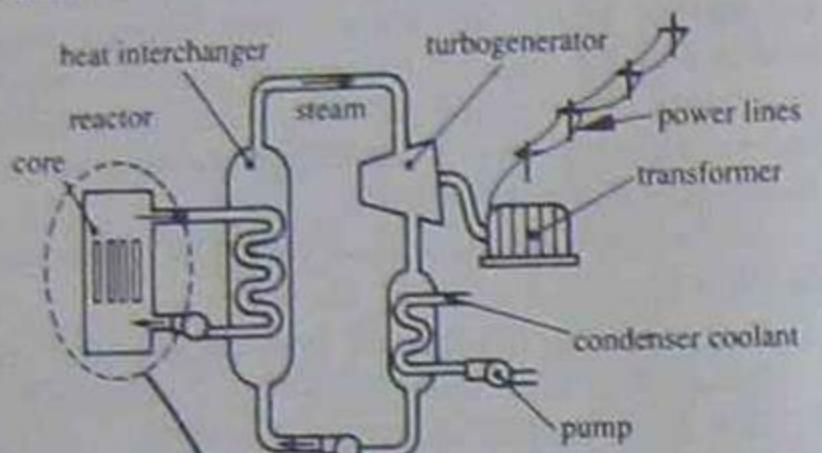
Uranium ore contains the black oxide U_3O_8 . The ore is purified and the oxide is concentrated and called yellowcake. Yellowcake is converted to an orange trioxide, UO_3 , which is reduced to the dioxide, UO_2 . This oxide is eventually converted to gaseous uranium hexafluorides. The hexafluorides are enriched in the U^{235}F_6 component by diffusion or centrifuging. The UF_6 is then made into uranium oxide pellets or uranium rods, and this is the fuel which is placed in the core of the reactor.

2. **Control system (rods).** The rate of nuclear fission in a thermal reactor is controlled by inserting or removing from the core of the reactor neutron-absorbing rods of boron, cadmium etc. Two arrays of rods are present in most reactors:
- (a) those for reactor control which are usually electrically inserted and removed from the reactor;
 - (b) those for reactor shutdown which are in a vertical position and are ready to fall freely under gravity.

3. **Moderators.** A moderator slows down fast neutrons to thermal speeds so that they are more effective in causing fission. Substances used as moderators are graphite, water, heavy water and beryllium.
4. **Coolant.** In the region of the core of the reactor, the coolant absorbs and transfers the KE of fission fragments from the core to where it can do useful work by converting water into steam. Useful coolants are water, sodium and helium.
5. **Shielding.** There are two categories:
 - (a) **Containment shielding.** To help ensure neutrons and nuclear radiation are contained in a stable environment, the core of the reactor is surrounded by concrete and steel to form a containment building.

(b) **Biological shielding.** To protect people from radiation, the containment building is encased in more concrete and high-carbon steel several metres thick.

A typical nuclear power plant is shown in Figure 7.35.



- 1 pump
- 2 fuel
- 3 control system
- 4 moderator
- 5 coolant
- 6 shielding—containment
- 7 shielding—biological